Chapter 3. Baseband Pulse and Digital Signaling

Chapter Objectives

- Analog-to-digital signaling
- Binary and multilevel digital signals
- Spectra and bandwidths of digital signals
- Prevention of intersymbol interference
- Time division multiplexing
- Packet transmission

3.1. Introduction

The following are the **four** main goals of this chapter

✧ To study how analog waveforms can be converted to digital waveform. The most popular technique is called *pulse code modulation* (**PCM**).

✧ To study how to compute the **spectrum** for digital signals.

✧ To examine how the filtering of pulse signals affects our ability to recover the digital information at the receiver. This filtering can produce what is called *intersymbol interference* (**ISI**) in the recovered data signal.

✧ To study how we can multiplex (combine) data from several digital bit streams into one high-speed digital stream for transmission over a digital system. One such technique, called *time-division multiplexing* (**TDM**), will be studied in this chapter.

3.2. Pulse Amplitude Modulation

✧ *Pulse amplitude modulation* **(PAM)** is an engineering term that is used to describe the conversion of the analog signal to a pulse-type signal in which the amplitude of the pulse denotes the analog information.

✧ The purpose of PAM signaling is to provide another waveform that looks like pulses, yet contains the information that was present in the analog waveform.

✧ There are two classes of PAM signals:

✓ PAM that uses **Natural Sampling** (**Gating**)

✓ PAM that uses **Instantaneous Sampling** (**Flat-Top PAM**)

3.2. Pulse Amplitude Modulation

Natural Sampling (Gating)

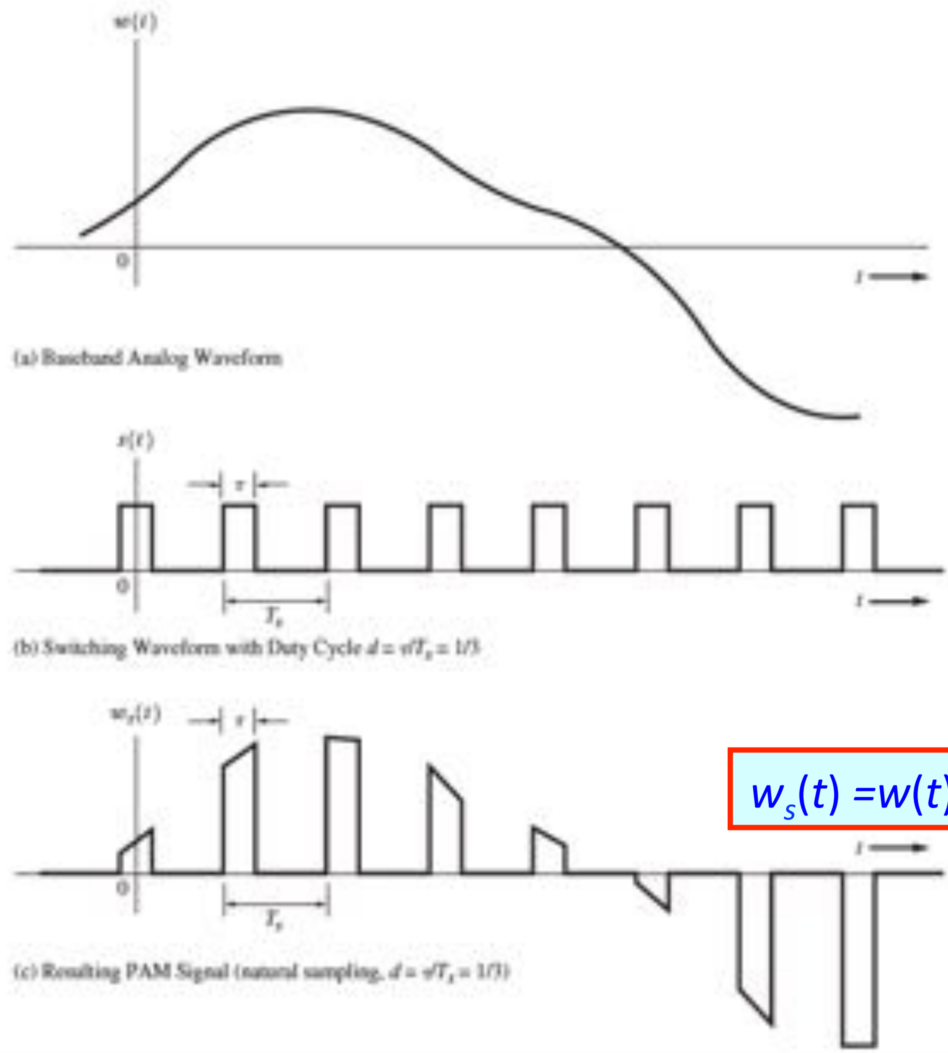
DEFINIATION: If *w*(*t*) is an analog waveform bandlimited to ***B*** hertz, the **PAM** signal that uses natural sampling is:

$$w_s(t) = w(t)s(t)$$

where $s(t) = \sum_{k=-\infty}^{\infty} \Pi(\frac{t-kT_s}{\tau})$ is a rectangular wave switching waveform and $f_s = 1/T_s \geq 2B$

3.2. Pulse Amplitude Modulation

Natural Sampling (Gating)



$w_s(t) = w(t)s(t)$

3.2. Pulse Amplitude Modulation

Natural Sampling (Gating)

THEOREM: The **spectrum** for a naturally sampled PAM signal is

$$W_s(f) = \Im[w_s(t)] = d \sum_{n=-\infty}^{\infty} \frac{\sin \pi n d}{\pi n d} W(f - n f_s)$$

where $f_s = 1/T_s$, $w_s = 2\pi f_s$, the duty cycle of $s(t)$ is

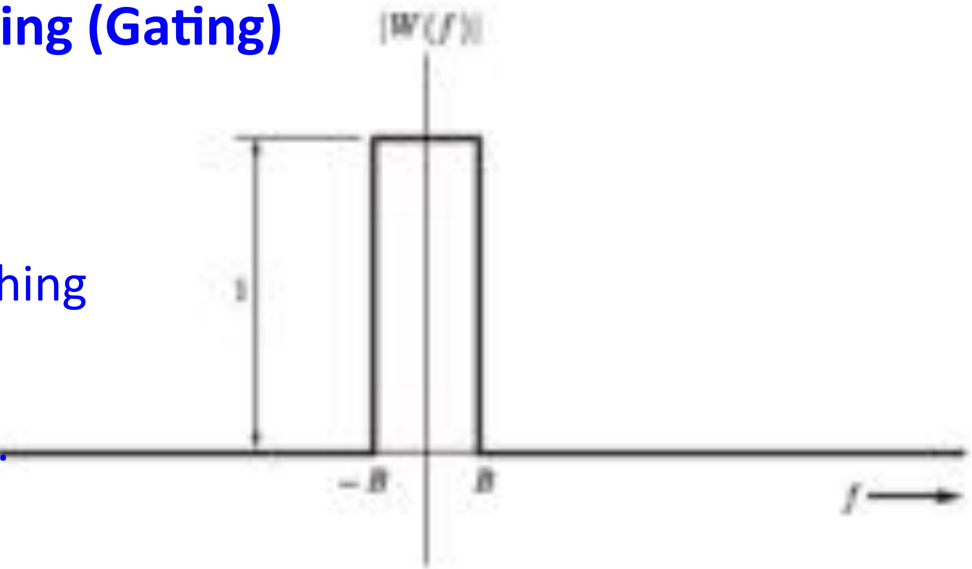
$d = \tau / T_s$, and $W(f) = \Im\left[w(t)\right]$ is the spectrum of the

original unsampled waveform.

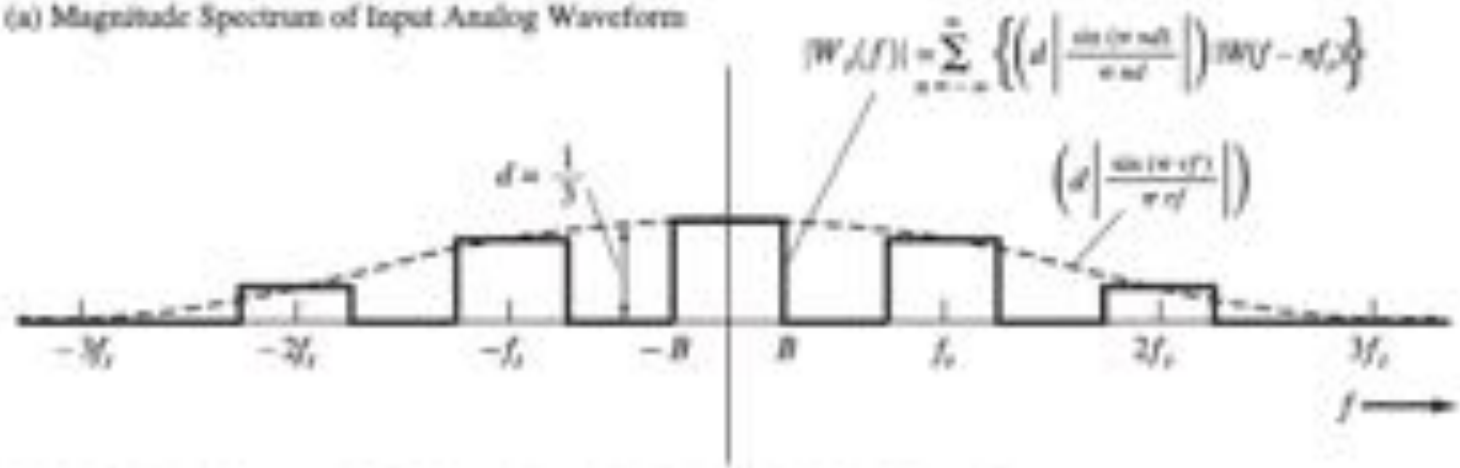
3.2. Pulse Amplitude Modulation

Natural Sampling (Gating)

- The duty cycle of the switching waveform is $d = \tau/T_s = 1/3$.
- The sampling rate is $f_s = 4B$.



(a) Magnitude Spectrum of Input Analog Waveform

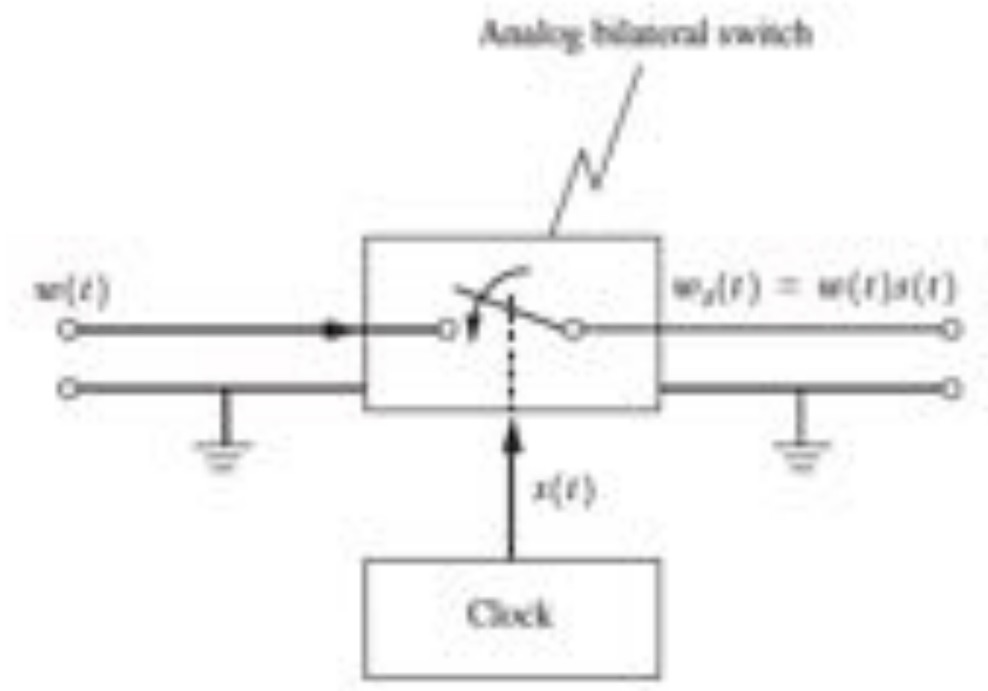


(b) Magnitude Spectrum of PAM (natural sampling) with $d = 1/3$ and $f_s = 4$ B

3.2. Pulse Amplitude Modulation

Generation of PAM with natural sampling (gating)

The PAM waveform with natural sampling can be generated using a CMOS (complementary metal-oxide-semiconductor) circuit consisting of A clock and analog switch as shown.



3.2. Pulse Amplitude Modulation

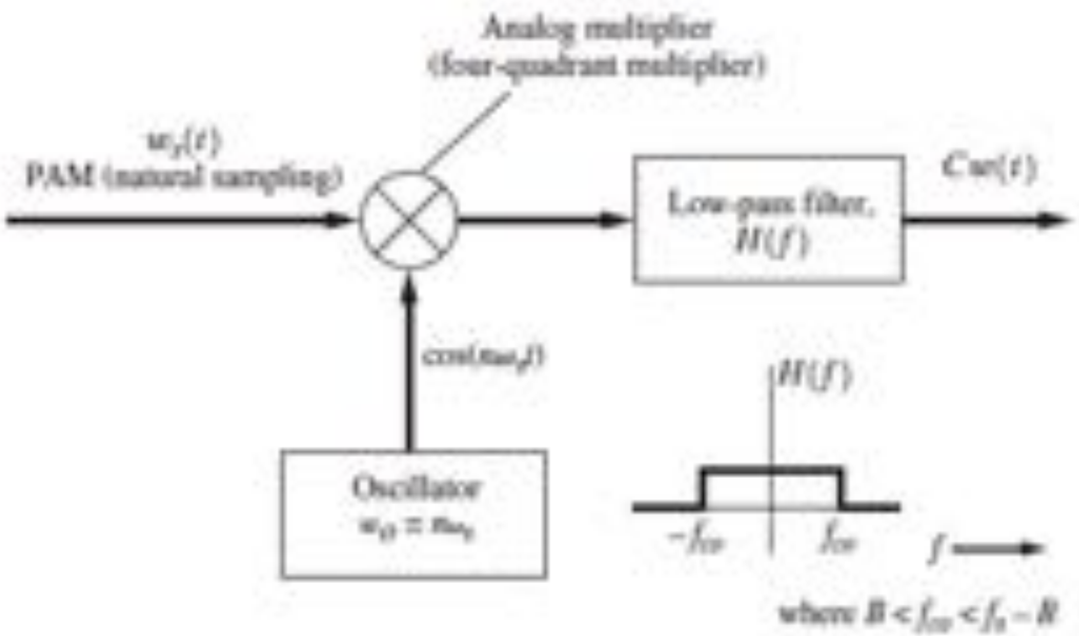
Recovering Naturally Sampled PAM

- At the receiver, the original analog waveform, ***w*(*t*)** can be recovered from the PAM signal, $w_s(t)$, by passing the PAM signal through a **low pass filter,** where the **cutoff frequency** is: $B < f_{cutoff} < f_s - B$.

- If the analog signal is under sampled $f_s < 2B$, the effect of spectral overlapping is called **Aliasing**. This results in a recovered analog signal that is distorted compared to the original waveform

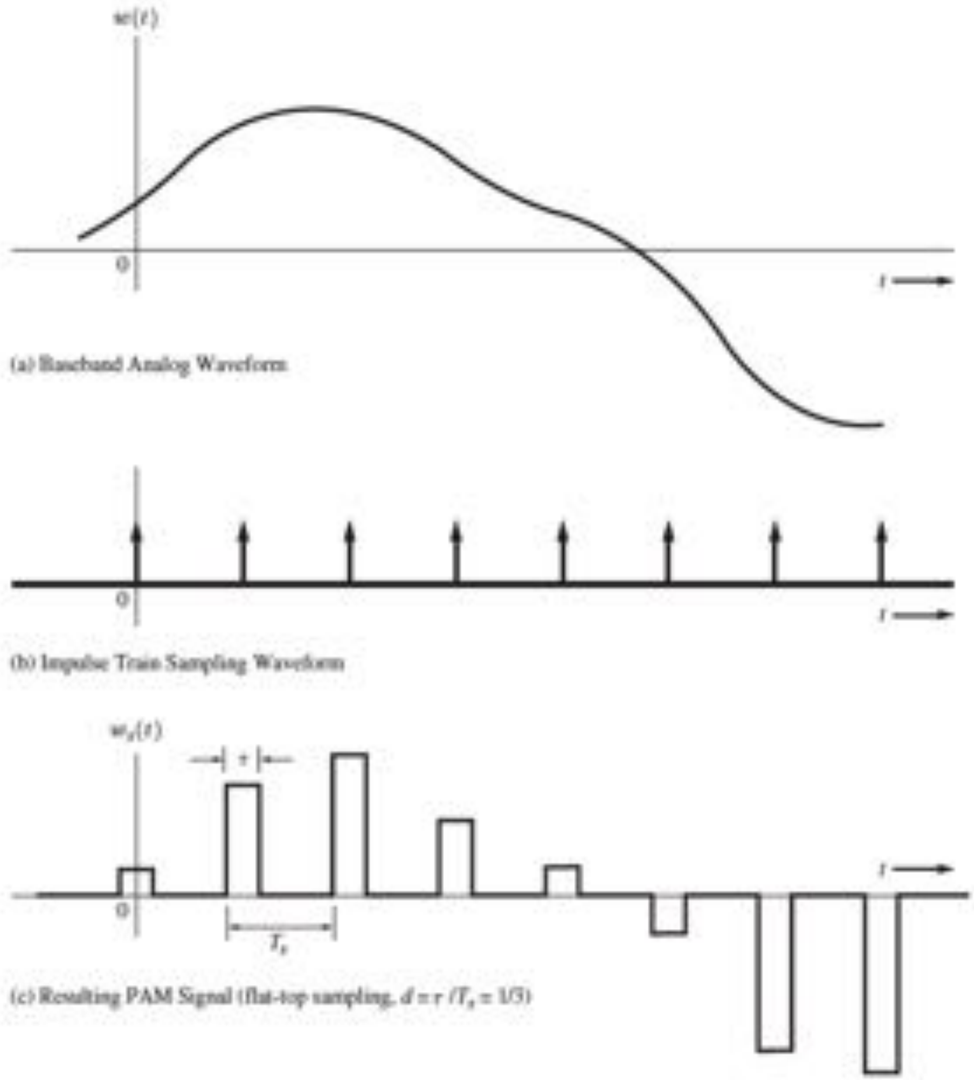
3.2. Pulse Amplitude Modulation

Recovering Naturally Sampled PAM



3.2. Pulse Amplitude Modulation

Instantaneous Sampling (Flat-Top PAM)



(a) Baseband Analog Waveform

(b) Impulse Train Sampling Waveform

(c) Resulting PAM Signal (flat-top sampling, $d = \tau / T_s = 1/3$)

3.2. Pulse Amplitude Modulation

Instantaneous Sampling (Flat-Top PAM)

DEFINIATION: If $w(t)$ is an analog waveform bandlimited to ***B*** hertz, the **PAM** signal that uses instantaneous sampling is:

$$w_s(t) = \sum_{k=-\infty}^{\infty} w(kT_s)h(t - kT_s)$$

where $h(t) = \prod\left(\frac{t}{\tau}\right) = \begin{cases} 1, & |t| < \tau/2 \\ 0, & |t| > \tau/2 \end{cases}$ is sampling pulse

shape, were $\tau \le T_s = 1/f_s$ and $f_s \ge 2B$

3.2. Pulse Amplitude Modulation

Instantaneous Sampling (Flat-Top PAM)

THEOREM: The spectrum for a flat-top PAM signal is

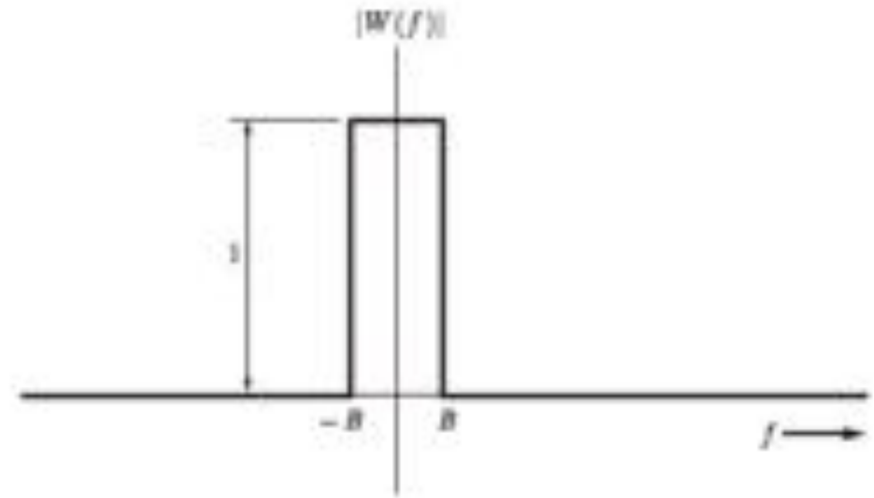
$$W_s(f) = \frac{1}{T_s} H(f) \sum_{k=-\infty}^{\infty} W(f - kf_s)$$

$$H(f) = \Im[h(t)] = \tau \left(\frac{\sin \pi\tau f}{\pi\tau f} \right)$$

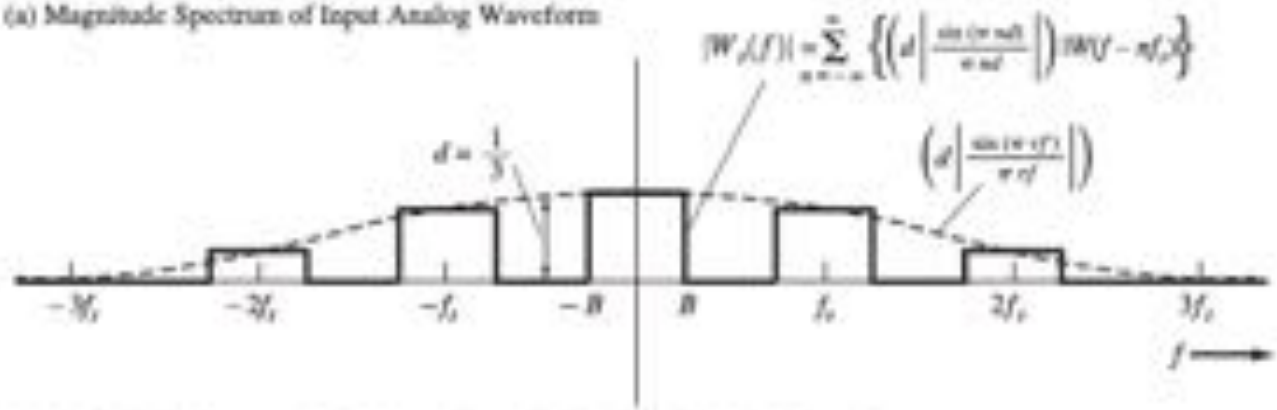
- ✧ This type of PAM signal consists of instantaneous samples
- ✧ *w(t)* is sampled at $t = kT_s$
- ✧ The sample values $w(kT_s)$ determine the amplitude of the flat-top rectangular pulses

3.2. Pulse Amplitude Modulation

Instantaneous Sampling (Flat-Top PAM)



(a) Magnitude Spectrum of Input Analog Waveform



(b) Magnitude Spectrum of PAM (natural sampling) with $d = 1/3$ and $f_s = 4$ B

3.2. Pulse Amplitude Modulation

Limitations of PAM

- ✧ The transmission of either naturally of instantaneously sampled PAM over a channel requires a very wide frequency response.
- ✧ The bandwidth required is much larger than that of the original analog signal.
- ✧ The noise performance of the PAM system can never be better than achieved by transmitting the analog signal directly.
- ✧ PAM is not very good for long-distance transmission.

3.3. Pulse Code Modulation

DEFINITION: ***Pulse code modulation*** (**PCM**) is essentially analog-to-digital conversion of a special type where the information contained in the instantaneous samples of an analog signal is represented by digital words in a ***serial bit stream.***

ADVANTAGES:

- ✧ Relatively inexpensive digital circuitry may be used extensively.
- ✧ PCM signals derived from all types of analog sources may be merged with data signals and transmitted over a common high-speed digital communication system.
- ✧ In long-distance digital telephone systems requiring repeaters, a clean PCM waveform can be regenerated at the output of each repeater, where the input consists of a noisy PCM waveform.
- ✧ The noise performance of a digital system can be superior to that of an analog system.
- ✧ The probability of error for the system output can be reduced even further by the use of appropriate coding techniques.

3.3. Pulse Code Modulation

PCM signal is generated by carrying out ***three basic operations***:

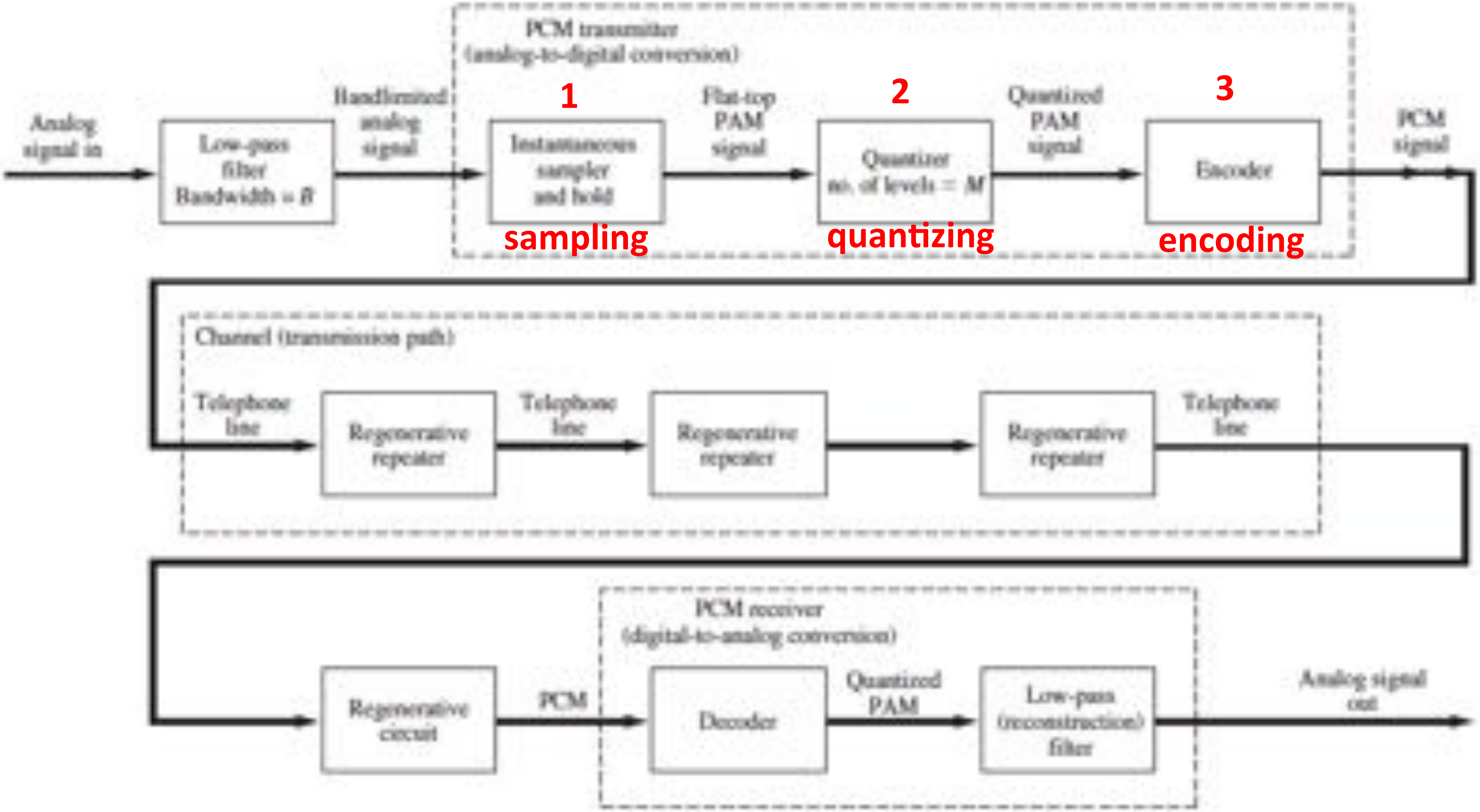
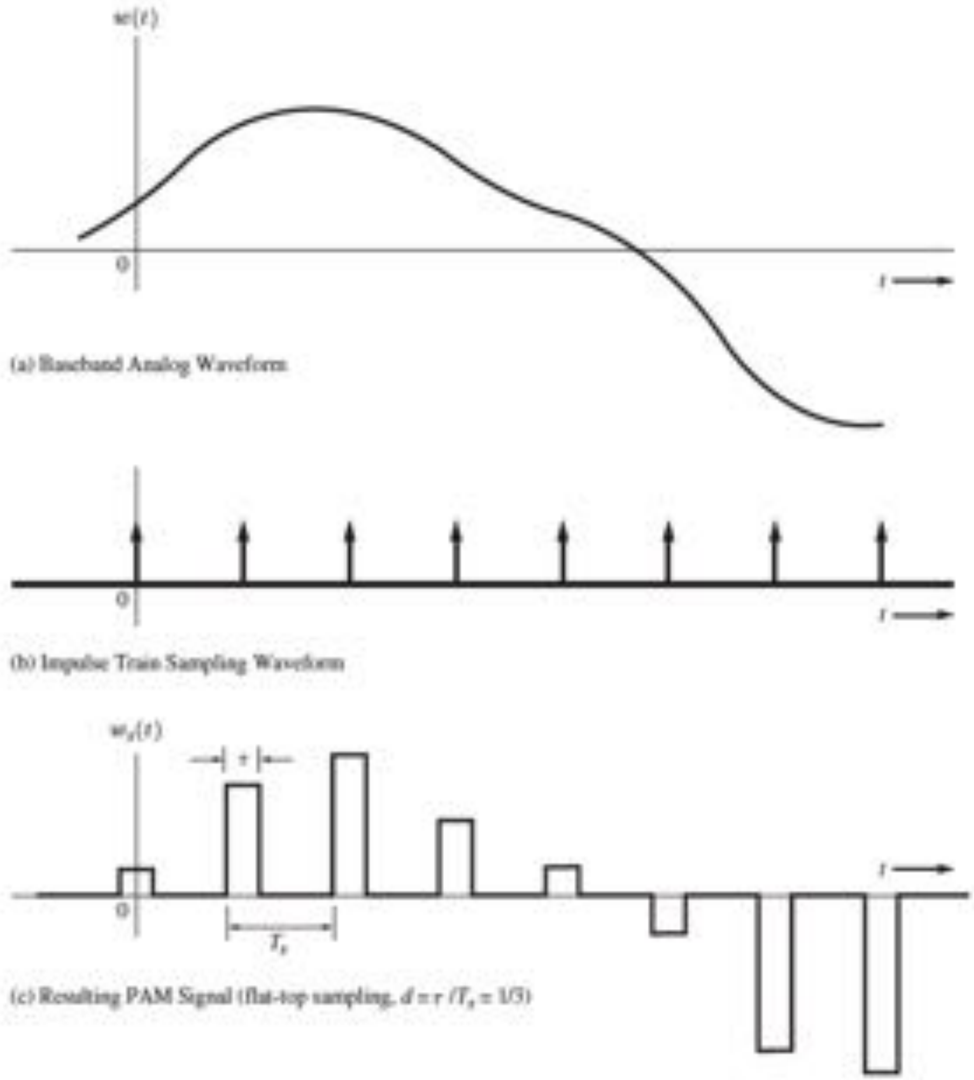


Figure 3–7 PCM transmission system.

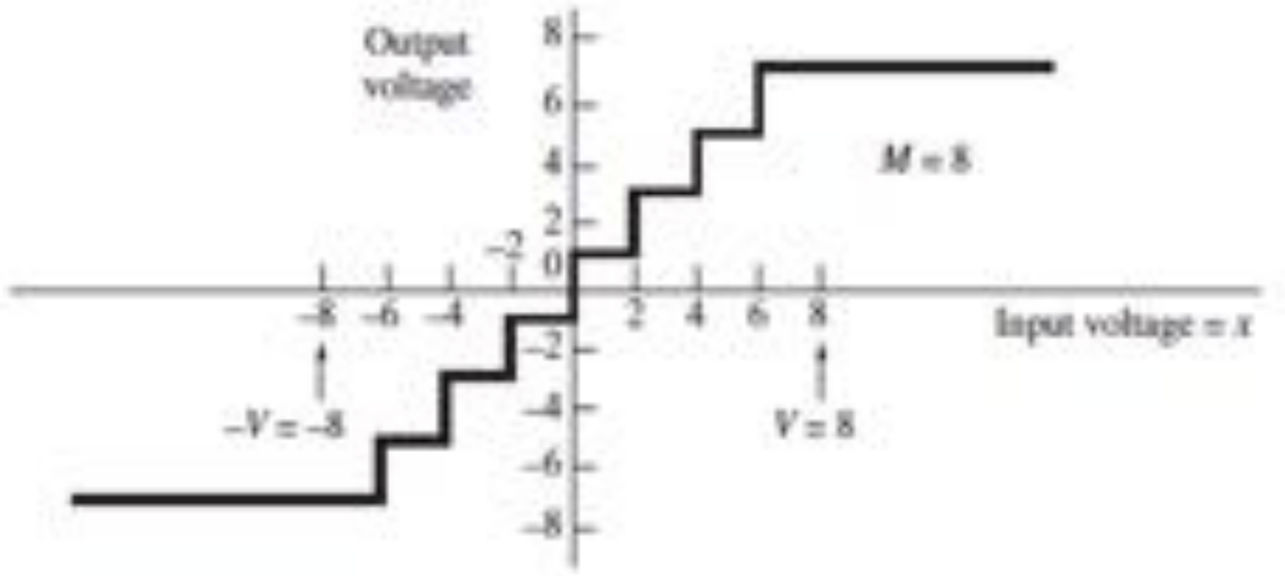
3.3. Pulse Code Modulation

1. Sampling (Flat-top PAM Signal)



3.3. Pulse Code Modulation

2. Quantizing



Example of M = 8 = 2^3 uniform quantizer

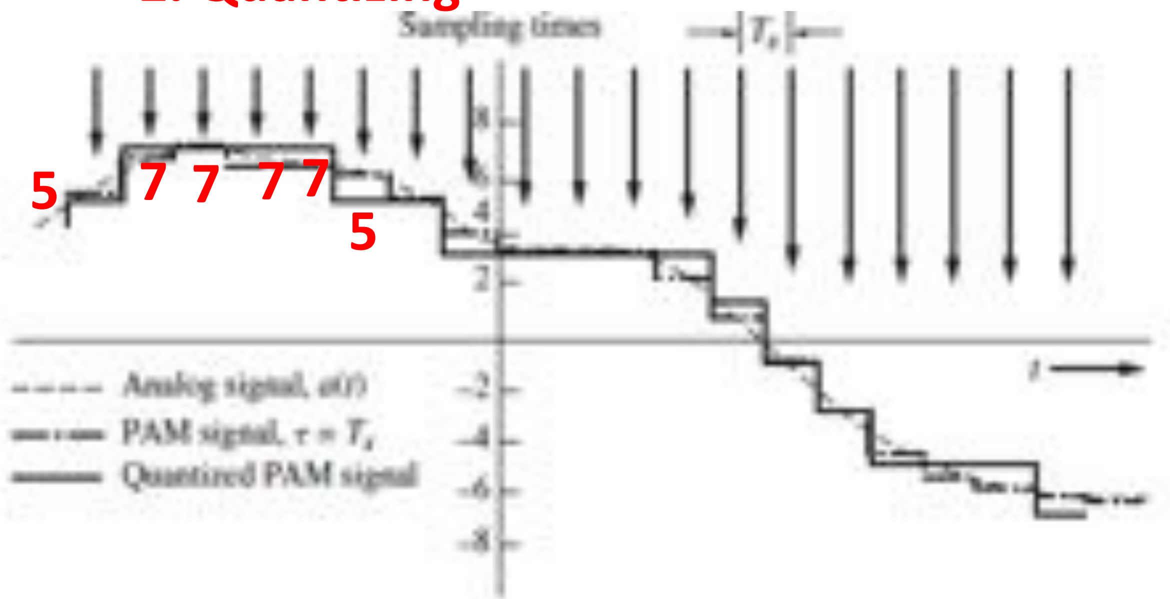
M = 8 **(M = 2^n)** levels are used to approximate the analog sample values (infinite number):

-7, -5, -3, -1, 1, 3, 5, 7

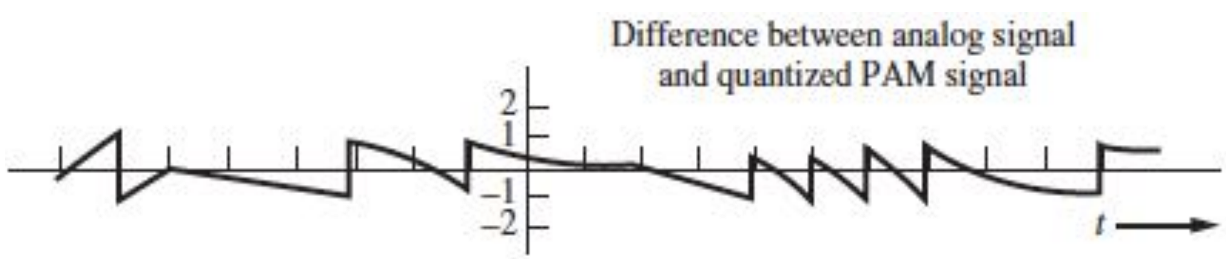
3.3. Pulse Code Modulation

2. Quantizing

The PAM signal is "rounding off" to get quantized PAM signal



errors are introduced because of the finite number of levels (***M = 8***) is used in the quantizing (*quantizing noise*)



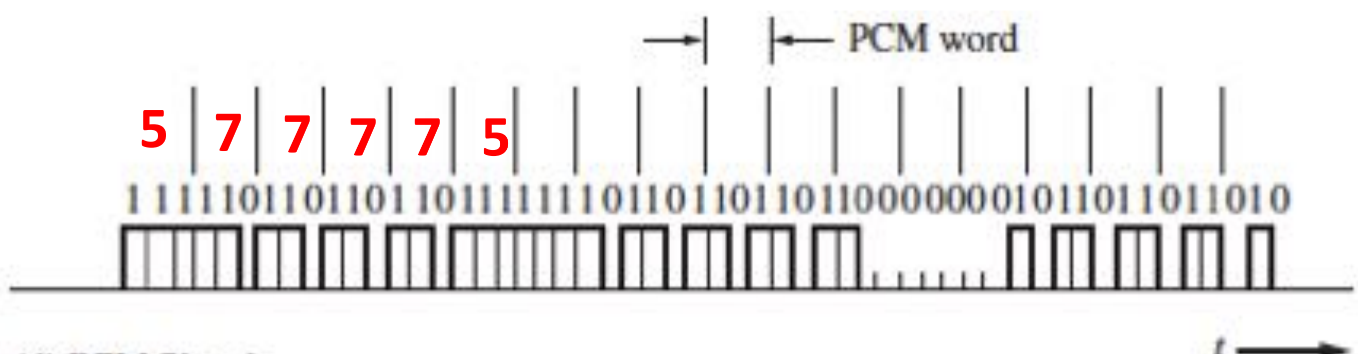
3.3. Pulse Code Modulation

3. Encoding

The PCM signal is obtained from the quantized PAM signal by encoding each quantized sample value into a digital word.

Three –bit M = 8 Gray code

-7	-5	-3	-1	1	3	5	7
010	011	001	000	100	101	111	110



(d) PCM Signal

3.3. Pulse Code Modulation

Practical PCM Circuits

- Three popular techniques are used to implement the analog-to-digital converter (ADC) encoding operation:
  1. The counting or ramp, (Maxim ICL7126 ADC)
  2. Serial or successive approximation, (AD 570)
  3. Parallel or flash encoders. (CA3318)

- The objective of these circuits is to generate the PCM word.

- Parallel digital output obtained (from one of the above techniques) needs to be serialized before sending over a 2-wire channel

- This is accomplished by parallel-to-serial converters [Serial Input-Output (SIO) chip]

- UART,USRT and USART are examples for SIO's

3.3. Pulse Code Modulation

Bandwidth of PCM Signals

- The bandwidth of PAM signal can be obtained as a function of the spectrum of the input analog signal (Linear function).

- The bandwidth of PCM signal is not directly related to the spectrum of input analogy signal. (Nonlinear function: 3 steps)

- The bandwidth of (serial) binary PCM waveforms depends on the ***bit rate R*** and the ***waveform pulse shape*** used to represent the data.

- The ***bit rate R*** is:

$$R = nf_s$$

where n is the number of bits in the PCM word ($M = 2^n$) and f_s is the sampling rate (PAM).

3.3. Pulse Code Modulation

Bandwidth of PCM Signals

- For no aliasing case (f_s *>=2B, B is the bandwidth of the analog signal, that is to be converted to PCM signal*), the **MINIMUM** bandwidth of PCM is:

  $$B_{PCM} = R/2 = nf_s/2$$

  The Minimum bandwidth of $nf_s/2$ is obtained only when sin(x)/x pulse is used to generate the PCM waveform.

- For PCM waveform generated by rectangular pulses, the First-null bandwidth is:

  $$B_{PCM} = R = nf_s$$

- For a reasonable value of n, the bandwidth of the serial PCM signal will be significantly larger than the bandwidth of the corresponding analog signal that it represents.

- If the bandwidth of the PCM signal is reduced by improper filtering of poor frequency response, one bit will smear into adjacent bit slot. This is called ***intersymbol interference (ISI).***

3.3. Pulse Code Modulation

Effects of Noise

The analog signal that is recovered at the PCM system output is corrupted by noise. Two main effects produce this noise or distortion:

- ✧ Quantizing noise that is caused by the M-step quantizer at the PCM transmitter.

- ✧ Bit errors in the recovered PCM signal. The bit errors are caused by *channel noise*, as well as *improper channel filtering*, which causes ISI (*intersymbol interference*).

The input analog signal needs to be sufficiently bandlimited (with a low-pass antialiasing filter) and sampled fast enough so that the aliasing noise on the recovered analog signal is negligible.

3.3. Pulse Code Modulation

Effects of Noise

The **ratio** of the recovered analog **PEAK signal power** to **the total average noise power** is given by:

$$\left(\frac{S}{N}\right)_{pkout} = \frac{3M^2}{1+4(M^2-1)P_e}$$

The **ratio** of the **AVERAGE signal power** to **the average noise power** is:

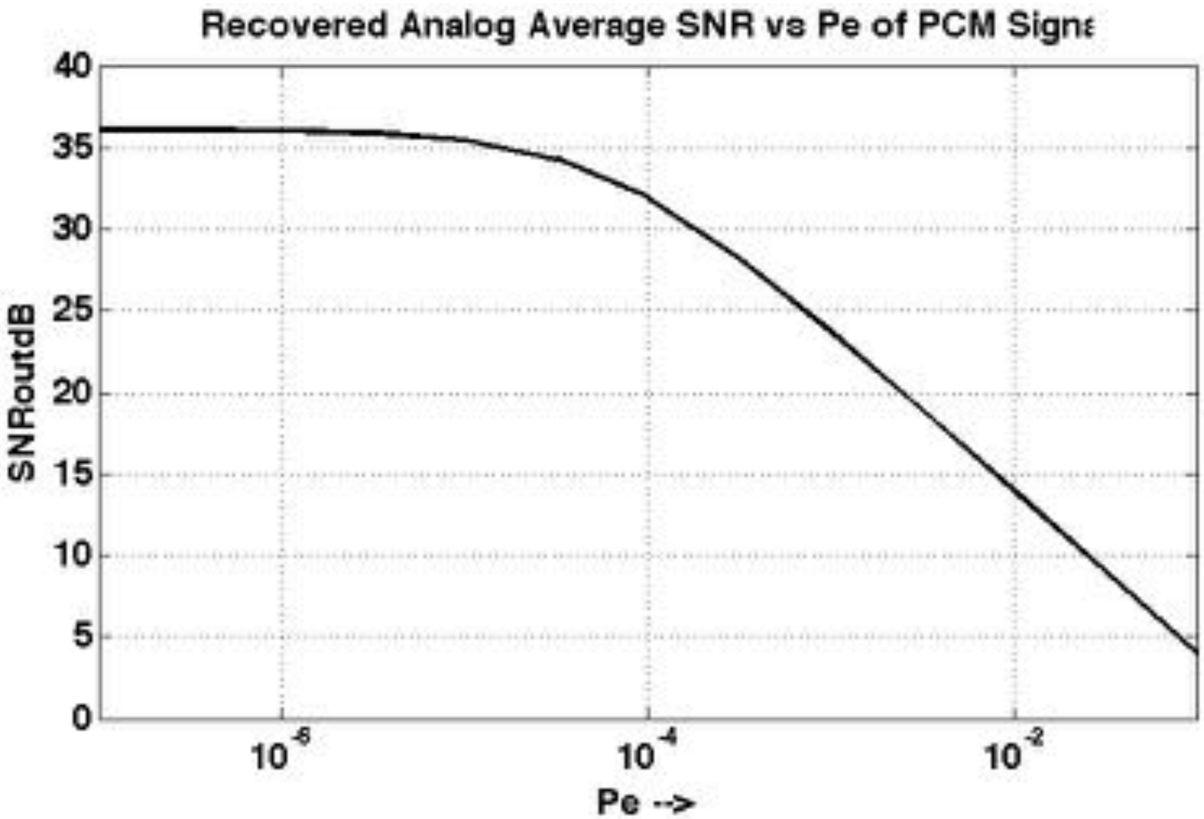
$$\left(\frac{S}{N}\right)_{pkout} = \frac{M^2}{1+4(M^2-1)P_e}$$

Where $\boldsymbol{M}$ is the number of quantized levels used in the PCM system and $\boldsymbol{P_e}$ is the probability of bit error in the recovered binary PCM signal at the receiver DAC before it is converted back into an analog signal

3.3. Pulse Code Modulation

Example 3-3. Average signal-to-noise ratio for a recovered analog signal

Calculate the average $\boldsymbol{SNR_{dB}}$ of the analog signal that is recovered from a PCM signal that has error bits with a probability of error of $\boldsymbol{P_e}$. Plot the $\boldsymbol{SNR_{dB}}$ for $\boldsymbol{P_e}$ over a range from 10^{-7} to 10^{-1}.



$$\left(\frac{S}{N}\right)_{pkout} = \frac{M^2}{1+4(M^2-1)P_e}$$

$M = 2^n$

$n = 6$

3.3. Pulse Code Modulation

If $\boldsymbol{P_e}$ is negligible, there are no bit errors resulting from channel noise and no ISI, the **peak SNR** resulting from only quantizing error is

$$\left(\frac{S}{N}\right)_{pkout} = \frac{3M^2}{1+4(M^2-1)P_e} \quad \Longrightarrow \quad \left(\frac{S}{N}\right)_{pkout} = 3M^2$$

The **ratio** of the **AVERAGE signal power** to **the average noise power** is:

$$\left(\frac{S}{N}\right)_{out} = \frac{M^2}{1+4(M^2-1)P_e} \quad \Longrightarrow \quad \left(\frac{S}{N}\right)_{out} = M^2$$

Above equations can be expressed in decibels as:

$$\left(\frac{S}{N}\right)_{dB} = 6.02n + \alpha$$

Where, $M = 2^n$

α = 4.77 for peak SNR

α = 0 for average SNR

3.3. Pulse Code Modulation

Example 3-4. Design of a PCM signal for telephone systems

Assume that an analog audio voice-frequency(VF) telephone signal occupies a band from 300 to 3,400Hz. The signal is to be converted to a PCM signal for transmission over a digital telephone system. The minimum sampling frequency is 2x3.4 = 6.8 *ksample/*sec. To be able to use of a low-cost low-pass antialiasing filter, the VF signal is oversampled with a sampling frequency of 8ksamples/sec. This is the standard adopted by the Unites States telephone industry. Assume that each sample values is represented by 8 bits; then the bit rate of the binary PCM signal is:

$$R = (f_s \text{ samples/s})(n \text{ bits/sample})$$

$$= (8k \text{ samples/s})(8 \text{ bits/sample}) = 64\ k\ bits/s$$

This 64k bits/s is called a DS-0 signal (digital signal, type zero)

The **minimum** absolute bandwidth of the binary PCM signal is

$$B_{\text{PCM}} \geq \frac{R}{2} = \frac{nf_s}{2} = 32kHZ$$

This bandwidth is realized when a *(sinx)/x* pulse shape is used.

3.3. Pulse Code Modulation

Example 3-4. Design of a PCM signal for telephone systems

If a rectangular pulse for sampling, the absolute bandwidth is infinity, and the first null bandwidth is :

$$B_{\mathrm{PCM}} = R = 64kHZ$$

We require a bandwidth of 64k HZ to transmit this digital voice PCM signal, whereas the bandwidth of the original analog voice signal was, at most, 4k Hz.

We observe that the peak signal-to-quantizing noise power ratio is:

$$\left(\frac{S}{N}\right)_{pkout} = 3(2^8)^2 = 52.9dB$$

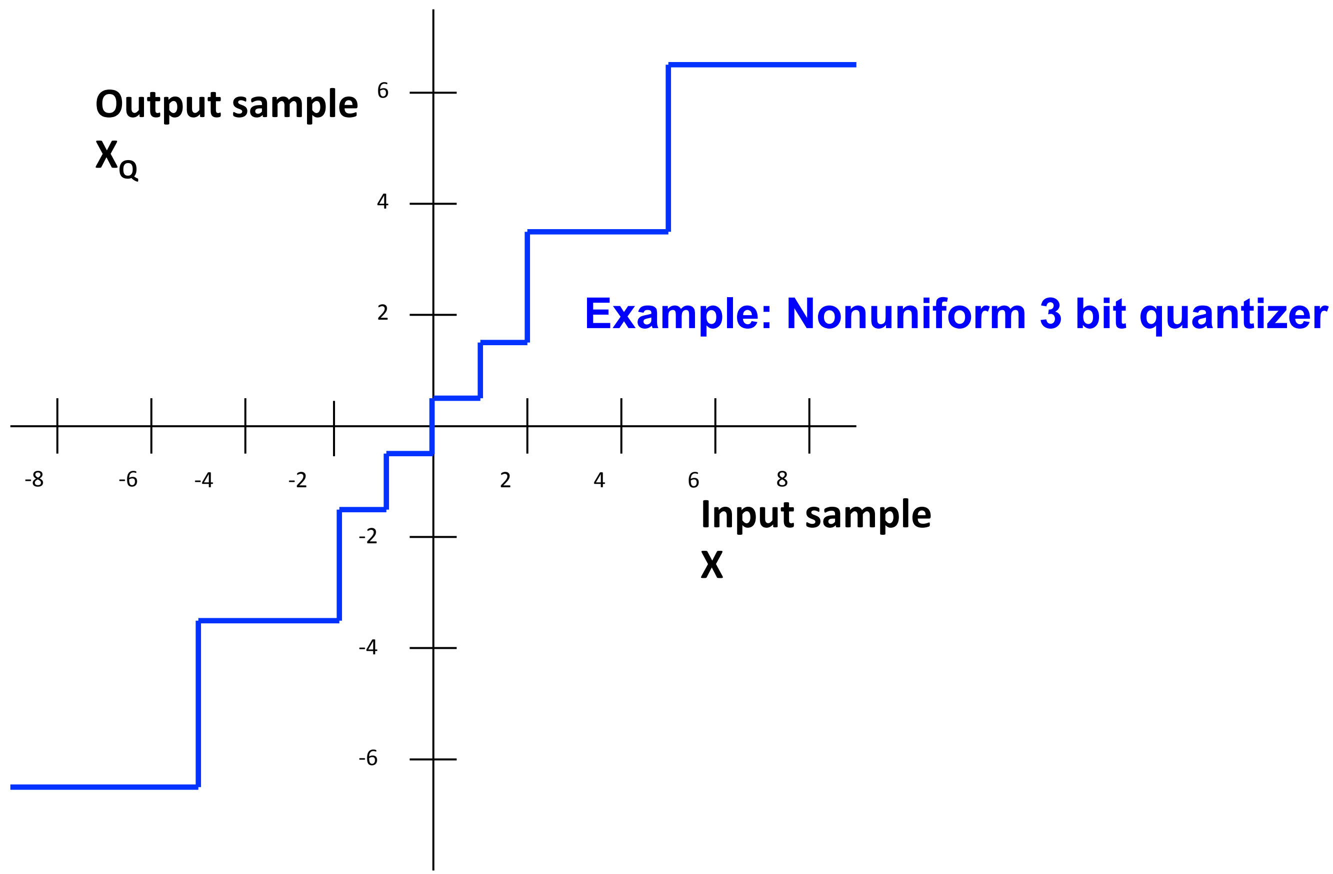
3.3. Pulse Code Modulation

Nonuniform Quantization

Many signals (e.g. voice analog) have a non-uniform distribution

-- The amplitude is more likely to be close to zero than to be at higher levels

-- Ex. If the peak value allowed is 1 V, weak passages my have voltage levels on the order of 01. V (20 dB down)



3.3. Pulse Code Modulation

Nonuniform Quantization

Analog signal $w_1(t)$ → Compression (nonlinear) → PCM circuit (uniform quantizer)

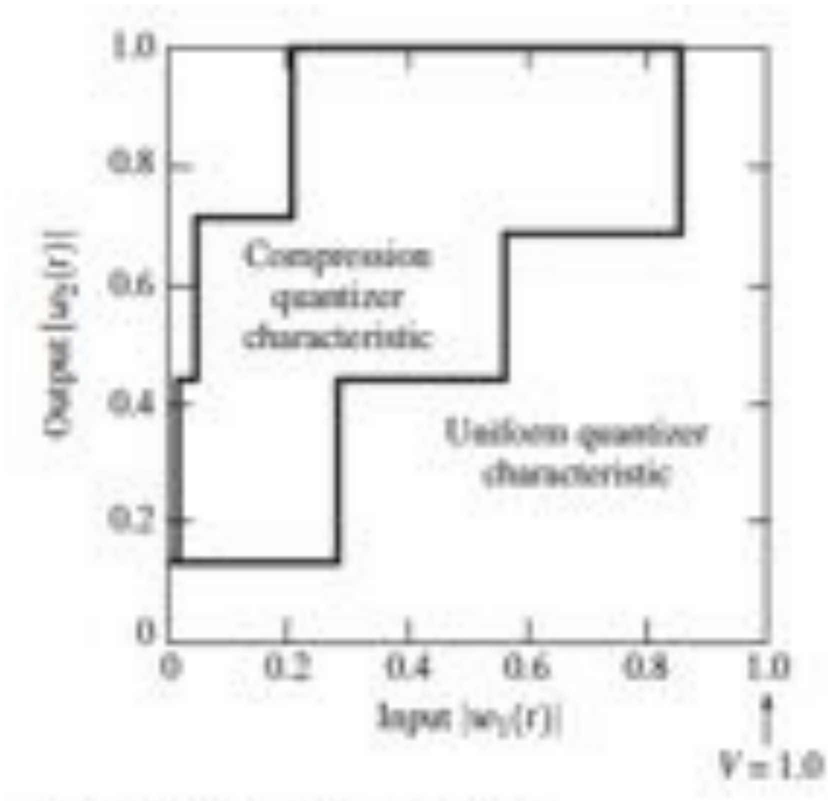
μ-law type:

$$|w_2(t)| = \frac{\ln\left(1+\mu|w_1(t)|\right)}{\ln(1+\mu)}$$

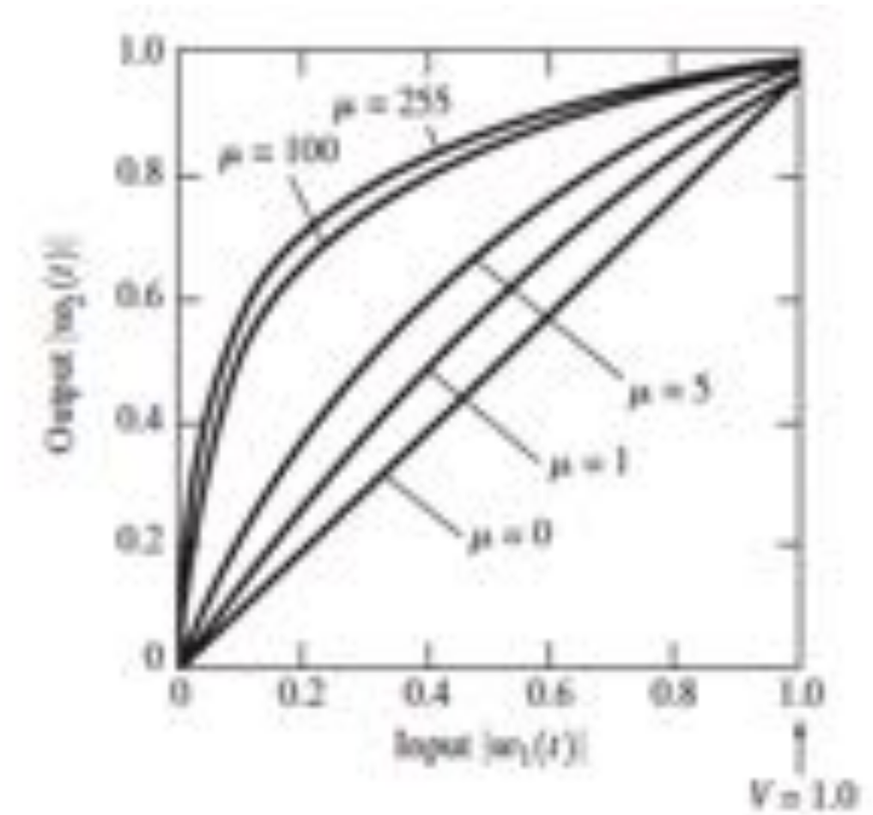
In the United States, Canada, and Japan, the telephone companies use a $\mu = 255$ compression characteristic in their PCM systems.

3.3. Pulse Code Modulation

Nonuniform Quantization



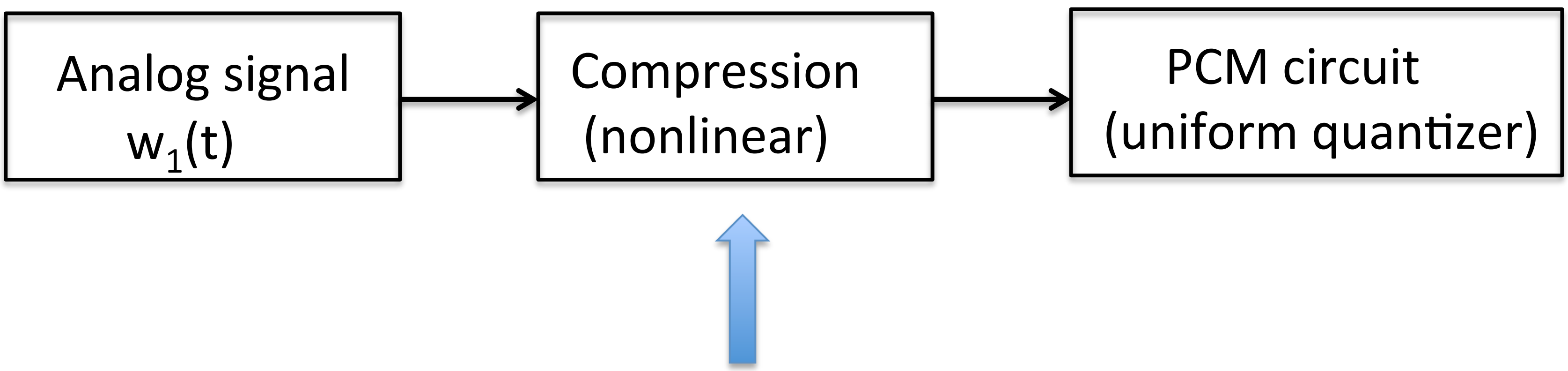
M = 8 Quantizer Characteristic



μ-law characteristic

3.3. Pulse Code Modulation

Nonuniform Quantization



A-law type: $$|w_2(t)| = \begin{cases} \dfrac{A|w_1(t)|}{1+\ln A} & 0 \le |w_1(t)| \le \dfrac{1}{A} \\[2ex] \dfrac{1+\ln(A|w_1(t)|)}{1+\ln A} & \dfrac{1}{A} \le |w_1(t)| \le 1 \end{cases}$$

In Europe, the A-law is generally used.

3.3. Pulse Code Modulation

Nonuniform Quantization

The Output SNR follows the 6-dB law

$$\left(\frac{S}{N}\right)_{dB} = 6.02n + \alpha$$

Where $\alpha = 4.77 - 10\log\left(V/x_{rms}\right)$ (uniform quantizing)

$\alpha \approx 4.77 - 10\log\left[\ln(1+\mu)\right]$ (μ-law quantizing)

$\alpha \approx 4.77 - 10\log\left[1 + \ln A\right]$ (A-law quantizing)