The output of an ADC can be transmitted over a baseband channel.

- The digital information must first be converted into a physical signal.
- The physical signal is called a **line code**.

Line coders use the terminology **mark** to mean binary one and **space** to mean binary zero.
3.5. Line Codes and Spectra

Binary Line Coding

DEFINITION: Binary 1’s and 0’s, such as in PCM signaling, may be represented in various serial-bit signaling formats called line codes.

There are two major categories: return-to-zero (RZ) and nonreturn-to-zero (NRZ).

With RZ coding, the waveform returns to a zero-volt level for a portion (usually on-half) of the bit interval.
3.5. Line Codes and Spectra

Unipolar signaling: “1” -- +A (high level)  
“0” -- 0 (zero level)  
Also called: on-off keying

polar signaling: “1” -- +A  
“0” -- -A

Bipolar signaling: “1” -- alternately positive or negative values  
“0” -- 0

Manchester signaling: “1” -- positive half-bit followed by a negative half-bit period pulse.  
“0” -- negative half-bit followed by a positive half-bit period pulse
3.5. Line Codes and Spectra

Desirable properties of a line code:

- **Self-synchronisation**: there is enough time information built in.
- **Low probability of error**: receivers can recover the binary data when the input data signals is corrupted.
- **A spectrum that is suitable for the channel**: the signal bandwidth needs to be sufficiently small compared to the channel bandwidth.
- **Transmission bandwidth**: this should be as small as possible.
- **Error detection capability**: this feature is easily implemented by the addition of channel encoders and decoders.
- **Transparency**: every possible sequence of data is faithfully and transparently received.
3.5. Line Codes and Spectra

Binary Line Coding

Digital Data $a_n$ → Line Coder → Physical Waveform

$s(t) = \sum_{n=-\infty}^{\infty} a_n f(t - nT_s)$

✧ The input to the line coder is a sequence of values, $a_n$, that is a function of a data bit or an ADC output bit.

✧ The output of the line coder is a waveform $s(t) = \sum_{n=-\infty}^{\infty} a_n f(t - nT_s)$

Where $f(t)$ is the symbol pulse shape and $T_s$ is the duration of one symbol. For binary signaling, $T_s = T_b$, where $T_b$ is the time that it takes to send 1 bit. For multilevel signaling, $T_s = lT_b$. The amplitude, “A” or “0” for NRZ, for example.
3.5. Line Codes and Spectra

Types of Line Codes

✧ Each line code is described by a symbol mapping function $a_n$, and a pulse shape $p(t)$ through

$$s(t) = \sum_{n=-\infty}^{\infty} a_n f(t - nT_s)$$

✧ Categories of line codes:

- Symbol mapping functions ($a_n$):
  - Unipolar
  - Polar
  - Bipolar

- Pulse shape ($p(t)$):
  - NRZ (Nonreturn-to-zero)
  - RZ (Return to Zero)
  - Manchester (split phase)
3.5. Line Codes and Spectra

Unipolar signaling: “1” -- +A (high level)  
“0” -- 0 (zero level)  
Also called: on-off keying

Polar signaling: “1” -- +A  
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Bipolar signaling: “1” -- alternately positive or negative values  
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3.5. Line Codes and Spectra

Power Spectra for Binary Line Code

✧ A digital signal is represented by:

\[ s(t) = \sum_{n=-\infty}^{\infty} a_n f(t - n T_s) \]

where \( a_n \) is the digital number (e.g. “0” or “1”), and \( f(t) \) is the symbol pulse shape (e.g. \( f(t) = \prod \left( \frac{t}{T_s} \right) \) For unipolar NRZ )

✧ PSD can be calculated using the autocorrelation function, and it depends on:

1.) the pulse shape
2.) statistical properties of data expressed by the autocorrelation function
3.5. Line Codes and Spectra

Power Spectra for Binary Line Code

The general expression for the **PSD of a digital signal** is:

\[
p_s(f) = \frac{|F(f)|^2}{T_s} \sum_{k=-\infty}^{\infty} R(k)e^{j2\pi kfT_s}
\]

Where \(F(f)\) is the Fourier transform of the pulse shape \(f(t)\) and \(R(k)\) is the autocorrelation of the data.

\[
R(k) = \sum_{i=1}^{I} (a_n a_{n+k})_i P_i
\]

Where \(a_n\) and \(a_{n+k}\) are the (voltage) levels of the data pulses at the \(n^{th}\) and \((n+k)^{th}\) symbol positions, respectively. \(P_i\) is the probability of having the \(i^{th}\) \(a_n a_{n+k}\) product.
3.5. Line Codes and Spectra

PSD for Unipolar NRZ signaling

Line Code of Unipolar NRZ

PSD of Unipolar NRZ

\[ P_{\text{unipolar NRZ}}(f) = \frac{A^2 T_b}{4} \left( \frac{\sin \pi f T_b}{\pi f T_b} \right)^2 \left[ 1 + \frac{1}{T_b} \delta(f) \right] \]

\[ A = \sqrt{2} \quad 1/T_b = R \]
3.5. Line Codes and Spectra

PSD for Polar NRZ signaling

Line Code of Polar NRZ

PSD of Polar NRZ

\[ P_{\text{PolarNRZ}}(f) = A^2 T_b \left( \frac{\sin \pi f T_b}{\pi f T_b} \right)^2 \]
3.5. Line Codes and Spectra

Differential Coding

Potential issues is unipolar NRZ, polar NRZ, and Manchester NRZ:

✧ The waveform is often *unintentionally inverted* (happens in a twisted-pair transmission line channel by switching the two leads)

✧ **Results:** 1 - > 0, 0 - > 1

✧ This is not a issue for bipolar signal
3.5. Line Codes and Spectra

Differential Coding

The technology called differential coding can be used to ameliorate this problem.

\[ e_n = d_n \oplus e_{n-1} \]
\[ \tilde{d}_n = \tilde{e}_n \oplus \tilde{e}_{n-1} \]

Where \( \oplus \) is a modulo 2 adder or an exclusive-OR gate (XOR) operation.
3.5. Line Codes and Spectra

**Differential Coding**

**Encoding**

\[ e_n = d_n \oplus e_{n-1} \]

- **Input sequence** \( d_n \) = 1 1 0 1 0 0 0 1
- **Encoded sequence** \( e_n \) = 1 0 1 1 0 0 0 1
- **Reference digit**

**Decoding (with correct channel polarity)**

\[ \tilde{d}_n = \tilde{e}_n \oplus \tilde{e}_{n-1} \]

- **Receiver sequence** = 1 0 1 1 0 0 0 1
  - (Correct polarity)
- **Decoded sequence** = 1 1 0 1 0 0 1

**Decoding (with inverted channel polarity)**

- **Received sequence** = 0 1 0 0 1 1 1 1 0
  - (Inverted polarity)
- **Decoded sequence** = 1 1 0 1 0 0 1
3.5. Line Codes and Spectra

Regenerative Repeaters

When a signal (waveform) is transmitted over a hardwire channel, it is filtered, attenuated, and corrupted by noise. Consequently, for long distance, the data cannot be recovered at the receiving end unless **repeaters** are utilized.

✧ For analog signal (such as PAM), only linear amplifiers with appropriate filter could be used. Only increase the **amplitude**, the in-band **distortion** (such as random noise) could accumulate.

✧ For digital signal (such as PCM), nonlinear processing can be used to regenerate a “noise-free” digital signal. This type of nonlinear processing is called a **regenerative repeater**.
3.5. Line Codes and Spectra

**Regenerative Repeaters**

- Increases the amplitude
- Produces a sample value
- Minimize the effect of channel noise & ISI
- Generates a clocking signal
- Produces a high level o/p if sample value $> V_T$
3.5. Line Codes and Spectra

Regenerative Repeaters

(a) Transmitted signal. (b) Received distorted signal (without noise). (c) Received distorted signal (with noise). (d) Regenerated signal (delayed).
In long-distance digital communication systems, many repeaters may be used in cascade, and the distance between the repeaters is governed by the path loss of the transmission medium and the amount of noise that is added.

A repeater is required when the SNR at a point along the channel becomes lower than the value that is needed to maintain the overall probability-of-bit-error specification.

For $m$ repeaters in cascade, the overall probability of bit errors $P_{me}$ is

$$P_{me} = mP_e$$

where $P_e$ is the probability of bit error for a single repeater.
3.5. Line Codes and Spectra

**Bit Synchronization**

✧ Synchronization signals are clock-type signals that are necessary within a receiver (or repeater) for detection (or regeneration) of the data from the corrupted input signal.

✧ These clock signals have precise frequency and phase relationship with respect to the received input signal, and they are delayed compared to the clock signals at the transmitter.

✧ At least three types of synchronization signals are needed:
  - *bit sync*
  - *frame sync*
  - *carrier sync*
3.5. Line Codes and Spectra

Bit Synchronization

Why sync signal is necessary??

clock-type signal
3.5. Line Codes and Spectra

Square-law bit synchronizer for NRZ signal
3.5. Line Codes and Spectra

Power Spectra for Multilevel Polar NRZ signals

Why do we need multilevel signal?

\[ w(t) = \sum_{k=1}^{N} w_k \varphi_k(t) \quad 0 < t < T_0 \]

\( w_k \) is the digital number (e.g. “0” and “1”), \( T_0 \) is the time period to send out a message

For **binary signal**: \( N \) (dimension number) = \( n \) (bit number)

For **multilevel signal**: \( N \) (dimension number) = \( n/l \) (bit number)
3.5. Line Codes and Spectra

Power Spectra for Multilevel Polar NRZ signals

Binary to multilevel conversion is used to reduce the bandwidth required by the binary signaling.

✧ Multiple bits (l number of bits) are converted into words having SYMBOL durations $T_s = lT_b$ where the Symbol Rate or the BAUD rate $D = 1/T_s = 1/lT_b$

✧ The symbols are converted to a L level ($L = 2^l$) multilevel signal using an l-bit DAC.

✧ Note that now the Baud rate is reduced ($D=R/l$), thus the bandwidth is also reduced.
3.5. Line Codes and Spectra

Power Spectra for Multilevel Polar NRZ signals

Bandwidth (minimum) of the waveform representing the digital signal:

\[ B = \frac{N}{2T_0} = \frac{1}{2} D \] Hz

For binary signal: \[ B = \frac{N}{2T_0} = \frac{1}{2} D \]

For multilevel signal: \[ B = \frac{N}{2T_0L} = \frac{1}{2L} D \]
3.5. Line Codes and Spectra

Power Spectra for Multilevel Polar NRZ signals

(c) $L = 8 = 2^3$ Level Polar NRZ Waveform Out
3.5. Line Codes and Spectra

Spectral Efficiency

**DEFINITION:** The spectral efficiency of a digital signal is given by the number of bits per second of data that can be supported by each hertz of bandwidth.

\[
\eta = \frac{R}{B} \text{ (Bit/s)}
\]

\[
\eta_{\text{max}} = \frac{C}{B} = \log_2 \left( 1 + \frac{S}{N} \right)
\]

<table>
<thead>
<tr>
<th>Code Type</th>
<th>First Null Bandwidth (Hz)</th>
<th>Spectral Efficiency ( \eta = \frac{R}{B} \text{ [(bits/s)/Hz]} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unipolar NRZ</td>
<td>( R )</td>
<td>1</td>
</tr>
<tr>
<td>Polar NRZ</td>
<td>( R )</td>
<td>1</td>
</tr>
<tr>
<td>Unipolar RZ</td>
<td>( 2R )</td>
<td>1</td>
</tr>
<tr>
<td>Bipolar RZ</td>
<td>( R )</td>
<td>1</td>
</tr>
<tr>
<td>Manchester NRZ</td>
<td>( 2R )</td>
<td>1</td>
</tr>
<tr>
<td>Multilevel polar NRZ</td>
<td>( R/\ell )</td>
<td>( \ell )</td>
</tr>
</tbody>
</table>