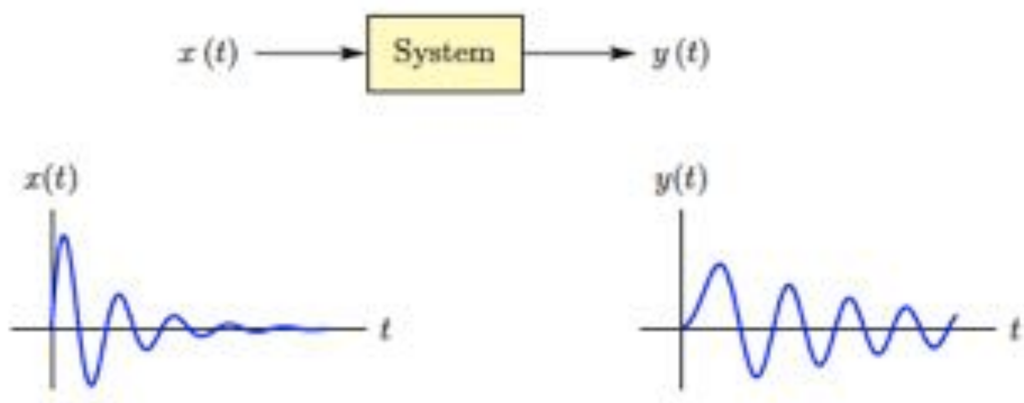
Chapter 2. Analyzing Continuous-Time Systems in the Time Domain

Chapter Objectives

- ✧ Develop the notion of a ***continuous-time*** system.
- ✧ Learn simplifying assumptions made in the analysis of systems. Discuss the concepts of ***linearity*** and ***time invariance***, and their significance.
- ✧ Explore the use of ***differential equations*** for representing continuous-time systems.
- ✧ Develop methods for solving differential equations to compute the output signal of a system in response to a specified input signal.
- ✧ Learn to represent a differential equation in the form of a **block diagram** that can be used as the basis for simulating a system.
- ✧ Discuss the significance of the **impulse response** as an alternative description form for linear and time-invariant systems.
- ✧ Learn how to compute the output signal for a linear and time-invariant system using ***convolution***
- ✧ Learn the concepts of **causality and stability** as they relate to physically realizable and useable systems.

2.1 Introduction

In general, a system is any physical entity that takes in a set of one or more physical signals and, in response, produces a new set of one or more physical signals.



The mathematical model of a system is a function, formula or algorithm (or a set of functions, formulas, algorithms) to approximately recreate the Same cause-effect relationship between the mathematical models of the Input and the output signals.

2.2.1 Linearity in Continuous-time Systems

Conditions for linearity

$$Sys\{x_1(t)+x_2(t)\} = Sys\{x_1(t)\} + Sys\{x_2(t)\}$$

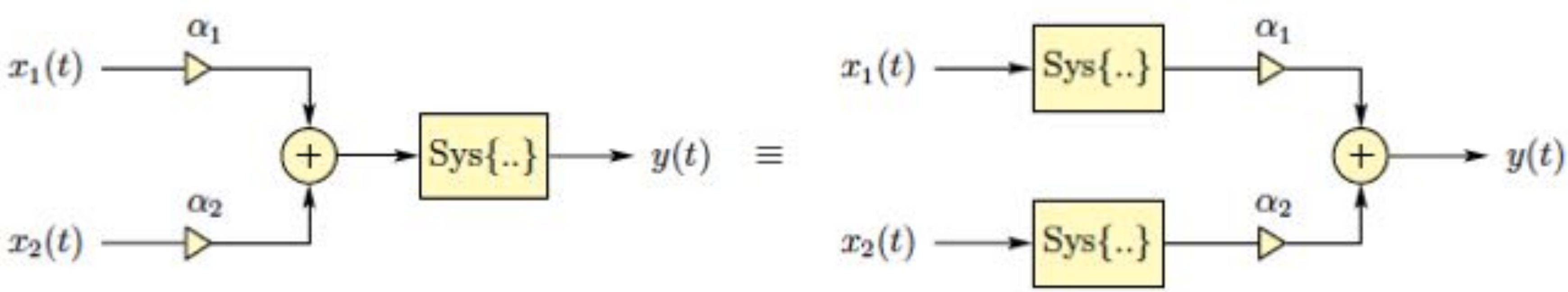
$$Sys\{\alpha_1 x_1(t)\} = \alpha_1 Sys\{x_1(t)\}$$

$x_1(t)$, $x_2(t)$: Any two input signals; α_1 :Arbitrary constant gain factor

Superposition principle (combine the two conditions into one)

$$Sys\{\alpha_1 x_1(t)+\alpha_2 x_2(t)\} = \alpha_1 Sys\{x_1(t)\} +\alpha_2\, Sys\{x_2(t)\}$$

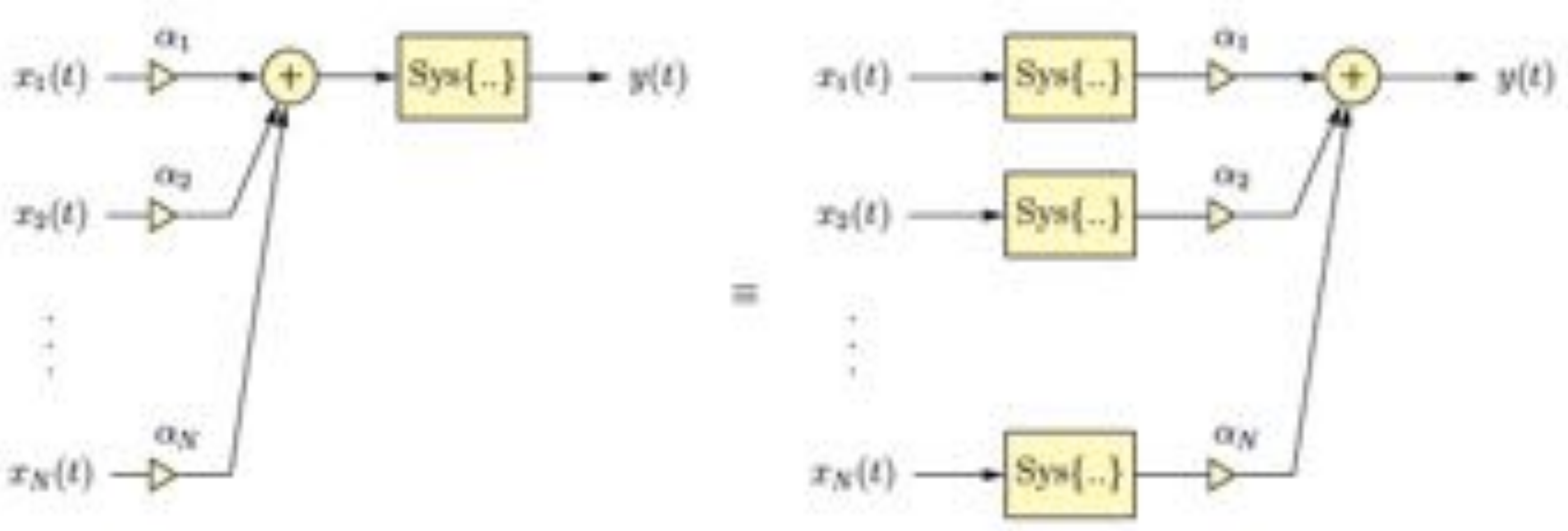
$x_1(t)$, $x_2(t)$: Any two input signals; $\alpha_{1,}\alpha_2$ Arbitrary constant gain factors



2.2.1 Linearity in Continuous-time Systems

If superposition works for the weighted sum of any two input signals, it also works for any arbitrary number of input signals.

$$Sys\{\sum_{i=1}^{N}\alpha_i x_i(t)\} = \sum_{i=1}^{N}\alpha_i Sys\{x_i(t)\} = \sum_{i=1}^{N}\alpha_i y_i(t)$$



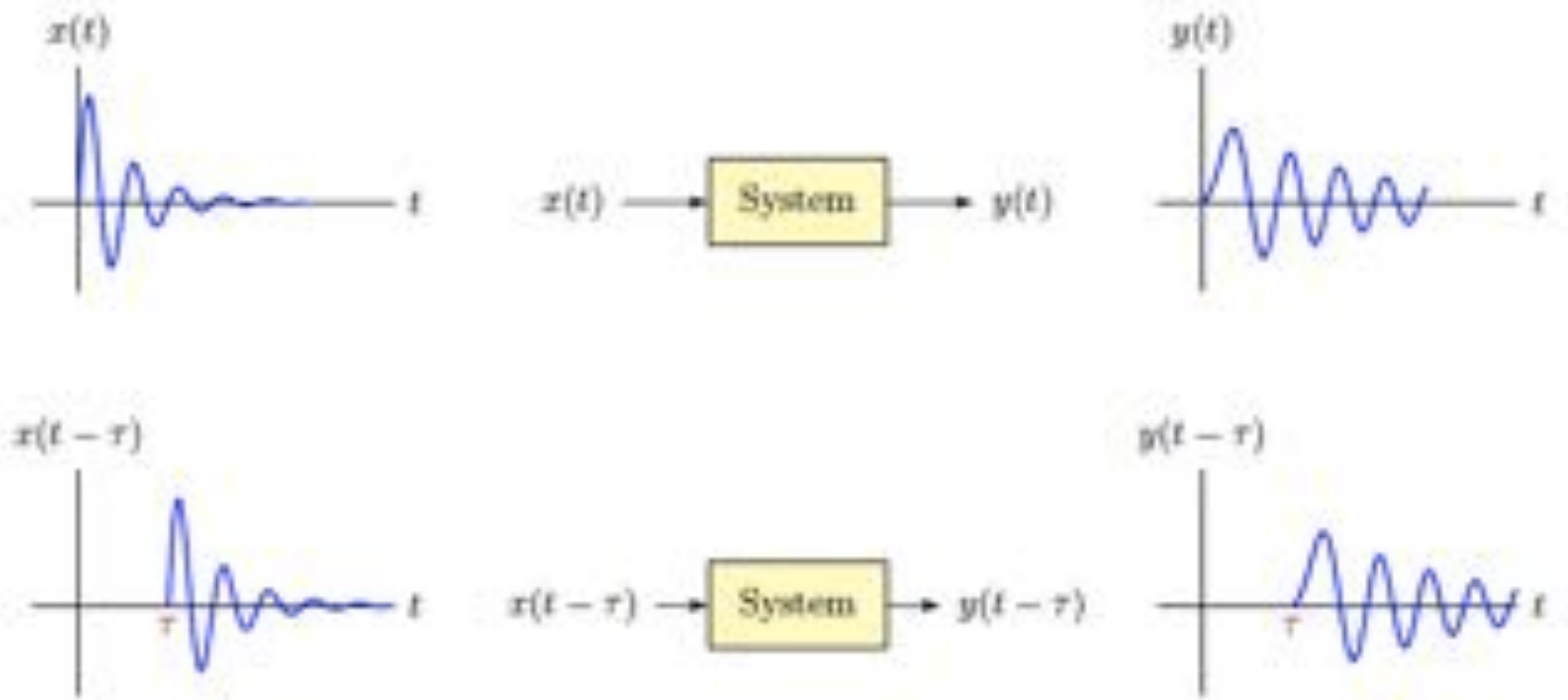
2.2.1 Linearity in Continuous-time Systems

Example 2.1 Testing linearity of continuous-time systems

2.2.2 Time Invariance in Continuous-time Systems

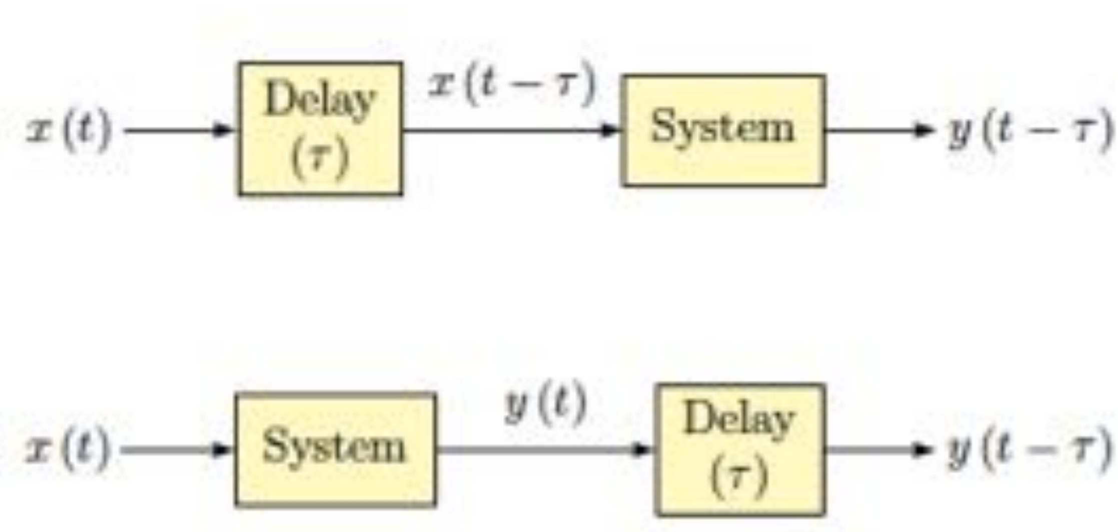
Conditions for time-invariance

$Sys\{x(t)\} = y(t)$ implies that $Sys\{x(t-\tau)\} = y(t-\tau)$



2.2.2 Time Invariance in Continuous-time Systems

time-invariance can be explained by the equivalence of the two system configurations



CTLTI systems: ***continuous-time linear and time-invariant*** **system**

2.2.2 Time Invariance in Continuous-time Systems

Example 2.2 Testing time invariance of continuous-time systems