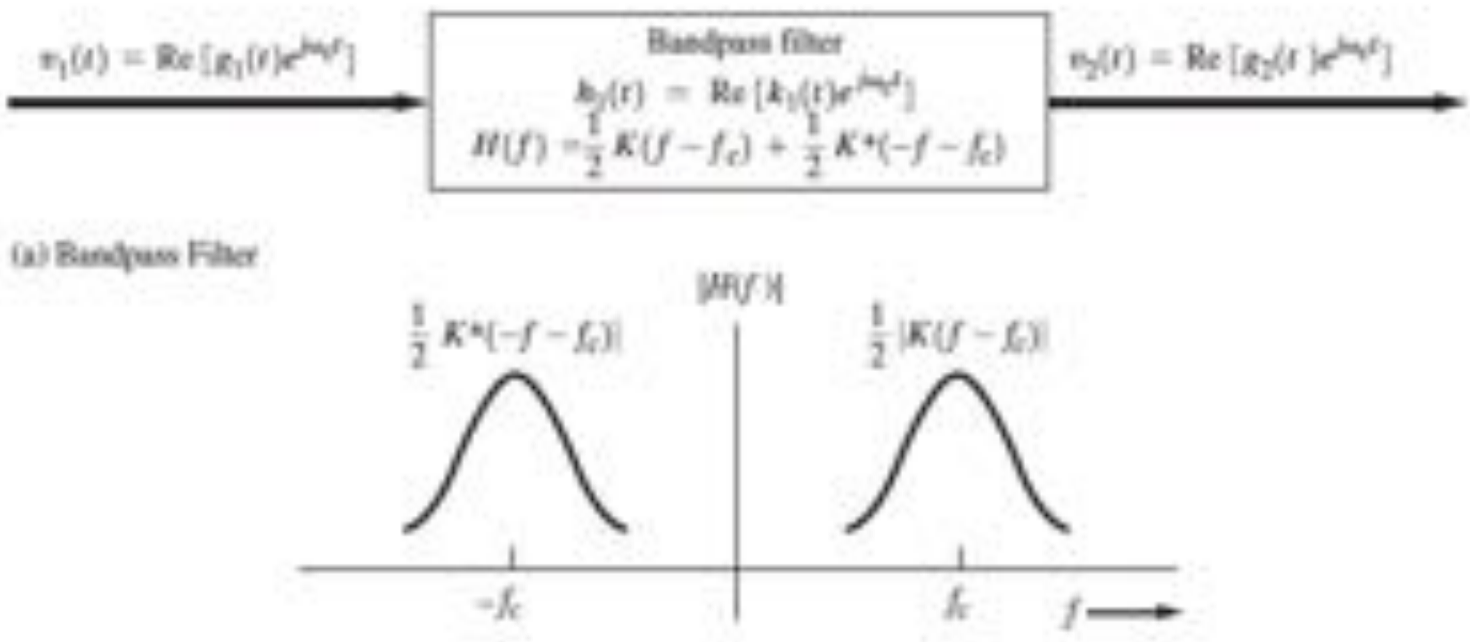
4.5. Bandpass Filtering and Linear Distortion

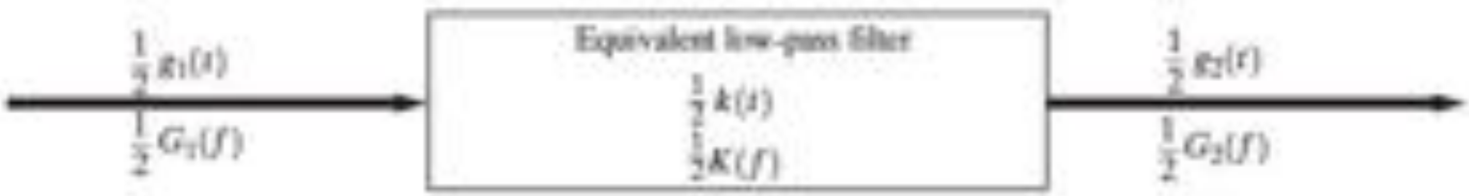
BandPass Filter



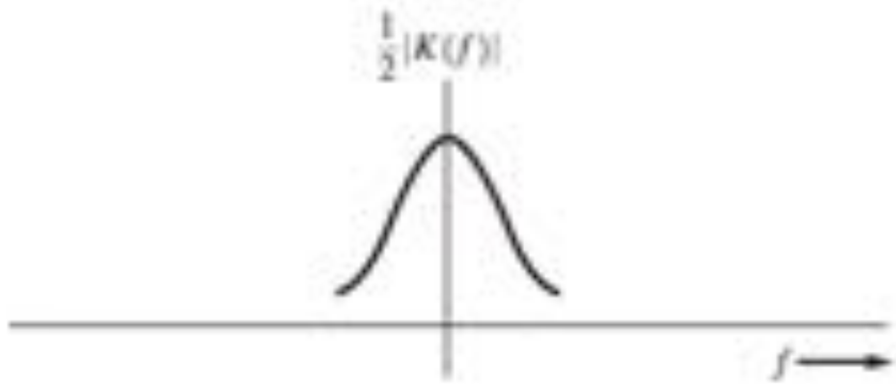
(b) Typical Bandpass Filter Frequency Response

4.5. Bandpass Filtering and Linear Distortion

Equivalent low-pass Filter



(c) Equivalent (Complex Impulse Response) Low-pass Filter



(d) Typical Equivalent Low-pass Filter Frequency Response

4.5. Bandpass Filtering and Linear Distortion

Equivalent Low-Pass Filter: Modeling a bandpass filter by using an ***equivalent low-pass filter*** that has a complex-valued impulse response.

Why?

Equations and analysis for equivalent low-pass filter are usually much less complicated than those for bandpass filters.

4.5. Bandpass Filtering and Linear Distortion

Theorem: The complex envelopes for the input, output, and impulse response of a bandpass filter are related by

$$\frac{1}{2}g_2(t) = \frac{1}{2}g_1(t) * \frac{1}{2}k(t);$$

Also,

$$\frac{1}{2}G_2(f) = \frac{1}{2}G_1(f)\frac{1}{2}K(f)$$

$g_1(t)$ – complex envelope of input

$k(t)$ – complex envelope of impulse response

4.5. Bandpass Filtering and Linear Distortion

Proof: Spectrum of the output is

$$V_2(f) = V_1(f)H(f)$$

Spectra of bandpass waveforms are related to that of their complex enveloped

$$\frac{1}{2}\left[G_2(f-f_c)+G_2^*(-f-f_c)\right] = \frac{1}{2}\left[G_1(f-f_c)+G_1^*(-f-f_c)\right]\frac{1}{2}\left[K(f-f_c)+K^*(-f-f_c)\right]$$

$$= \frac{1}{4}\begin{bmatrix} G_1(f-f_c)K(f-f_c)+G_1(f-f_c)K^*(-f-f_c) \\ +G_1^*(-f-f_c)K(f-f_c)+G_1^*(-f-f_c)K^*(-f-f_c) \end{bmatrix}$$

But $G_1(f-f_c)K^*(-f-f_c)=0,$

$G_1^*(-f-f_c)K(f-f_c)=0.$

$$\left[\frac{1}{2}G_2(f-f_c)\right]+\left[\frac{1}{2}G_2^*(-f-f_c)\right] = \left[\frac{1}{2}G_1(f-f_c)\frac{1}{2}K(f-f_c)\right]+\left[\frac{1}{2}G_1^*(-f-f_c)\frac{1}{2}K^*(-f-f_c)\right]$$

4.5. Bandpass Filtering and Linear Distortion

Linear Distortion

For distortionless transmission of bandpass signals, the channel transfer function $H(f) = |H(f)|e^{j\theta(f)}$ should satisfy the following requirements:

- The amplitude response is constant

$$|H(f)| = A$$

A- positive constant

- The derivative of the phase response is constant

$$\theta(f) = \angle H(f) = -2\pi f T_d$$

$$-\frac{1}{2\pi}\frac{d\theta(f)}{df} = T_g$$

T_g – complex envelope delay

$\theta(f) = \angle H(f)$

Integrating the above equation, we get

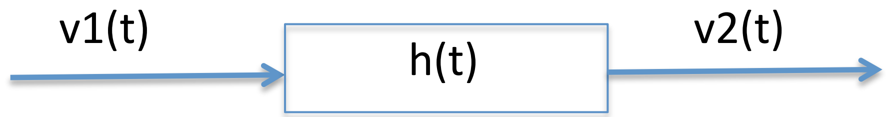
$$\theta(f) = -2\pi f T_g + \theta_0$$

θ_0 – phase shift constant

Are these requirements sufficient for distortionless transmission?

4.5. Bandpass Filtering and Linear Distortion

Linear Distortion



$$v_1(t) = x(t)\cos\omega_c t - y(t)\sin\omega_c t$$

$$\downarrow$$

$$H(f) = Ae^{j(-2\pi f T_g + \theta_0)} = \left(Ae^{j\theta_0}\right)e^{-j2\pi f T_g}$$

$$\downarrow$$

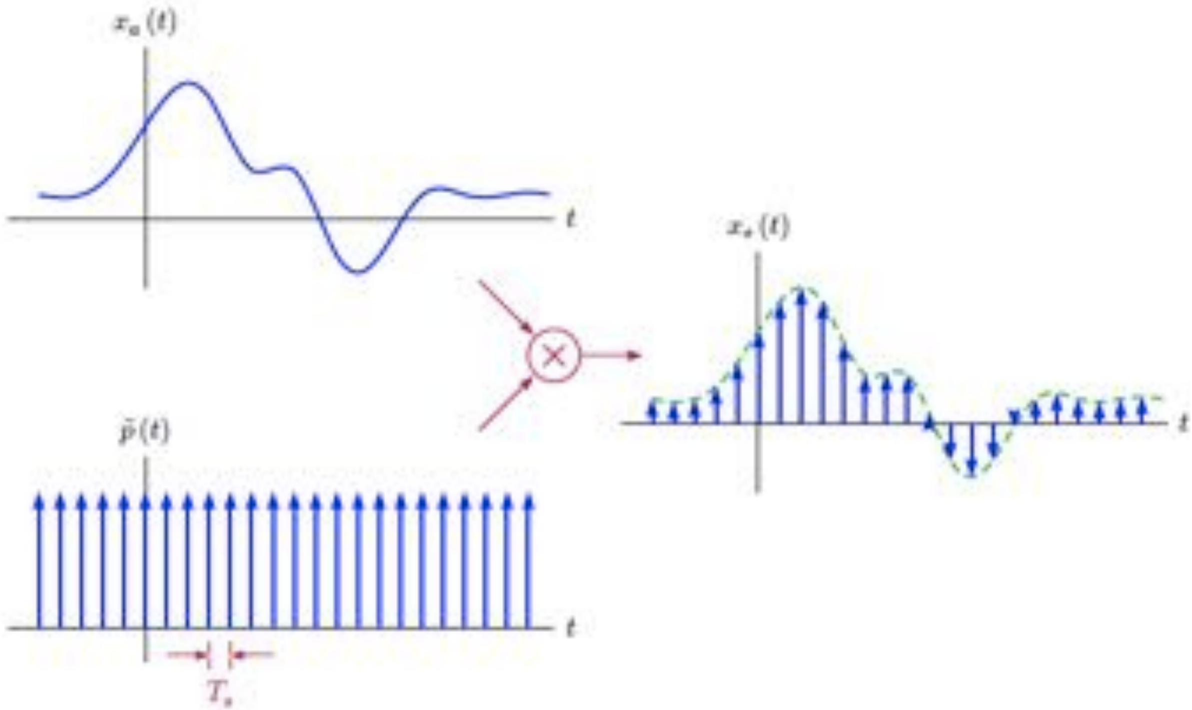
$$v_2(t) = Ax(t - T_g)\cos[\omega_c(t - T_d)] - Ay(t - T_g)\sin[\omega_c(t - T_d)]$$

T_g: group time delay
T_d: carrier time delay, or phase delay

$$\theta(f_c) = -2\pi f_c T_d\ ; \quad \theta(f_c) - \text{carrier phase shift} \Rightarrow \mathrm{T_d} - \text{phase delay}$$

4.6. Bandpass Sampling Theorem

Sampling Theorem:



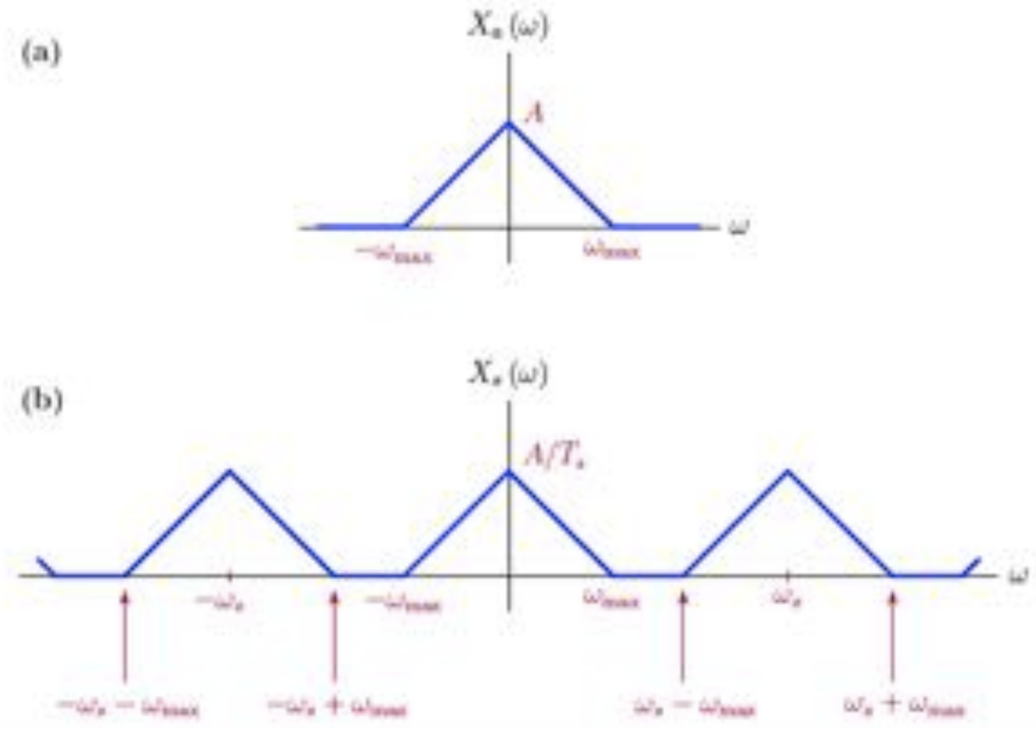
$$\tilde{p}(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

$$x_s(t) = x_a(t)\,\tilde{p}(t) = x_a(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s) = \sum_{n=-\infty}^{\infty} x_a(nT_s)\,\delta(t - nT_s)$$

4.6. Bandpass Sampling Theorem

Sampling Theorem:

$$x_s(t) = \sum_{n=-\infty}^{\infty} x_a(nT_s)\,\delta(t - nT_s) \qquad \Rightarrow \qquad X_s(\omega) = \frac{1}{T_s}\sum_{k=-\infty}^{\infty} X_a(\omega - k\omega_s)$$



$$\omega_s \geq 2\,\omega_{max}$$

4.6. Bandpass Sampling Theorem

Theorem:

If a (real) **bandpass** waveform has a non-zero spectrum only over the interval $f_1 < |f| < f_2$, where the transmission bandwidth B_T is taken to be same as absolute BW, $B_T = f_2 - f_1$, then the waveform may be reproduced by its sample values if the sampling rate is:

$$f_s >= 2B_T$$

4.6. Bandpass Sampling Theorem

Bandpass Dimensionality Theorem:

Assume that a **bandpass** waveform has a non-zero spectrum only over the interval $f_1 < |f| < f_2$, where the transmission bandwidth B_T is taken to be same as absolute BW, $B_T = f_2 - f_1$, and $B_T << f_1$. then the waveform may be completely specified over a T_0-second interval by independent pieces of information. N is said to be the number of dimensions required to specify the waveform

$$N = 2B_T T_0$$

4.7. Received Signal Plus Noise

The signal out of the transmitter

$$s(t) = \operatorname{Re}\left[g(t)e^{j\omega_c t}\right]$$

If the channel is LTI , then received signal + noise

$$r(t) = s(t) * h(t) + n(t)$$

Where *h(t)* is the impulse response, and *n(t)* is the noise at the receiver input

4.7. Received Signal Plus Noise

If the channel is distorntionless

Signal + noise at the receiver input

A – gain of the channel

$$r(t) = \mathrm{Re}\left[A g\left(t - T_g\right) e^{j(\omega_c t + \theta(f_c))} + n(t) \right]$$

$\theta(f_c)$ - carrier phase shift caused by the channel,
T_g – channel group delay.

Receiver circuits are designed to estimate the $\theta(f_c)$ and T_g

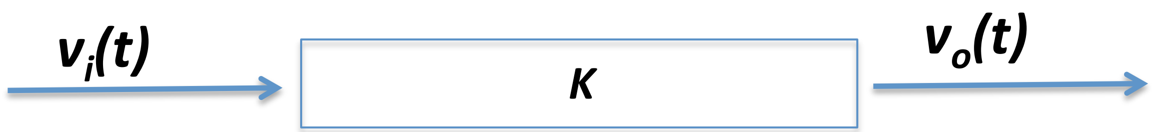
Signal + noise at the receiver input

$$r(t) = \mathrm{Re}\left[g(t) e^{j\omega_c t} \right] + n(t)$$

4.9. Nonlinear Distortion

Linear Distortion

Linear Amplifier



$$v_o(t) = Kv_i(t)$$

K is the voltage gain of the amplifier

4.9. Nonlinear Distortion

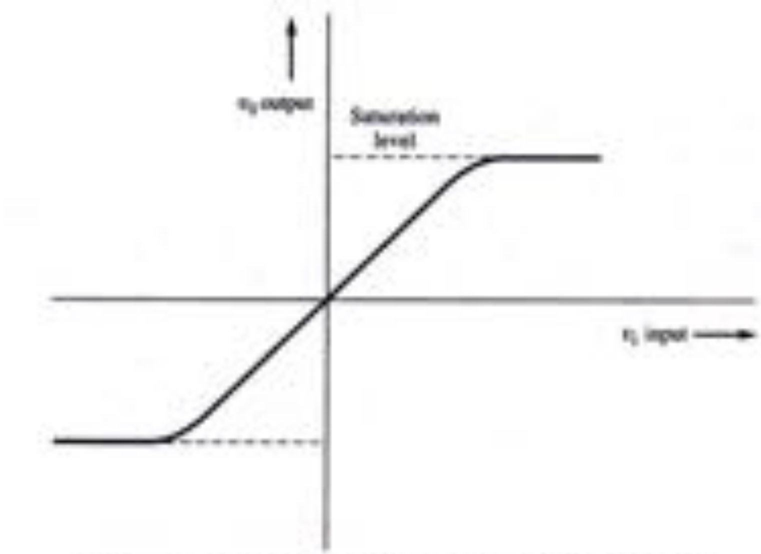


Figure 4-5 Nonlinear amplifier output-to-input characteristic.

In practice, amplifier output becomes saturated as the amplitude of the input signal is increased.

Nonlinear Amplifier

$v_i(t)$ → $K_0, K_1, K_2, \ldots$ → $v_o(t)$

$$v_o(t) = K_0 + K_1 v_i + K_2 v_i^2 + \ldots = \Sigma K_n v_i^n$$

Where

$$K_n = \frac{1}{n!}\left(\frac{d^n v_0}{dv_i^n}\right)\Bigg|_{v_i=0}$$

K_0 is the output DC offset level, $K_1 v_i$ is the first-order (linear) term, others are nonlinear term

4.9. Nonlinear Distortion

Harmonic Distortion associated with the amplifier output can be determined by applying a single sinusoidal test tone to the amplifier input.

$$v_i(t) = A_0 \sin \omega_0 t$$

Then the second-order output term is

$$K_2 v_i^2 = K_2 (A_0 \sin \omega_0 t)^2 = \frac{K_2 A_0^2}{2}(1 - \cos 2\omega_0 t)$$

2nd Harmonic Distortion with amplitude of $\frac{K_2 A_0^2}{2}$

In general, for a single-tone input, the output will be

$$v_{out}(t) = V_0 + V_1 \cos(\omega_0 t + \varphi_1) + V_2 \cos(2\omega_0 t + \varphi_2) + V_3 \cos(3\omega_0 t + \varphi_3) + \cdots$$

V_n – peak value of the output at the frequency nf_0

4.9. Nonlinear Distortion

The **Percentage Total Harmonic Distortion (THD)** of an amplifier is defined by

$$\boxed{\mathrm{THD}(\%) = \frac{\sqrt{\sum_{\mathrm{n}=2}^{\infty} V_n^2}}{\mathrm{V}_1} \times 100}$$

4.9. Nonlinear Distortion

Intermodulation distortion (IMD) of the amplifier can be determined by a two-tone input

$$v_i(t) = A_1 \sin \omega_1 t + A_2 \sin \omega_2 t$$

the second-order output term is:

$$K_2\left(A_1 \sin \omega_1 t + A_2 sn \omega_2 t\right)^2 = K_2\left(A_1^2 \sin^2 \omega_1 t + 2 A_1 A_2 \sin \omega_1 t \sin \omega_2 t + A_2^2 \sin^2 \omega_2 t\right)$$

$$= \underbrace{\boxed{K_2 A_1^2 \sin^2 \omega_1 t + K_2 A_2^2 \sin^2 \omega_2 t}}_{\textbf{Harmonic distortion at } 2f_1 \text{ \& } 2f_2} + \underbrace{\boxed{K_2 2 A_1 A_2 \sin \omega_1 t \sin \omega_2 t}}_{\textbf{IMD}}$$

Second-order IMD is:

$$2K_2 A_1 A_2 \sin \omega_1 t \sin \omega_2 t = K_2 A_1 A_2 \left\{\cos\left[(\omega_1 - \omega_2)t\right] - \cos\left[(\omega_1 + \omega_2)\right]\right\}$$