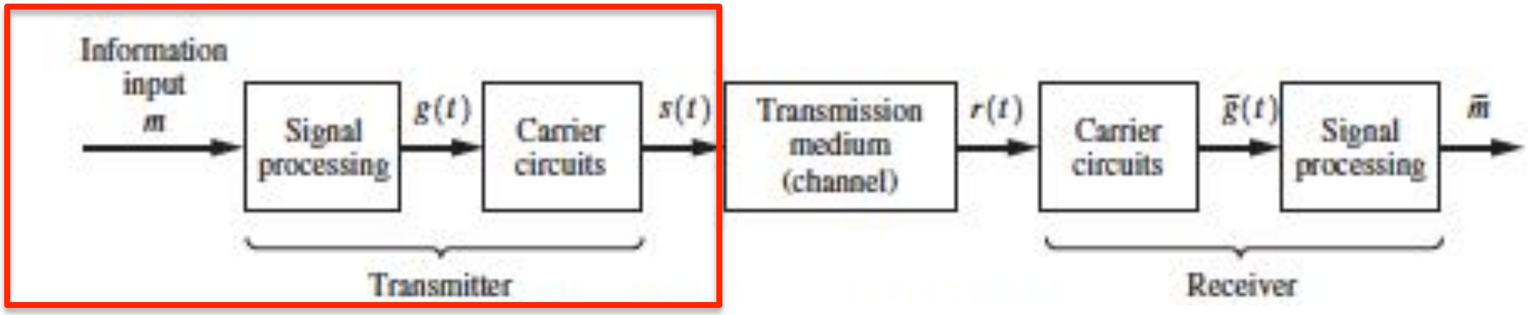
5.1. Amplitude Modulation



The **complex envelope** of an AM signal is given by

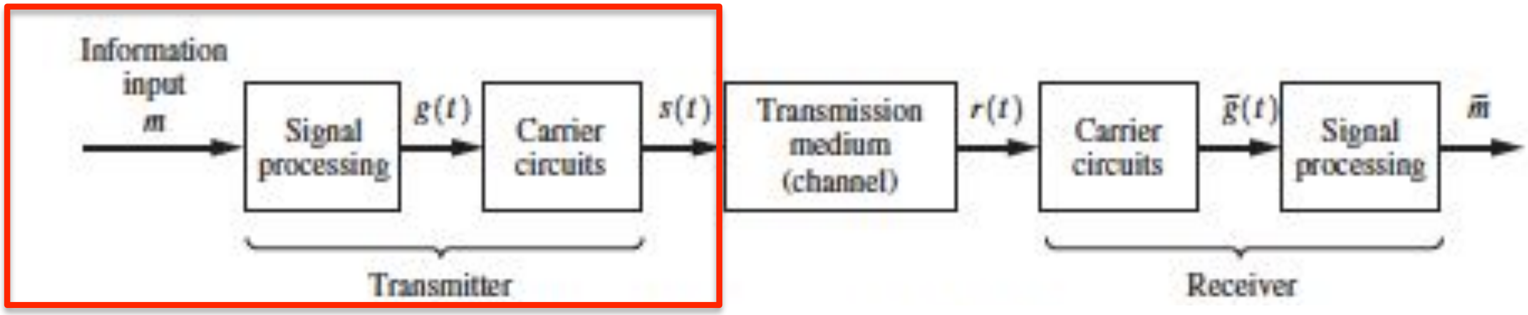
$$g(t) = A_c[1 + m(t)]$$

where the constant A_c has been included to specify the power level and m(t) is the modulating signal (analog or digital).

The **amplitude modulated** signal ***s(t)***

$$s(t) = A_c[1 + m(t)]\cos w_c t$$

5.1. Amplitude Modulation



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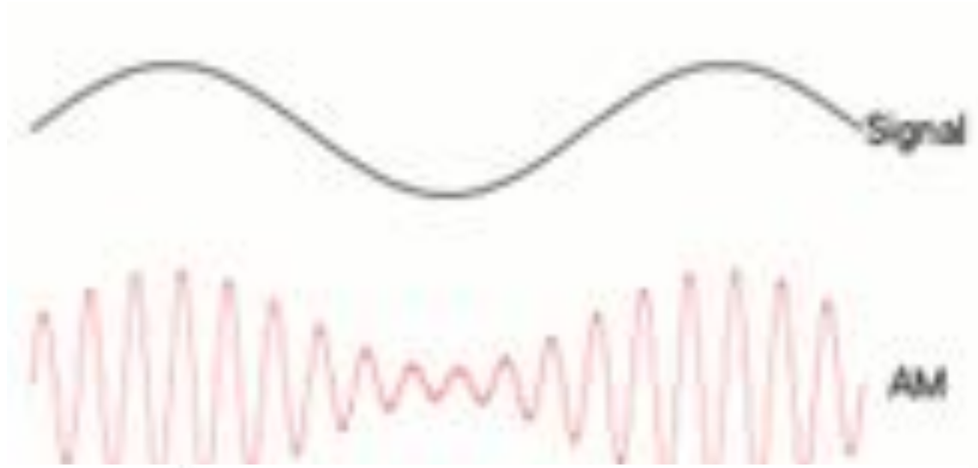
where the constant A_c has been included to specify the power level and $m(t)$ is the modulating signal (analog or digital).

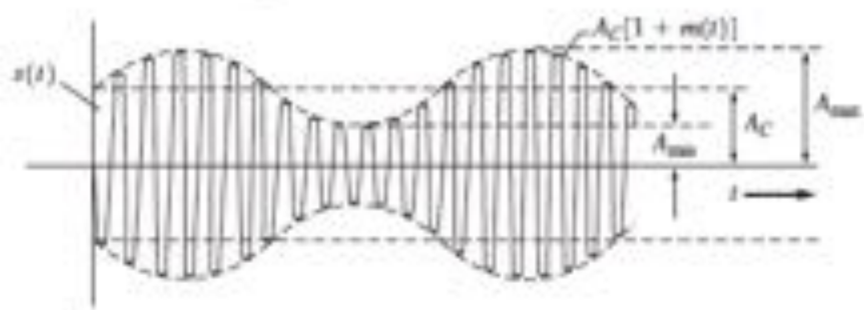
The **spectrum** of the AM signal ***S(f)***

$$S(f) = \frac{A_c}{2}\left[\delta(f - f_c) + M(f - f_c) + \delta(f + f_c) + M(f + f_c)\right]$$

5.1. Amplitude Modulation



(a) Sinusoidal Modulating Wave



(b) Resulting AM Signal

m(t)

$$v(t) = \mathrm{Re}\left\{g(t)e^{jw_c t}\right\}$$

$$s(t) = A_c[1 + m(t)]\cos w_c t$$

5.1. Amplitude Modulation

DEFINATIONS

The percentage of ***positive modulation*** on an AM signal is

100% positive modulation = $(A_{max} - A_c)/A_c$*100 = max[$m(t)$]*100

The percentage of ***negative modulation*** on an AM signal is

100% negative modulation = $(A_c - A_{min})/A_c$*100 = -min[$m(t)$]*100

The percentage of ***overall modulation*** on an AM signal is

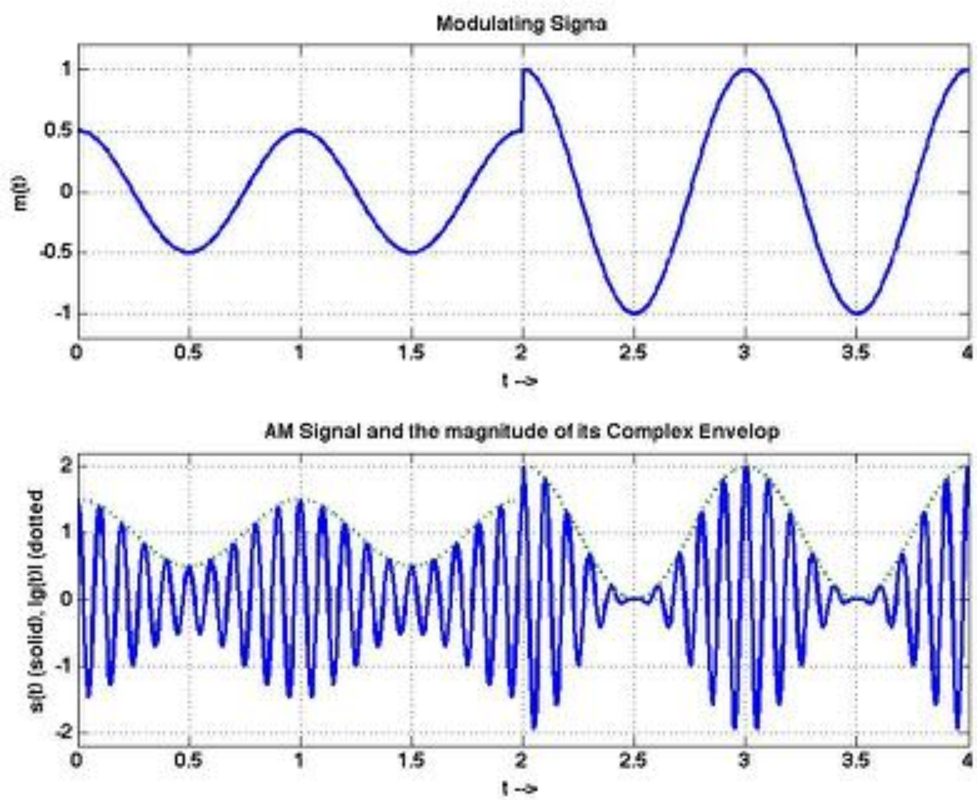
100% modulation = $(A_{max} - A_{min})/2A_c$*100

= (max[$m(t)$]-min[$m(t)$])/2*100

5.1. Amplitude Modulation

Example 5.1 AM Signal with 50% and 100% Modulation

Let an AM signal with a carrier frequency of 10 HZ be modulated with a sinusoidal signal having a frequency of 1 Hz. Furthermore, let the percentage of modulation be 50% over the time interval $0 < t < 2$ sec and then changed to 100% over $2 < t < 4$ sec. Plot the AM signal waveform over the interval of $0 < t < 4$ sec.



5.1. Amplitude Modulation

Four-quadrant multiplier :The percentage of overall modulation can be over 100% when the A_{min} shows negative value.

Two-quadrant multiplier :The percentage of overall modulation has maximum value of 100%.

$$s(t) = \begin{cases} A_c\left[1+m(t)\right]\cos w_c t, & m(t) \geq -1 \\ 0, & m(t) < -1 \end{cases}$$

5.1. Amplitude Modulation

Normalized average power of the AM signal is:

$$\left\langle s^2(t) \right\rangle = \frac{1}{2}\left\langle \left|g(t)\right|^2 \right\rangle = \frac{1}{2} A_c \left\langle \left[1 + m(t)\right]^2 \right\rangle$$

$$= \frac{1}{2} A_c^2 + A_c^2 \left\langle m(t) \right\rangle + \frac{1}{2} A_c^2 \left\langle m^2(t) \right\rangle$$

If there is no DC level in the modulation, the normalized power of the AM signal is:

$$\left\langle s^2(t) \right\rangle = \frac{1}{2} A_c^2 + \frac{1}{2} A_c^2 \left\langle m^2(t) \right\rangle$$

discrete carrier power

sideband power

5.1. Amplitude Modulation

DEFINATIONS

In AM signal, only the sideband components convey information, therefore, the ***modulation efficiency*** is the percentage of the total power of the modulated signal that conveys information.

$$E = \frac{\left\langle m^2(t) \right\rangle}{1 + \left\langle m^2(t) \right\rangle} \times 100\%$$

The ***higest efficiency*** can be generated is 50% for a pure AM signal when square-wave modulation is used.

5.1. Amplitude Modulation

DEFINATIONS

The ***normalized peak envelope power (PEP)*** is.

$$P_{PEP} = \frac{A_c^2}{2}\left\{1 + \max\left[m(t)\right]\right\}^2$$

The ***voltage spectrum*** is:

$$S(f) = \frac{A_c}{2}\left[\delta(f - f_c) + M(f - f_c) + \delta(f + f_c) + M(f + f_c)\right]$$

5.3. Double-Sideband Suppressed Carrier

A **double-sideband suppressed carrier (DSB-SC)** signal ***s*(*t*)** is and **AM** signal that has a suppressed discrete carrier.

$$s(t) = A_c m(t) \cos w_c t$$

The **spectrum** for DSB-SC signal ***S*(*f*)** is

$$S(f) = \frac{A_c}{2}\left[M(f - f_c) + M(f + f_c)\right]$$

5.6. Phase Modulation and Frequency Modulation

Angle-modulated Signal.

$$\left.\begin{aligned} s(t) &= \mathrm{Re}\left\{g(t)e^{jw_c t}\right\} \\ g(t) &= A_c e^{j\theta(t)} \end{aligned}\right\} \Rightarrow s(t) = A_c \cos\left[w_c t + \theta(t)\right]$$

Where $R(t) = \left|g(t)\right| = A_c$, is a constant, $\theta(t)$ is a linear function of $m(t)$.

The **Phase Modulation (PM) and Frequency Modulation (FM)** are special cases of angle-modulated signaling

5.6. Phase Modulation and Frequency Modulation

Phase Modulation (PM)

$$\theta(t) = D_p m(t) \quad \Rightarrow \quad s(t) = A_c \cos[w_c t + D_p m(t)]$$

D_p , ***the phase sensitivity of the phase modulator,*** is a ***constant***

Unit: radians/volt-seconds

Frequency Modulation (FM)

$$\theta(t) = D_f \int_{-\infty}^{t} m(\sigma) d\sigma \quad \Rightarrow \quad s(t) = A_c \cos[w_c t + D_f \int_{-\infty}^{t} m(\sigma) d\sigma]$$

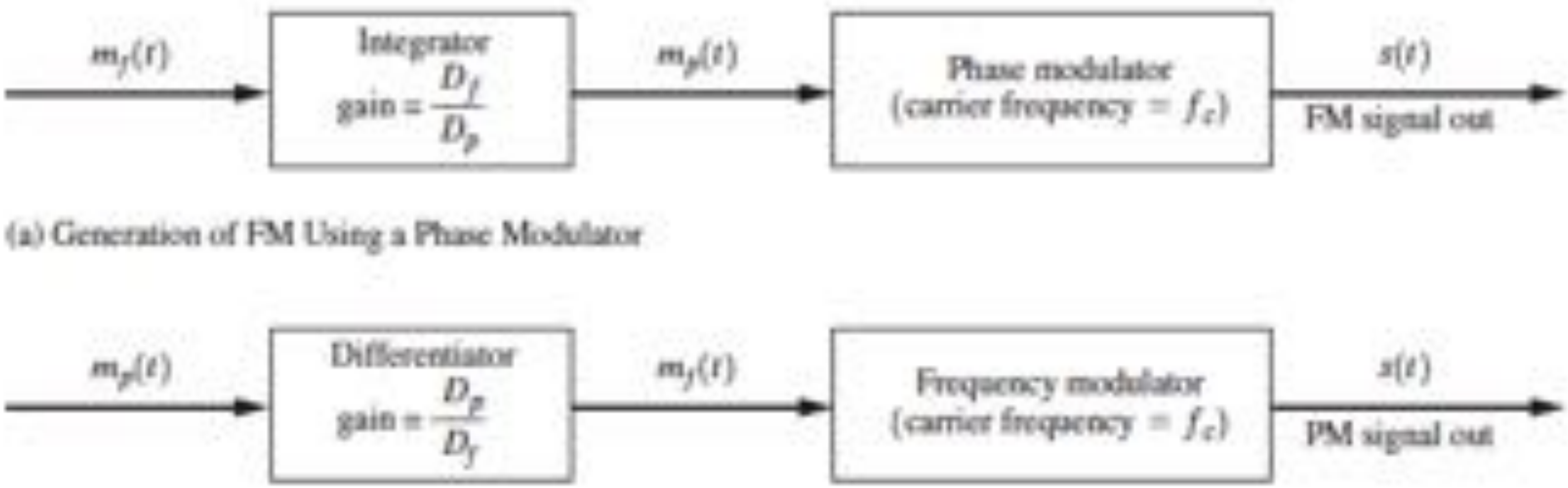
D_f , ***the frequency sensitivity of the phase modulator,*** is a ***constant***

Unit: radians/volt-seconds

5.6. Phase Modulation and Frequency Modulation

Relation between **Phase Modulation (PM)** and **Frequency Modulation (FM)**

$$m_f(t) = \frac{D_p}{D_f}\left[\frac{dm_p(t)}{dt}\right] \iff m_p(t) = \frac{D_f}{D_p}\int_{-\infty}^{t} m_f(\sigma)\,d\sigma$$



(a) Generation of FM Using a Phase Modulator

(b) Generation of PM Using a Frequency Modulator

5.6. Phase Modulation and Frequency Modulation

DEFINATIONS

If a bandpass signal is represented by

$$s(t) = \mathrm{R}(t)\cos\psi(t)$$

Where $\psi(t) = w_c t + \theta(t)$, then the instantaneous frequency of $s(t)$ is

$$f_i(t) = \frac{1}{2\pi} w_i(t) = \frac{1}{2\pi}\left[\frac{d\psi(t)}{dt}\right]$$

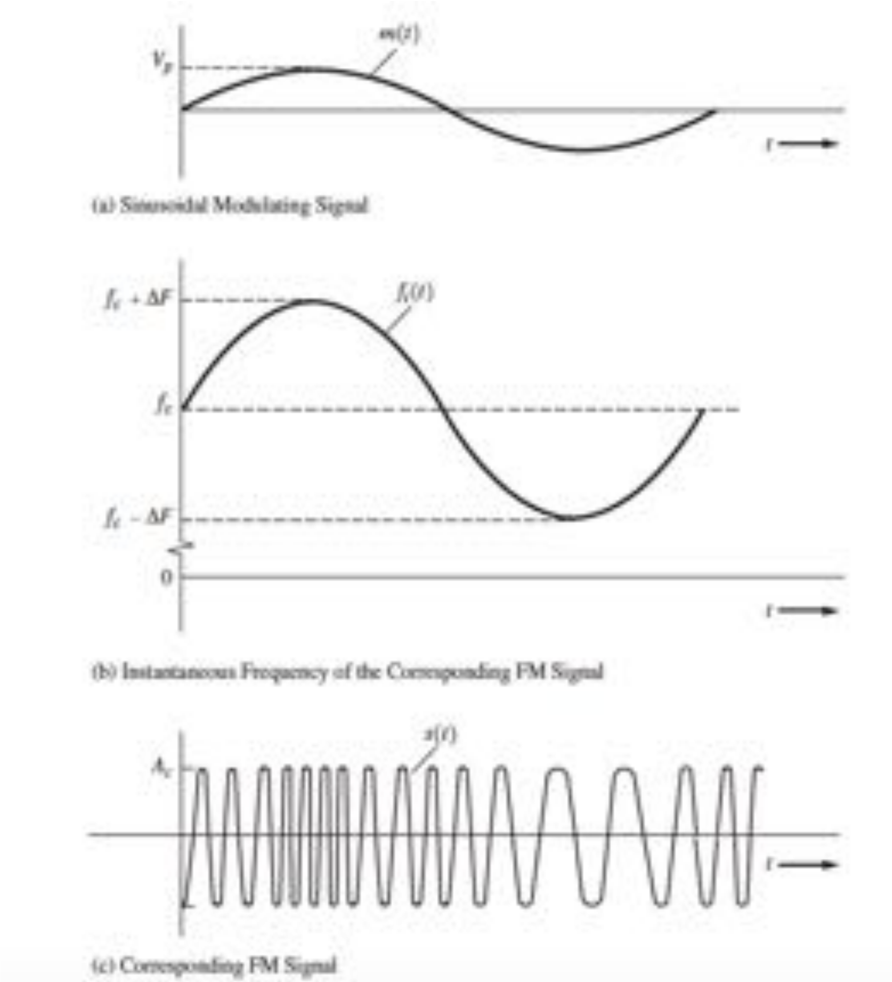
Or $$f_i(t) = f_c + \frac{1}{2\pi}\left[\frac{d\theta(t)}{dt}\right]$$

For FM case, the ***instantaneous frequency*** is

$$f_i(t) = f_c + \frac{1}{2\pi} D_f m(t)$$

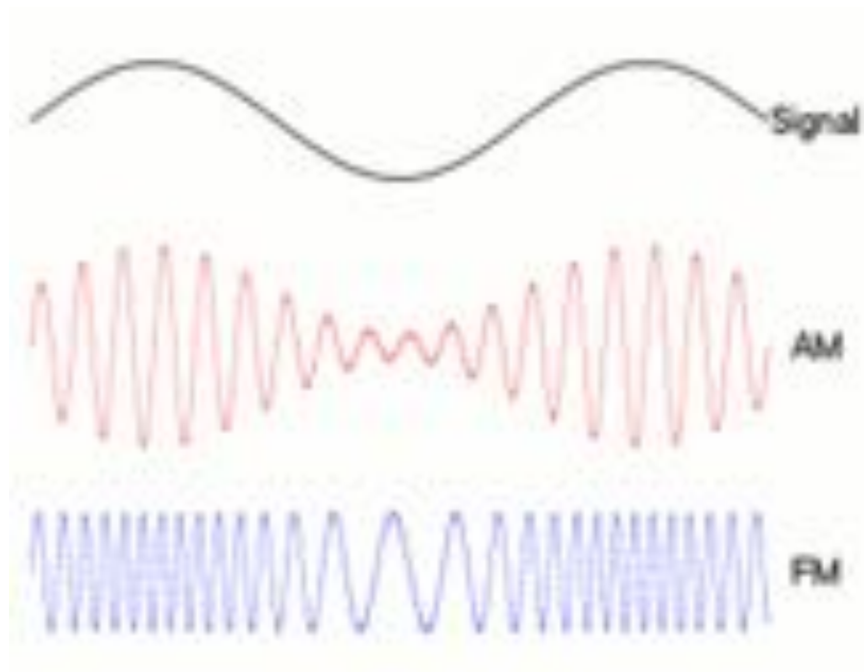
It is the frequency that is present at a particular instant of time

5.6. Phase Modulation and Frequency Modulation


(a) Sinusoidal Modulating Signal

(b) Instantaneous Frequency of the Corresponding FM Signal

(c) Corresponding FM Signal



$$s(t) = A_c \cos[w_c t + D_f \int_{-\infty}^{t} m(\sigma)\, d\sigma]$$

5.6. Phase Modulation and Frequency Modulation

Frequency Modulation

The ***frequency deviation*** from the carrier frequency is:

$$f_d(t) = f_i(t) - f_c = \frac{1}{2\pi}\left[\frac{d\theta(t)}{dt}\right]$$

The ***peak frequency deviation*** is:

$$\Delta F = \max\left\{\frac{1}{2\pi}\left[\frac{d\theta(t)}{dt}\right]\right\}$$

For FM signal, the ***peak frequency deviation*** is related to the peak Modulating voltage by:

$$\Delta F = \frac{1}{2\pi} D_f V_p \qquad \text{where} \qquad V_p = \max\left[m(t)\right]$$

5.6. Phase Modulation and Frequency Modulation

Phase Modulation

The ***peak phase deviation*** is:

$$\Delta\theta = \max[\theta(t)] = D_p \max[m(t)] = D_p V_p$$

5.6. Phase Modulation and Frequency Modulation

The ***frequency modulation index*** is:

$$\beta_f = \frac{\Delta F}{B}$$

Where B is the bandwidth of the modulating signal, which, for the case of sinusoidal modulation, is f_m, the frequency of the sinusoid.

The ***phase modulation index*** is:

$$\beta_p = \Delta\theta$$

5.6. Phase Modulation and Frequency Modulation

Spectra of Angle-Modulated Signals

$$S(f) = \frac{1}{2}\left[G(f - f_c) + G^*(-f - f_c)\right]$$

where:

$$G(f) = \Im[g(t)] = \Im[A_c e^{j\theta(t)}]$$

5.6. Phase Modulation and Frequency Modulation

FM VS AM

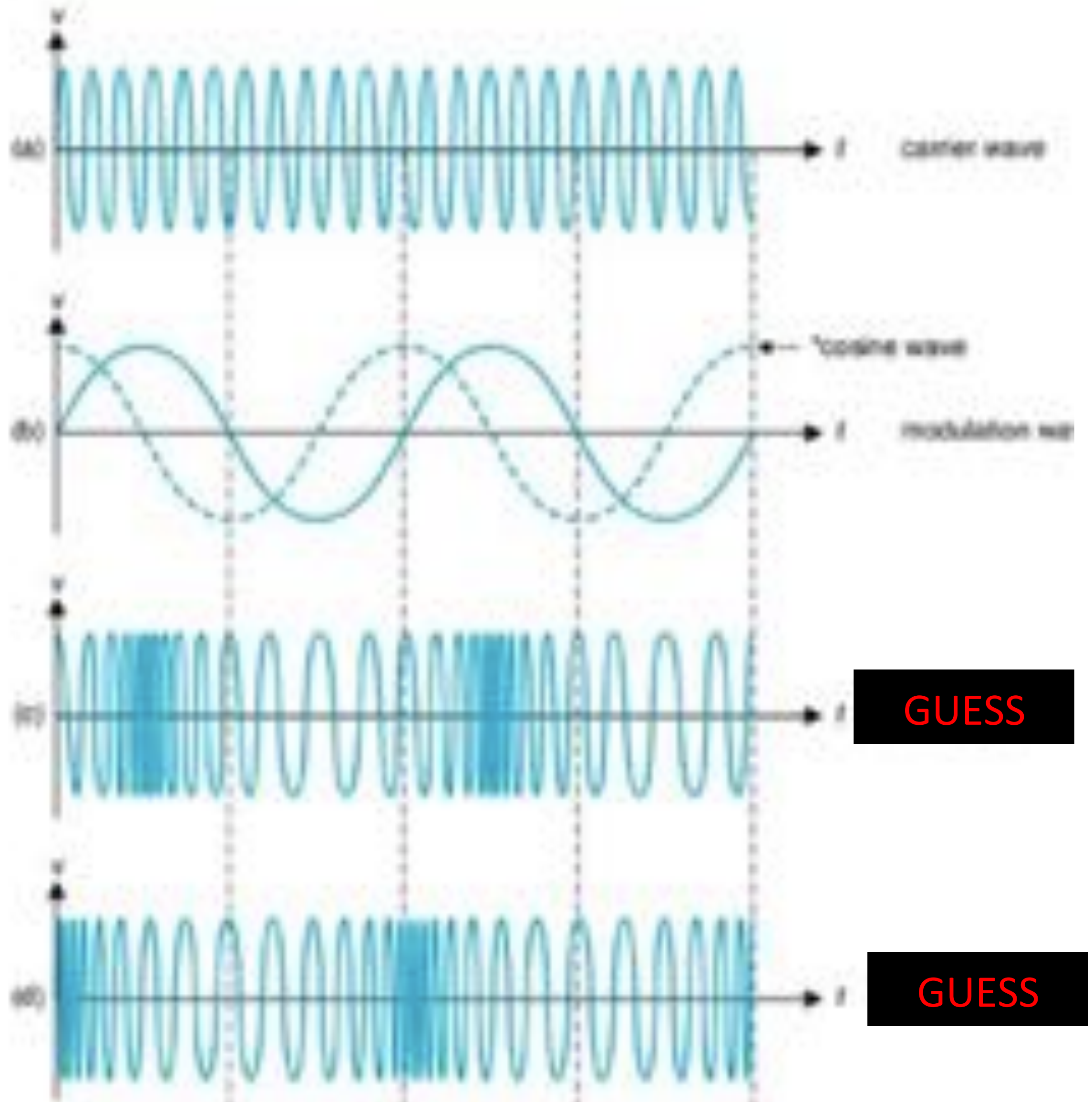
- FM is considered to be superior to AM.
- Transmission efficiency.
  - AM use linear amplifier to produced the final RF signal
  - AM has constant carrier amplitude so it is not necessary to use linear amplifier
- Fidelity (capture effect)
  - The stronger signal will be capture and eliminate the weaker
  - In AM, the weaker signal can be heard in the background
- Noise immunity (noise reduction)
  - Constant carrier amplitude
  - FM receiver have limiter circuit

5.6. Phase Modulation and Frequency Modulation

Disadvantages of FM

- ✧ Use too much spectrum space.
- ✧ Requiring a wider bandwidth.
- ✧ More complex circuitry

5.6. Phase Modulation and Frequency Modulation



5.6. Phase Modulation and Frequency Modulation

Bessel Function

5.6. Phase Modulation and Frequency Modulation

Narrowband Angle Modulation

When $\theta(t)$ is restricted to a small value, say, $|\theta(t)| < 0.2$ rad, the complex envelope $g(t) = A_c e^{j\theta}$ may be approximated by a Taylor's series in which only the first two terms are used.

$$g(t) \approx A_c[1 + j\theta(t)]$$

$$s(t) = \mathrm{Re}\left\{g(t)e^{jw_c t}\right\}$$

$$\Longrightarrow \quad s(t) = A_c \cos w_c t - A_c\theta(t)\sin w_c t$$

discrete carrier term

sideband term

5.6. Phase Modulation and Frequency Modulation

Narrowband Angle Modulation

The spectrum of narrowband angle modulation

$$S(f) = \frac{A_c}{2}\left\{\left[\delta(f - f_c) + \delta(f + f_c)\right] + j\left[\Theta(f - f_c) + \Theta(f + f_c)\right]\right\}$$

where $$\Theta(f) = \Im[\theta(t)] = \begin{cases} D_p M(f), & PM \\ \dfrac{D_f}{j2\pi f} M(f), & FM \end{cases}$$

5.6. Phase Modulation and Frequency Modulation

Wideband Frequency Modulation (WBFM)

THEOREM: For WBFM signaling, where

$$s(t) = A_c \cos[w_c t + D_f \int_{-\infty}^{t} m(\sigma)\, d\sigma]$$