3.4. Digital Signaling

Two questions we need to answer through this section:

- ✧ How to mathematically represent the waveform for a digital signal
- ✧ How to estimate the bandwidth of the waveform

Mathematical representation of the waveform: voltage (current) waveform for digital signal:

$$w(t) = \sum_{k=1}^{N} w_k \varphi_k(t) \qquad 0 < t < T_0$$

Where w_k represents the digital data, and $\varphi_k(t)$, $k = 1, 2, \ldots N$, are N orthogonal functions that give the waveform its waveshape. N is the number of dimensions required to describe the waveform.

3.4. Digital Signaling

DEFINIATION: The baud (symbol rate) is:

$$D = N/T_0 \text{ symbols/s}$$

Where N is the number of dimensions used in T_0 seconds

DEFINIATION: The bit rate is:

$$R = n/T_0 \text{ bits/s}$$

Where n is the number of data bits sent in T_0 seconds

For the case when the w_k's have binary values (e.g., "0" or "1"), $n = N$, and $w(t)$ is said to be a **binary signal**.

When the w_k's are assigned more then two possible values, $w(t)$ is said to be a **multilevel signal**.

3.4. Digital Signaling

How can a receiver be build to detect the data?

Input signal at the receiver:

$$w(t) = \sum_{k=1}^{N} w_k \varphi_k(t)$$

$\varphi_k(t)$ is the known orthogonal function that was used to generate the waveform

output signal at the receiver:

$$w_k = \frac{1}{K_k} \int_0^{T_0} w(t)\varphi_k^*(t)\,dt, \qquad k = 1,2,\ldots,N$$

3.4. Digital Signaling

Vector Representation

The orthogonal function space corresponds to the orthogonal vector space

Orthogonal function space $$w(t) = \sum_{k=1}^{N} w_k \varphi_k(t) \qquad 0 < t < T_0$$

Orthogonal vector space $$\mathbf{w} = \sum_{j=1}^{N} w_j \varphi_j$$

w is an N-dimensional vector in Euclidean vector space, and $\{\varphi_j\}$ is an orthogonal set of N-directional vecotr

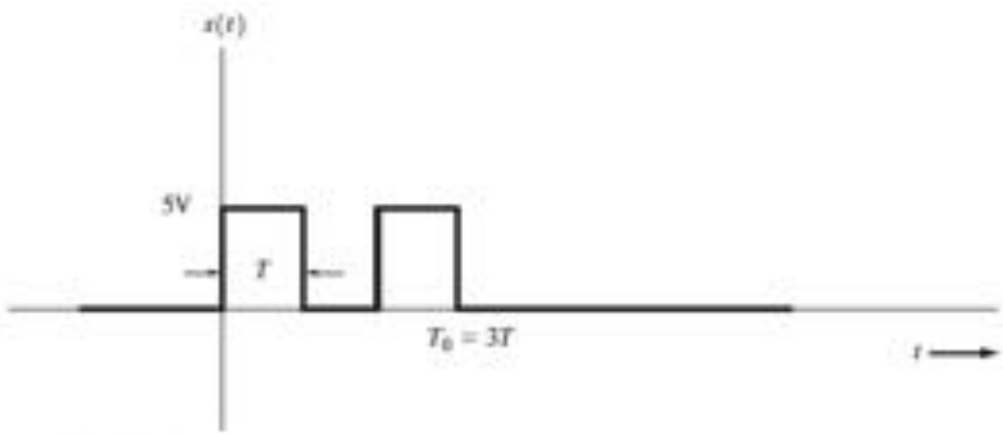
$$w = (w_1, w_2, \ldots., w_N)$$

3.4. Digital Signaling

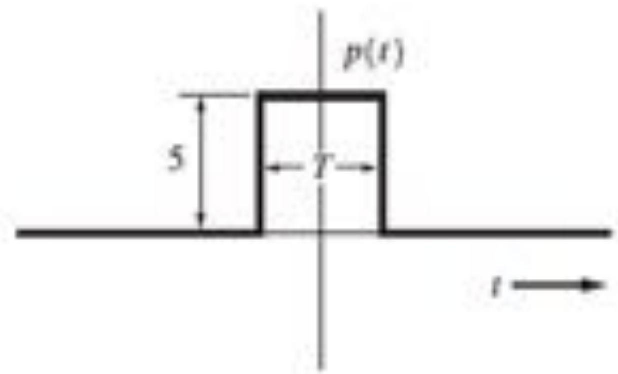
Example 3-6. VECTOR REPRESENTATION OF A BINARY SIGNAL

Examine the representation for the waveform of a 3-bit (binary) signal shown in Fig. 3-11a. This signal could be represented by

$$s(t) = \sum_{j=1}^{N=3} d_j p\left[t - (j - \frac{1}{2})T\right] = \sum_{j=1}^{N=3} d_j p_j(t) \qquad p_j(t) \equiv p\left[t - (j - 1/2)T\right]$$



(a) A Three-Bit Signal Waveform



(b) Bit Shape Pulse

3.4. Digital Signaling

Bandwidth Estimation

From the Dimensionality Theorem

The bandwidth of

$$w(t) = \sum_{k=1}^{N} w_k \varphi_k(t) \qquad 0 < t < T_0$$

is

$$B \geq \frac{N}{2T_0} = \frac{1}{2} D \quad Hz$$

if the φ_k are the sin(x)/x type, the ***lower bound absolute bandwidth*** of $N/(2T_0)$=D/2 will be achieved; otherwise, the bandwidth will be larger than this lower bound.

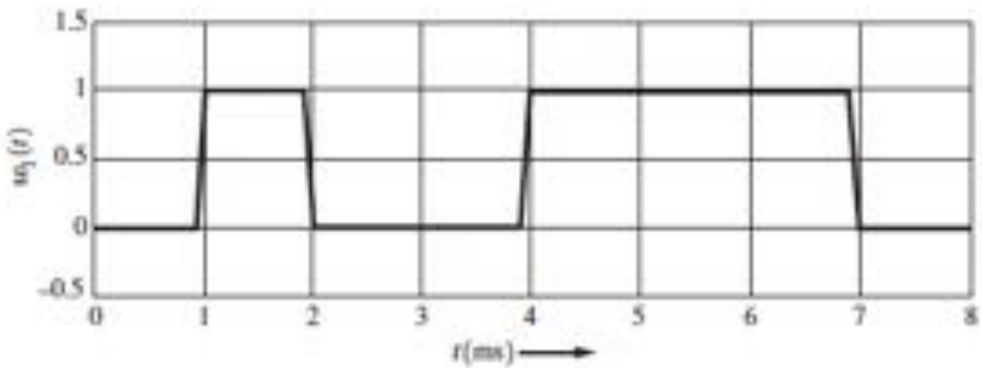
3.4. Digital Signaling

Example 3-7. BINARY SIGNAL

Digital source that can produce M = 256 distinct messages. Each message could be represented by n = 8-bit binary words because $M = 2^n = 2^8 = 256$. It takes T_0 = 8 ms to transmit one message 01001110
$w_1 = 0$, $w_2 = 1$, $w_3 = 0$, $w_4 = 0$, $w_5 = 1$, $w_6 = 1$, $w_7 = 1$, and $w_8 = 0$

CASE I

Rectangular pulse



(a) Rectangular Pulse Shape, T_b = 1 ms

Bit rate: $R = n/T_0$ =1 kbits/s   baud (symbol rate): $D = N/T_0$=1 kbaud

Lower bound bandwidth is 1/2D = 500 HZ.
The actual null bandwidth is $B = 1/T_s = D$ = 1,000 Hz

3.4. Digital Signaling

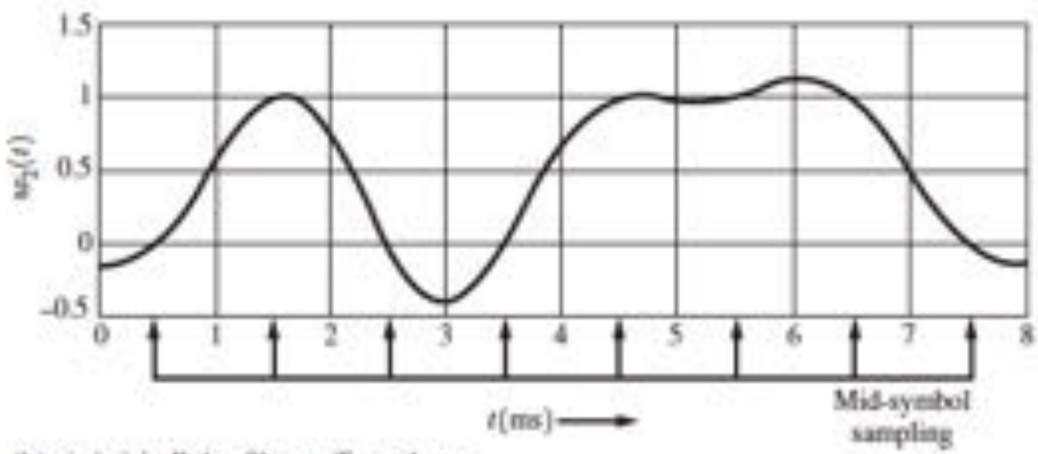
Example 3-7. BINARY SIGNAL

Digital source that can produce M = 256 distinct messages. Each message could be represented by n = 8-bit binary words because $M = 2^n = 2^8 = 256$. It takes T_0 = 8 ms to transmit one message 01001110
$w_1 = 0$, $w_2 = 1$, $w_3 = 0$, $w_4 = 0$, $w_5 = 1$, $w_6 = 1$, $w_7 = 1$, and $w_8 = 0$

CASE II

sinc pulse

$$\varphi_k(t) = \frac{\sin\left\{\frac{\pi}{T_s}(t-kT_s)\right\}}{\frac{\pi}{T_s}(t-kT_s)}$$



(b) sin(x)/x Pulse Shape, T_b = 1 ms

Bit rate: $R = n/T_0$ =1 kbits/s   baud (symbol rate): $D = N/T_0$=1 kbaud

bandwidth: $B = 1/2T_s$ = ½*D = 500 Hz

3.4. Digital Signaling

Multilevel Signaling

✧ Multilevel signal: w_k is multilevel word.

Given message: 01001110

binary: $w_1 = 0$, $w_2 = 1$, $w_3 = 0$, $w_4 = 0$, $w_5 = 1$, $w_6 = 1$, $w_7 = 1$, and $w_8 = 0$

$N = 8$

multilevel (L= 4-level): $w_1 = -3$, $w_2 = -1$, $w_3 = =3$, $w_4 = 1$

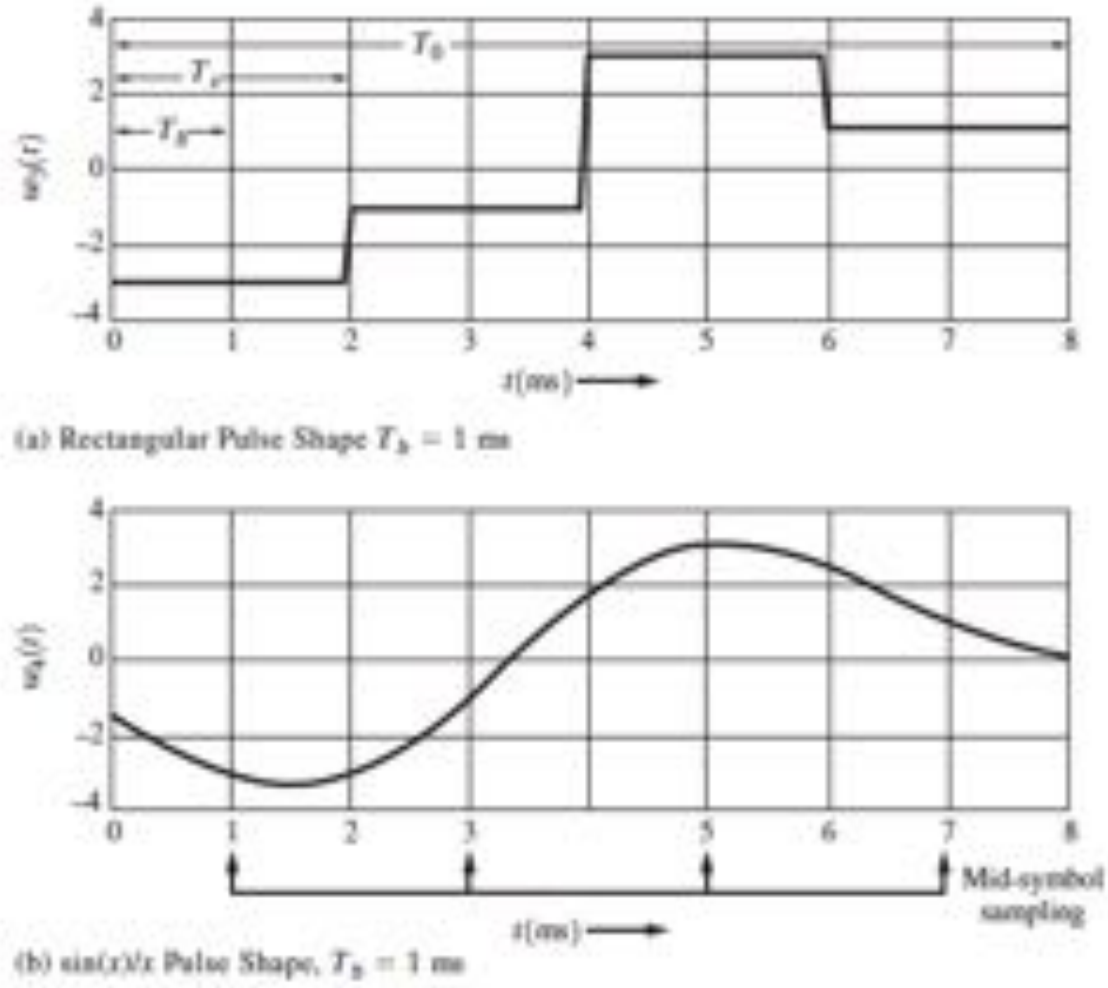
$N = 4$

TABLE 3–3 A 2-BIT DIGITAL-TO-ANALOG CONVERTER

Binary Input ($\ell = 2$ bits)	Output Level (V)
11	+3
10	+1
00	−1
01	−3

3.4. Digital Signaling

Multilevel Signaling



(a) Rectangular Pulse Shape $T_b = 1$ ms

(b) sin(x)/x Pulse Shape, $T_b = 1$ ms

T_s = 2 ms
(middle of symbol intervals)

D = N/T_0 = 4/8 = 0.5 kbaud
B = D/2 = 250 Hz