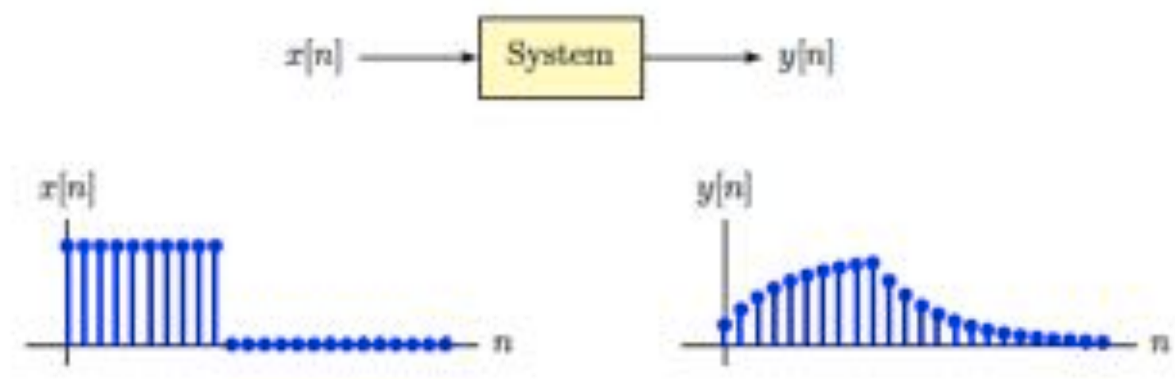
Chapter 3. Analyzing Discrete-Time Systems in the Time Domain

Chapter Objectives

- ✧ Develop the notion of a ***discrete-time*** system.
- ✧ Learn simplifying assumptions made in the analysis of systems. Discuss the concepts of ***linearity*** and ***time invariance***, and their significance.
- ✧ Explore the use of ***differential equations*** for representing discrete-time systems.
- ✧ Develop methods for solving differential equations to compute the output signal of a system in response to a specified input signal.
- ✧ Learn to represent a differential equation in the form of a block diagram that can be used as the basis for simulating a system.
- ✧ Discuss the significance of the ***impulse response*** as an alternative description form for linear and time-invariant systems.
- ✧ Learn how to compute the output signal for a linear and time-invariant system using ***convolution***
- ✧ Learn the concepts of causality and stability as they relate to physically realizable and useable systems.

3.1 Introduction

In general, a discrete-time system is a mathematical formula, method or algorithm that defines a cause-effect relationship between a set of discrete-time input signals and a set of discrete-time output signals.



$$y[n] = Sys\{x[n]\}$$

Sys{….} represents the transformation that defines the system in the time domain.

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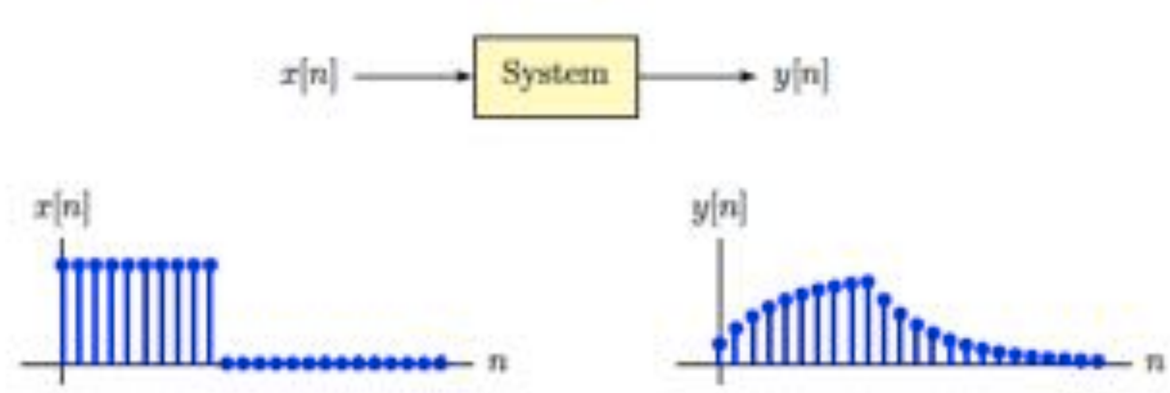


Examples:

$$y[n] = K\, x[n]$$

$$y[n] = x[n-m]$$

$$y[n] = K\, [x[n]]^2$$

3.2.1 Linearity

Conditions for linearity

$$Sys\{x_1[n]+x_2[n]\} = Sys\{x_1[n]\} + Sys\{x_2[n]\}$$

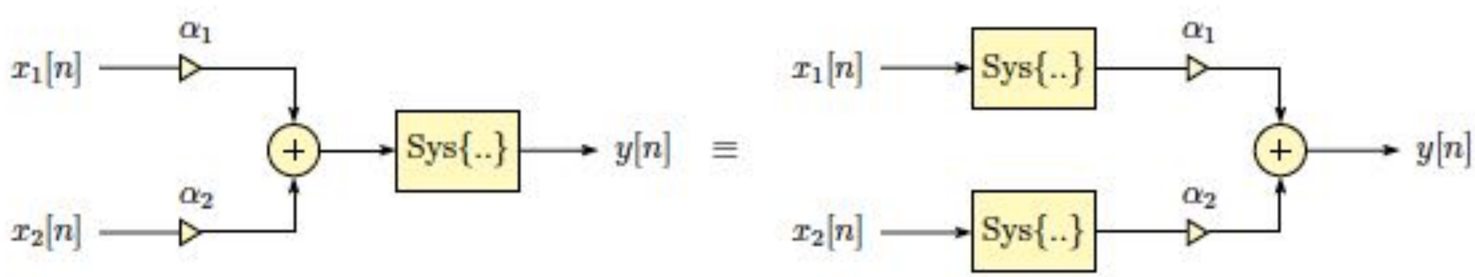
$$Sys\{\alpha_1 x_1[n]\} = \alpha_1 Sys\{x_1[n]\}$$

$x_1[n]$, $x_2[n]$: Any two input signals; α_1 :Arbitrary constant gain factor

Superposition principle (combine the two conditions into one)

$$Sys\{\alpha_1 x_1[n]+\alpha_2 x_2[n]\} = \alpha_1 Sys\{x_1[n]\} +\alpha_2\, Sys\{x_2[n]\}$$

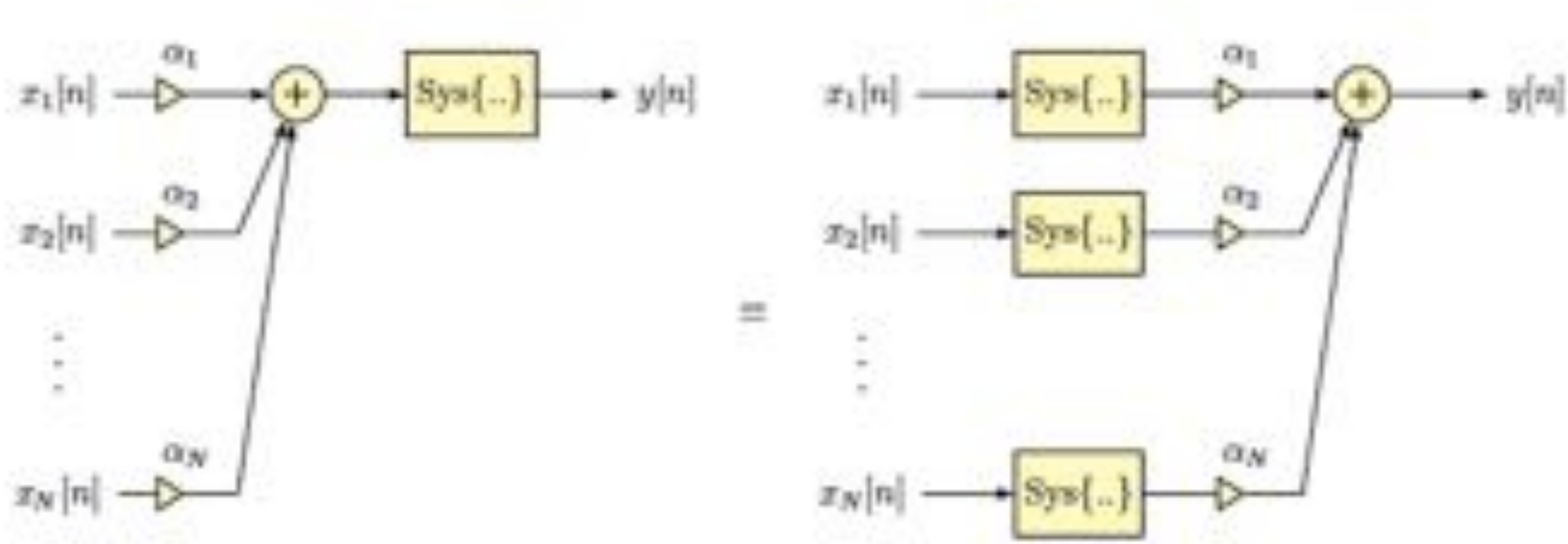
$x_1[n]$, $x_2[n]$: Any two input signals; α_1, α_2 Arbitrary constant gain factors



3.2.1 Linearity

If superposition works for the weighted sum of any two input signals, it also works for any arbitrary number of input signals.

$$Sys\{\sum_{i=1}^{N}\alpha_i x_i[n]\} = \sum_{i=1}^{N}\alpha_i Sys\{x_i[n]\} = \sum_{i=1}^{N}\alpha_i y_i[n]$$



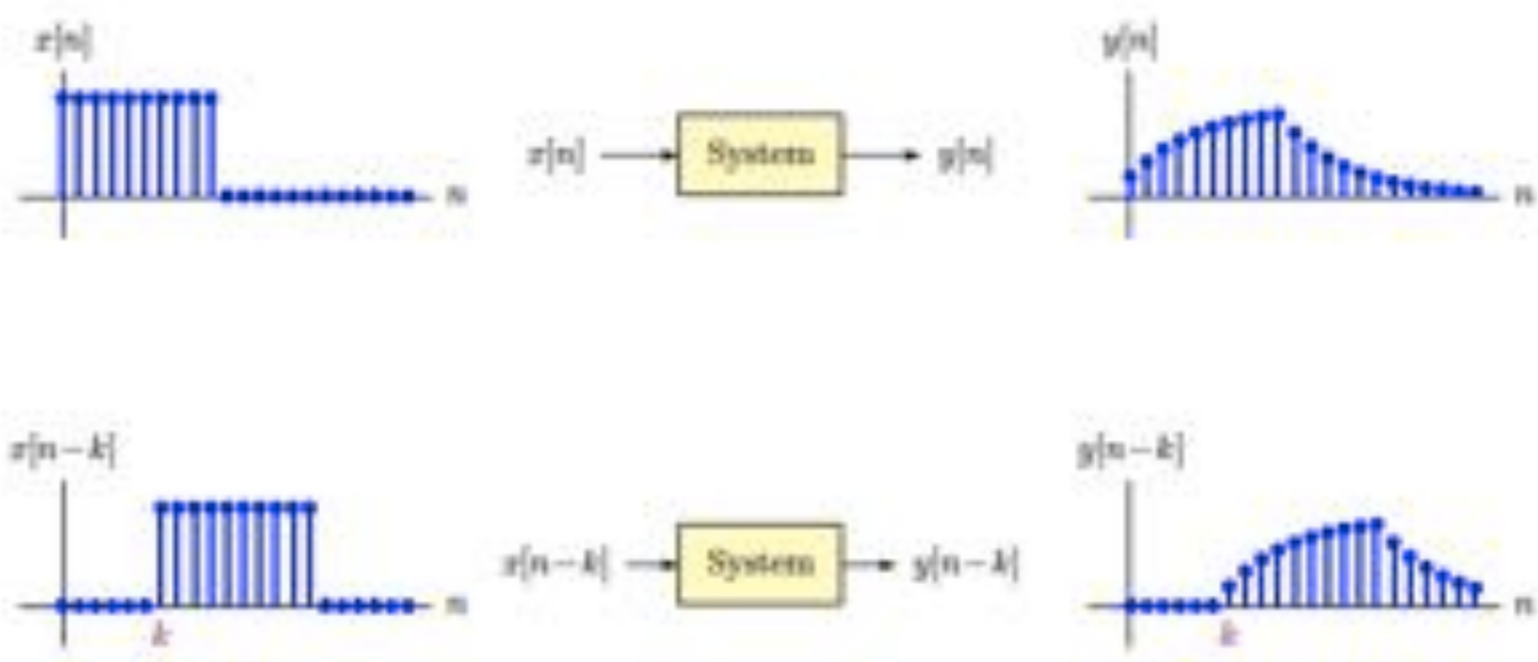
3.2.1 Linearity

Example 3.1 Testing linearity of discrete-time systems

3.2.2 Time Invariance in Continuous-time Systems

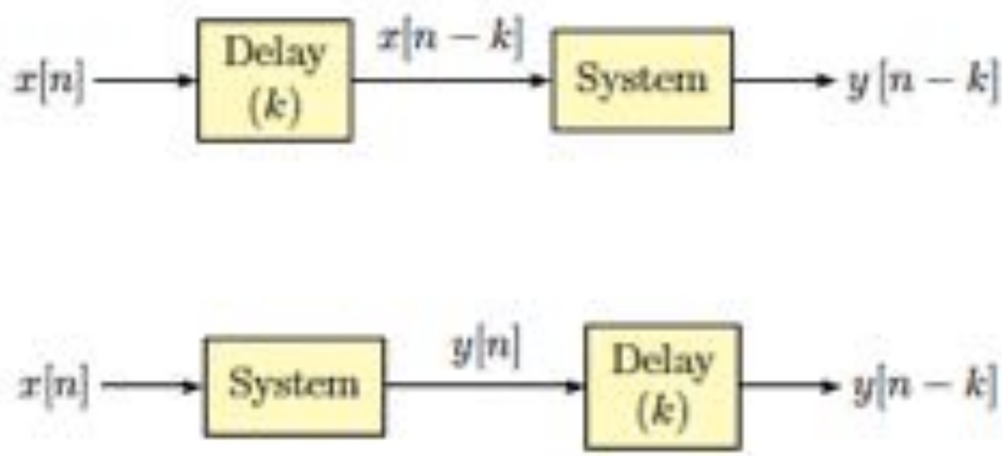
Conditions for time-invariance

$Sys\{x[n]\} = y[n]$ implies that $Sys\{x[n-k]\} = y[n-k]$



3.2.2 Time Invariance in Continuous-time Systems

time-invariance can be explained by the equivalence of the two system configurations



DTLTI systems: ***discrete-time linear and time-invariant*** **system**

3.2.2 Time Invariance in Discrete-time Systems

Example 3.2 Testing time invariance of discrete-time systems