3.7 Impulse Response and Convolution

$$h[n] = Sys\{\delta[n]\}$$

$$\delta[n] \longrightarrow \boxed{\text{Sys\{..\}}} \longrightarrow h[n]$$

For a DTLTI system: the impulse response also constitutes a complete description of the system

- ✧ Finding the impulse response of a DTLTI system amounts to finding the forced response of the system when the forcing function is a unit-impulse:

$$x[n] = \delta[n]$$

- ✧ In the case of difference equation with no feedback, the impulse response is found by direct substitution of the input signal $x[n] = \delta[n]$ into the difference equation.
- ✧ In the case of difference equation has feedback, then it is easier to first find the unit-step response of the system as the intermediate step, and to determine the impulse response from the unit-step response.
- ✧ A more practical method will be presented in Chapter 8 through the use of z-transform.

3.7 Impulse Response and Convolution

<u>Example 3.17 Impulse response of moving average filters</u>

Finding the impulse response of the length-2, length-4 and length-N moving average filters.

$$h_2[n] = \{1/2, 1/2\}$$

$$h_4[n] = \{1/4, 1/4, 1/4, 1/4\}$$

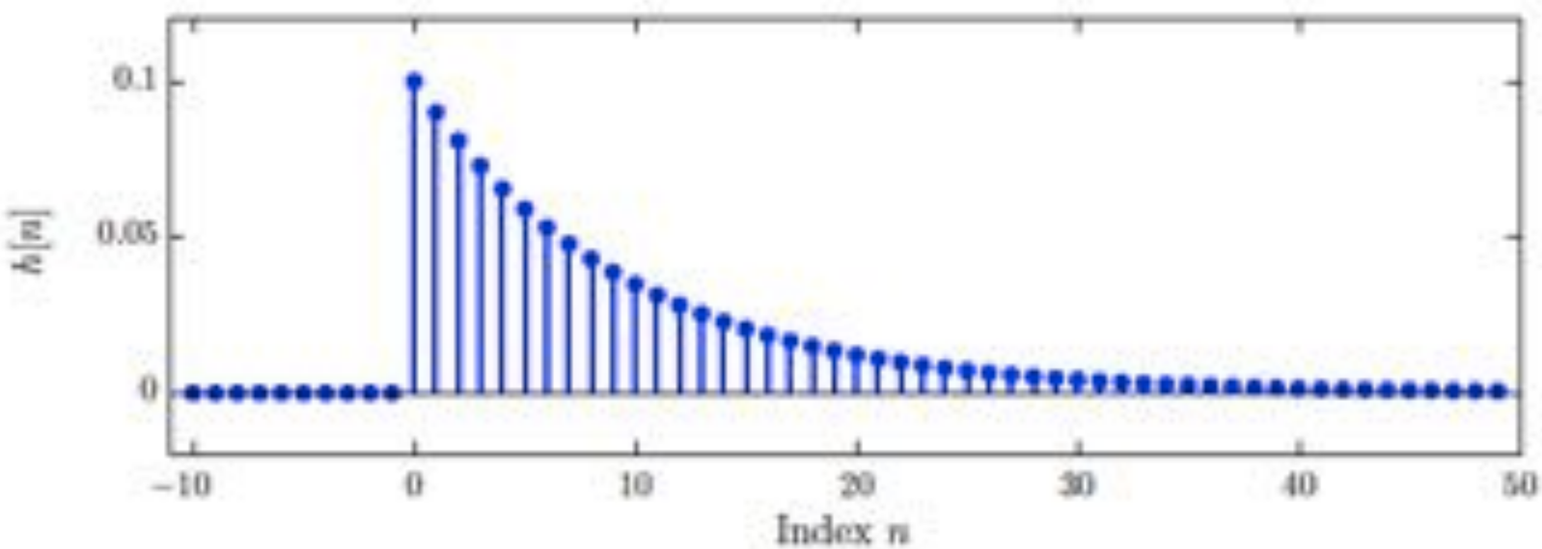
$$h_N[n] = \begin{cases} \dfrac{1}{N} & n = 0, 1, \dots, N-1 \\ 0, & otherwise \end{cases}$$

3.7 Impulse Response and Convolution

Example 3.18 Impulse response of exponential smoother

Finding the impulse response of the exponential smoother of

$$y[n] = (1-\alpha)\, y[n-1] + \alpha x[n] \text{ with } y[-1] = 0$$



3.7.2 Convolution Operation for DTLTI systems

The output signal ***y[n]*** of a DTLTI system is equal to the ***convolution*** of Its impulse response ***h[n]*** with the input signal ***x[n]***

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$= h[n] * x[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

The symbol * represents ***convolution operator***

3.7.2 Convolution Operation for DTLTI systems

Example 3.19 A simple discrete-time convolution example

A discrete-time system is described through the impulse response

$$h[n] = \{\underset{\substack{\uparrow \\ n=0}}{4}, 3, 2, 1\}$$

Use the convolution operation to find the response of the system to the input signal

$$x[n] = \{\underset{\substack{\uparrow \\ n=0}}{-3}, 7, 4\}$$

3.7.2 Convolution Operation for DTLTI systems

Example 3.20 A simple discrete-time convolution example

A discrete-time system is described through the impulse response

$$h[n] = \{\underset{\substack{\uparrow \\ n=N_2}}{4}, 3, 2, 1\}$$

Use the convolution operation to find the response of the system to the input signal

$$x[n] = \{\underset{\substack{\uparrow \\ n=N_1}}{-3}, 7, 4\}$$

Assume the starting indices N_1 and N_2 are known constant

3.8 Causality in Discrete-time Systems

A system is said to be ***causal*** if the current value of the output signal depends only on current and past values of the input signal, but not on its future values

CTLTI system

$$y[n] = h[n] * x[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

For $k < 0$, the term *x[n-k]* refers to future values of the input signal

Causality in DTLTI systems

For a DTLTI system to be causal, the impulse response of the system must be equal to zero for all negative values of its argument.

$$h[n] = 0 \quad \text{for all } n < 0$$

3.9 Stability in Discrete-time Systems

A system is said to be ***stable*** in the ***bounded-input bounded-output (BIBO)*** sense if any bounded input signal is guaranteed to produce a bounded output signal

$$|x[n]| < B_x < \infty \quad \text{Implies that} \quad |y[n]| < B_y < \infty$$

For $\lambda < 0$, the term $x(t-\lambda)$ refers to future values of the input signal

Stability in DTLTI systems

For a DTLTI system to be stable, the impulse response of the system must be ***absolute summable.***

$$\sum_{k=-\infty}^{\infty} |h[k]| < \infty$$