Chapter 2. Signals And Spectra

Chapter Objectives

- Basic signal properties (DC, RMS, dBm and power)
- Fourier transform and spectra
- Linear systems and linear distortion
- Bandlimited signals and sampling
- Discrete Fourier transform
- Bandwidth of signals

2.1 Properties of signals and noise

✧ In communication systems, the received waveform is usually categorized into two parts:

- the **desired part**, called **SIGNAL**, contains the **Information**
- the **undesired part**, called **NOISE**

✧ Properties of waveforms include:

- DC value
- Root-mean-square (RMS) value
- Normalized power
- Magnitude spectrum
- Phase spectrum
- Power spectral density
- Bandwidth

2.1 Properties of signals and noise

Physically realizable waveforms

Practical waveform that that ***physically realizable*** waveforms (i.e. measurable in a laboratory) satisfy several conditions:

- ✧ The waveform ***has significant nonzero values*** over a composite time interval that is finite. --- *produce finite amount of energy*
- ✧ The spectrum of the waveform has ***significant values over a composite frequency interval*** that is finite. -- *transmission medium has restricted bandwidth*
- ✧ The waveform is a ***continuous function of time***. -- *restricted bandwidth*
- ✧ The waveform has a ***finite peak*** value. -- *physical devices protection*
- ✧ The waveform has ***only real values***. That is, at any time, it cannot have a complex value $a + jb$, where b is nonzero. -- *real waveform in real word*

2.1 Properties of signals and noise

Time Average Operator

DEFNITION. $$\langle [\bullet] \rangle = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} [\bullet] dt$$

The time average operator is a ***linear*** operator

$$\langle \alpha_1 w_1(t) + \alpha_2 w_2(t) \rangle = \alpha_1 \langle w_1(t) \rangle + \alpha_2 \langle w_2(t) \rangle$$

DEFNITION. A waveform $w(t)$ is ***periodic*** with period T_0 if

$$w(t) = w(t + T_0) \qquad \text{for all } t$$

THEOREM. If the waveform involved is periodic, the time average operator can be reduced to:

$$\langle [\bullet] \rangle = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} [\bullet] dt$$

2.1 Properties of signals and noise

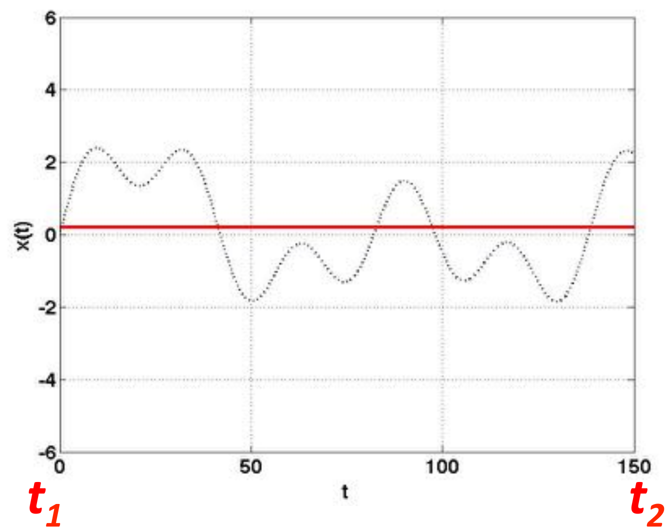
DC Value

DEFNITION. The ***DC*** (Direct "Current") value of a waveform $w(t)$ is given by its time average, <$w(t)$>

$$W_{dc} = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} w(t)dt$$

For any ***physical waveform***, we are actually interested in evaluating the DC value only over a ***finite interval*** of interest, say, from t_1 to t_2, so that the DC Value is

$$\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} w(t)dt$$

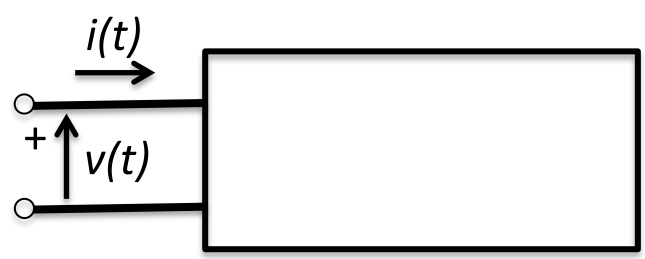


2.1 Properties of signals and noise

Power

DEFNITION. Let ***v*(*t*)** denote the voltage across a set of circuit terminals, and let ***i*(*t*)** denote the current into the terminal, as shown below. the ***instantaneous power*** (incremental work divided by incremental time) associated with the circuit is given by

$$p(t) = v(t)i(t)$$



where the instantaneous power flows into the circuit when $p(t)$ is positive and flow out of the circuit when $p(t)$ is negative

the ***average power***

$$P = \langle p(t) \rangle = \langle v(t)i(t) \rangle$$

2.1 Properties of signals and noise

Example 2-2. let the circuit of Fig. 2-2 (in textbook) contain a 120-V, 60-Hz fluorescent lamp wired in a high-power-factor configuration. Assume that the voltage and current are both sinusoids and in phase (unity power factor), as shown below. Find

1.) the DC value of this (periodic) voltage waveform?
2.) the instantaneous power?
3.) the average power?

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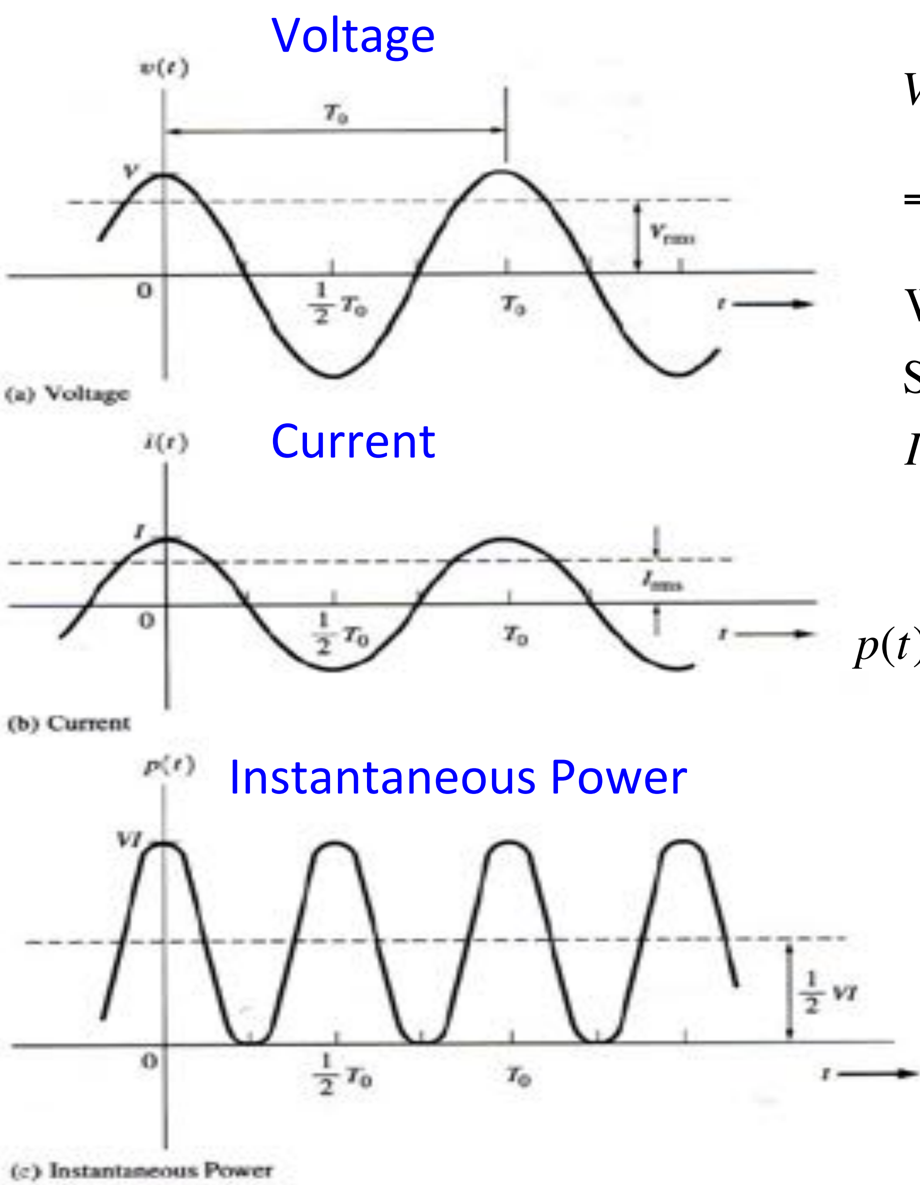
2.1 Properties of signals and noise



The DC values

$$V_{dc} = \langle v(t) \rangle = \langle V \cos \omega_0 t \rangle$$

$$= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} V \cos \omega_0 t \cdot dt = 0$$

Where, $\omega_0 = 2\pi / T_0$, and $f_0 = 1/T_0 = 60 Hz$

Similarly,

$$I_{dc} = 0$$

The instantaneous power

$$p(t) = (V \cos \omega_0 t)(I \cos \omega_0 t) = \frac{1}{2} VI(1 + \cos 2\omega_0 t)$$

The average power

$$P = \left\langle \frac{1}{2} VI(1 + \cos 2\omega_0 t) \right\rangle = \frac{VI}{2T_0} \int_{-T_0/2}^{T_0/2} (1 + \cos 2\omega_0 t)$$

$$= \frac{VI}{2}$$

2.1 Properties of signals and noise

RMS Value and Normalized Power

DEFNITION. The root-mean-square ***(RMS)*** value of $w(t)$ is

$$W_{rms} = \sqrt{\left\langle w^2(t) \right\rangle}$$

THEOREM. If a load is resistive (with unity power factor), the ***average power*** is

$$P = \frac{\left\langle v^2(t) \right\rangle}{R} = \left\langle i^2(t) \right\rangle R = \frac{V_{rms}^2}{R} = I_{rms}^2 R = V_{rms} I_{rms}$$

DEFNITION. The ***average normalized power*** is

$$P = \left\langle w^2(t) \right\rangle = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} w^2(t)\, dt$$

where $w(t)$ represents a real voltage or current waveform

2.1 Properties of signals and noise

Energy and Power Waveforms

DEFNITION. *w*(*t*) is a ***power waveform*** if and only if the ***normalize average power P*** is ***finite*** and ***nonzero*** (i.e. 0 < P < ∞)

DEFNITION. The ***total normalized power*** is

$$E = \lim_{T -> \infty} \int_{-T/2}^{T/2} w^2(t)\, dt$$

DEFNITION. *w*(*t*) is a ***energy waveform*** if and only if the ***total normalize energy E*** is **finite** and **nonzero** (i.e. 0 < E < ∞)

2.1 Properties of signals and noise

Energy and Power Waveforms

- If a waveform is classified as either one of these types, it cannot be of the other type
- If w(t) has finite energy, the power averaged over infinite time is zero
- If the power (averaged over infinite time) is finite, the energy is infinite
- However, mathematical functions can be found that have both infinite energy and infinite power and, consequently, cannot be classified into either of these two categories. ($w(t) = e^{-t}$).
- Physically realizable waveforms are of the energy type.
  We can find a finite power for these!!

2.1 Properties of signals and noise

Decibel

The **decibel** is a base 10 logarithmic measure of power ratios. For example, the ratio of the power level at the output of circuit compared with that at the input is often specified by the decibel gain instead of the actual ratio

DEFNITION 1. The **decibel gain** of a circuit is.

$$dB = 10\log\left(\frac{average \quad power \quad out}{average \quad power \quad in}\right) = 10\log\left(\frac{P_{out}}{P_{in}}\right)$$

2.1 Properties of signals and noise

Decibel

DEFNITION 2. The **decibel signal-to-noise ratio** .

$$(S/N)_{dB} = 10\log\left(\frac{P_{signal}}{P_{noise}}\right) = 10\log\left(\frac{\langle s^2(t)\rangle}{\langle n^2(t)\rangle}\right)$$

Since Signal power (S) = $\dfrac{\langle s^2(t)\rangle}{R} = \dfrac{V^2_{rms\ \ signal}}{R}$

noise power (N) = $\dfrac{\langle n^2(t)\rangle}{R} = \dfrac{n^2_{rms\ \ noise}}{R}$

This definition is equivalent to

$$(S/N)_{dB} = 20\log\left(\frac{V_{rms\ \ signal}}{V_{rms\ \ noise}}\right)$$

2.1 Properties of signals and noise

Decibel

DEFNITION 3. The **decibel power level with respect to 1 mW is**

$$dBm = 10\log(\frac{actual \quad power \quad level \quad (watts)}{10^{-3}})$$

=30 + 10log(Actual Power Level (watts))

Here the “m” in the dBm denotes a mlliwatt reference. When a 1-W reference level is used, the decibel level is denoted dBW; when a 1-kW reference level is used, the Decibel level is denoted dBK.

2.1 Properties of signals and noise

Phasors

DEFNITION. A complex number *c* is said to be a "phasor" if it is used to represent a sinusoidal waveform. That is,

$$w(t) = |c|\cos\left[\omega_0 t + \angle c\right] = \mathrm{Re}\left\{ c e^{j\omega_0 t} \right\}$$

where the phasor $c = |c|e^{j\angle c}$ and **Re{.}** denotes the real part of the complex quantity **{.}**.

The phasor (complex number) can also be written in either **Cartesian** form

$$c = x + jy$$

or **polar** form

$$c = |c|e^{j\varphi}$$