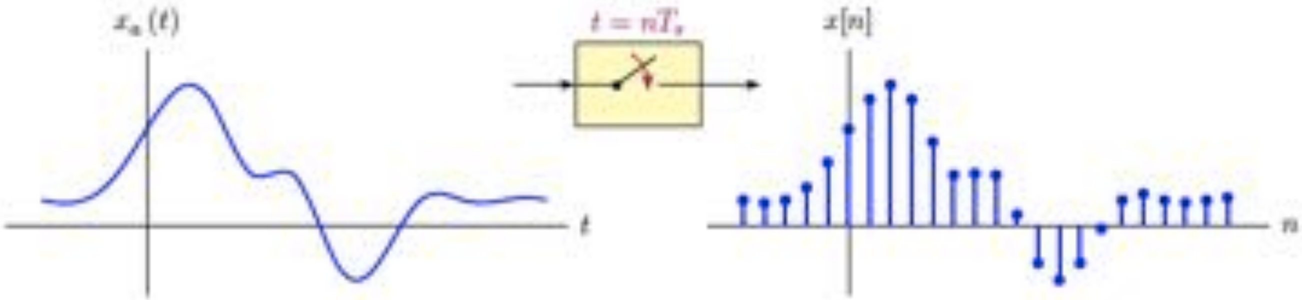
Chapter 6 Sampling and Reconstruction

It may be possible to represent a continuous-time signal without any loss of information by a discrete set of amplitude values measured at uniformly spaced time intervals.



$$x[n] = x_a(t)\Big|_{t=nT_s} = x_a(nT_s)$$

n: Integer, T_s : Sampling interval/period

$$f_s = \frac{1}{T_s} \text{ : Sampling rate/frequency}$$

6.2 Sampling of a Continuous-Time Signal

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

A periodic impulse train with period T_s

$$x_s(t) = x_a(t)p(t)$$
$$= \sum_{n=-\infty}^{\infty} x_a(nT_s)\delta(t - nT_s)$$

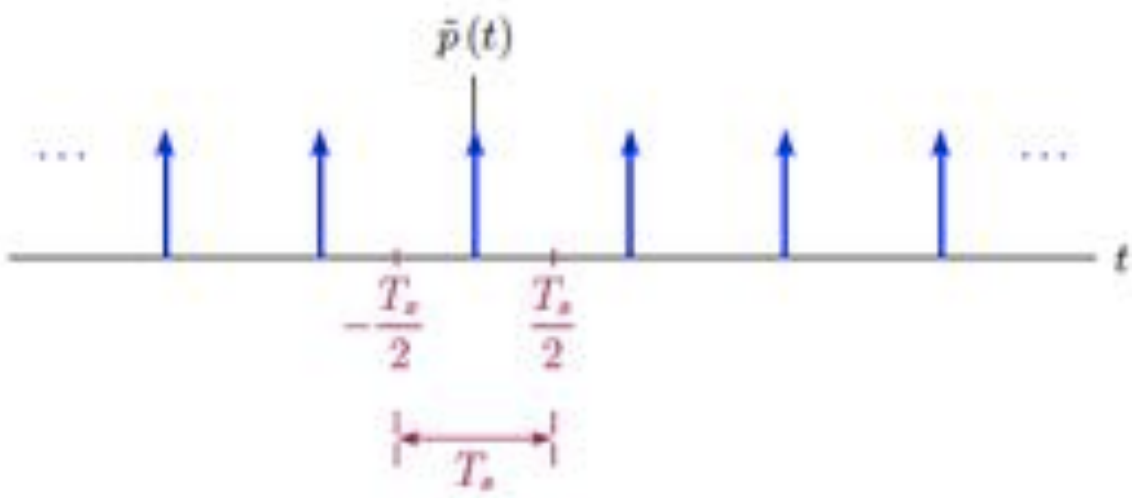
$x_s(t)$: the ***impulse-sampled signal (digital signal)***

$x_a(t)$: the ***original signal (analog signal)***

NOTE: the impulse-sampled signal $x_s(t)$ is still a continuous-time signal.

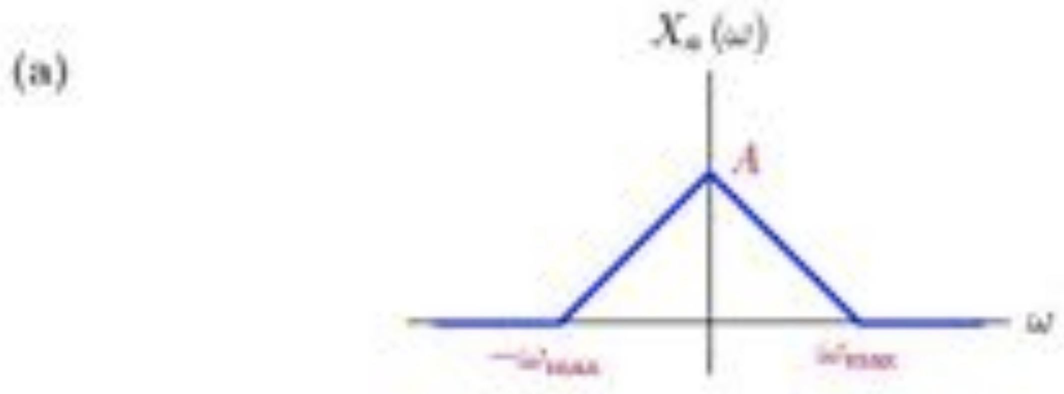
6.2 Sampling of a Continuous-Time Signal

Question: **How dense must the impulse train *p(t)* to be so that the impulse-sampled signal $x_s(t)$ is an accurate and complete representation of the original (analog) signal $x_a(t)$?**

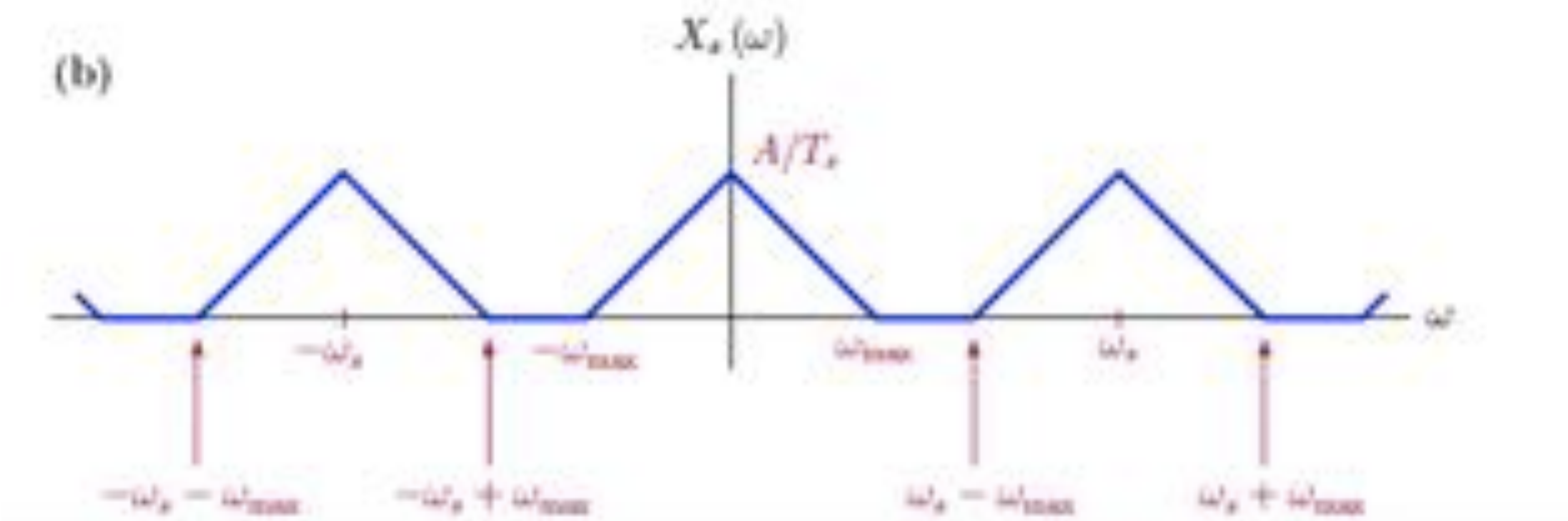


6.2 Sampling of a Continuous-Time Signal

$x_a(t)$ **CAN** be totally recovered from impulse-sampled signal $x_s(t)$

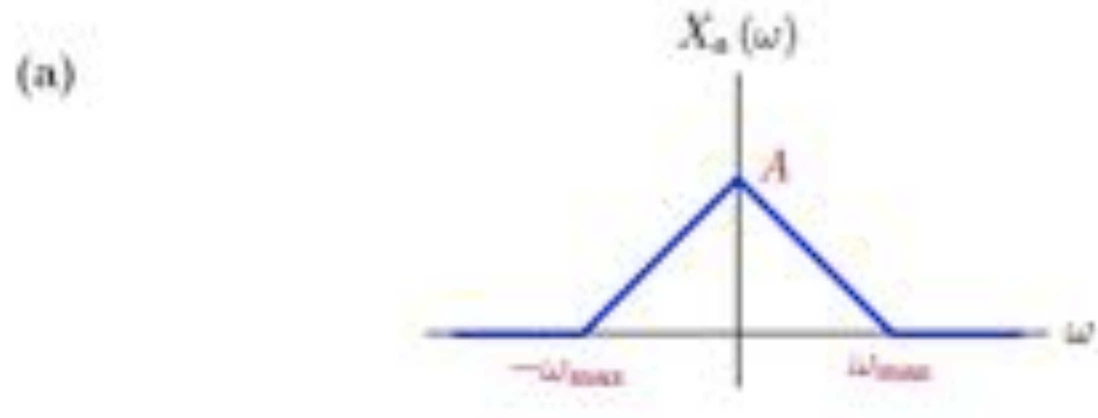


$$\omega_s \geq 2\omega_{max}$$

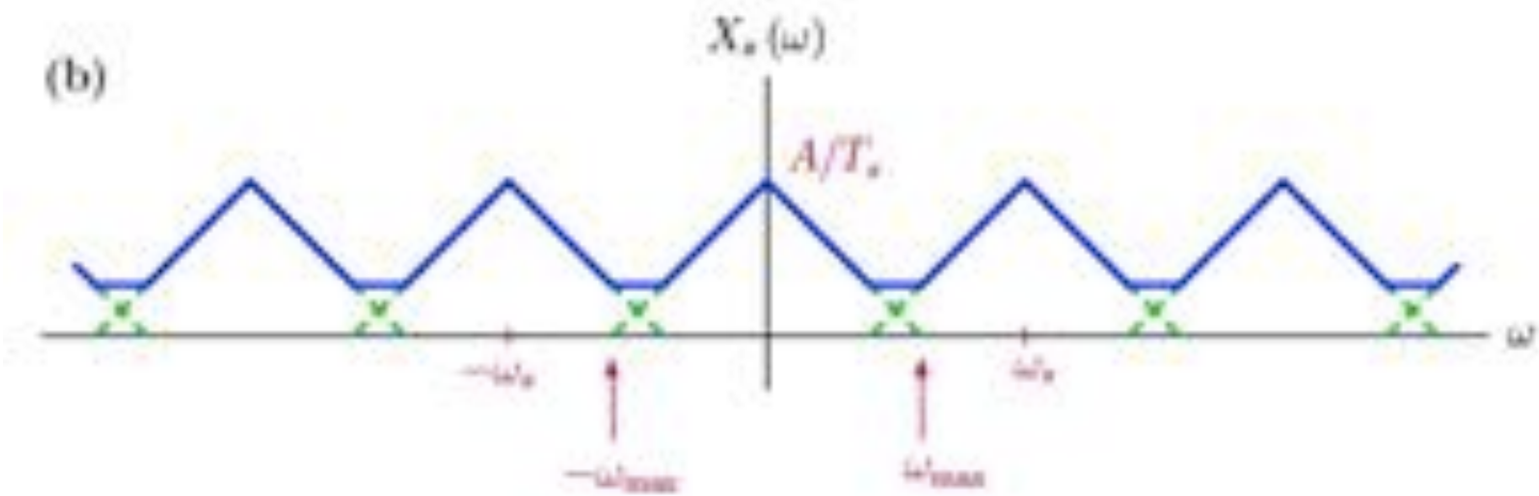


6.2 Sampling of a Continuous-Time Signal

$x_a(t)$ **CAN'T** be totally recovered from impulse-sampled signal ***$x_s(t)$***



$$\omega_s < 2\omega_{max}$$



6.2 Sampling of a Continuous-Time Signal

Impulse-sampling a right-sided exponential

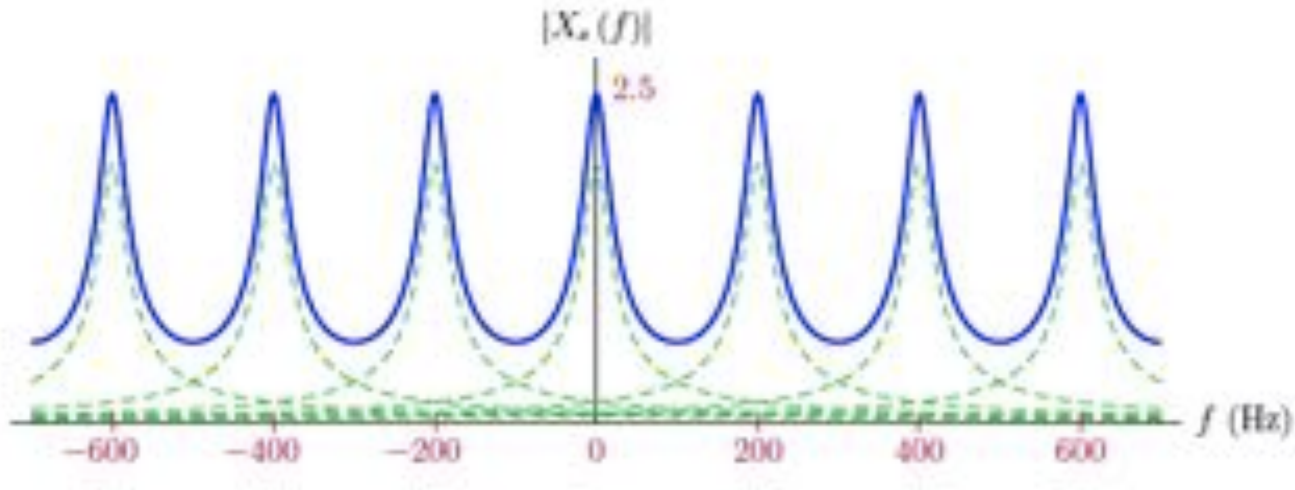
Consider a right-sided exponential signal

$$x_a(t) = e^{-100t}\, u(t)$$

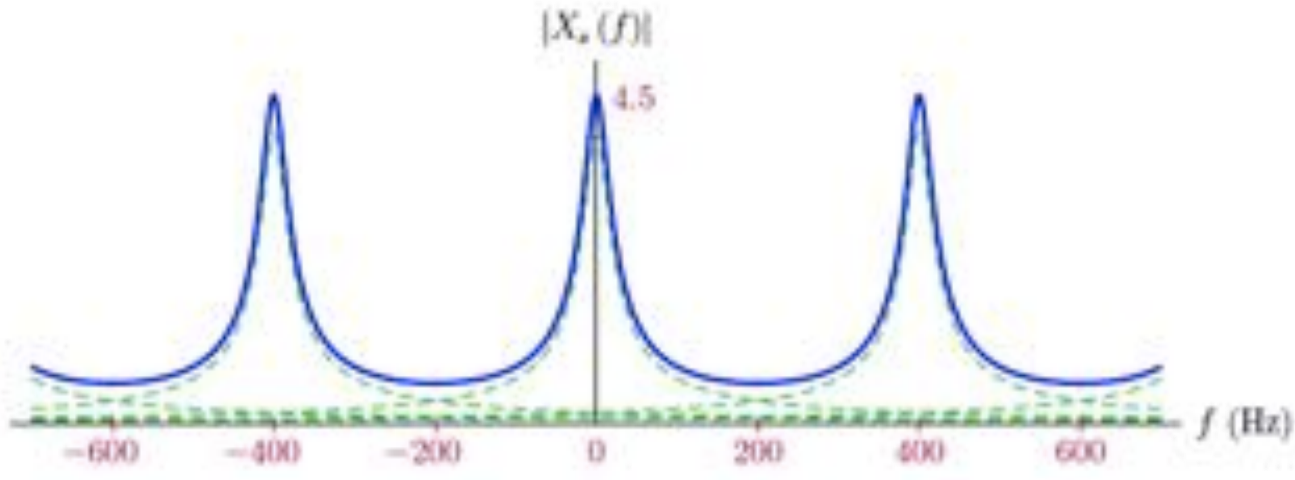
This signal is to be impulse sampled. Determine and graph the spectrum of the impulse sampled signal $x_s(t)$ for sampling rates $f_s = 200$ Hz, $f_s = 400$ Hz and $f_s = 600$ Hz.

6.2 Sampling of a Continuous-Time Signal

$f_s = 200$ Hz



$f_s = 400$ Hz



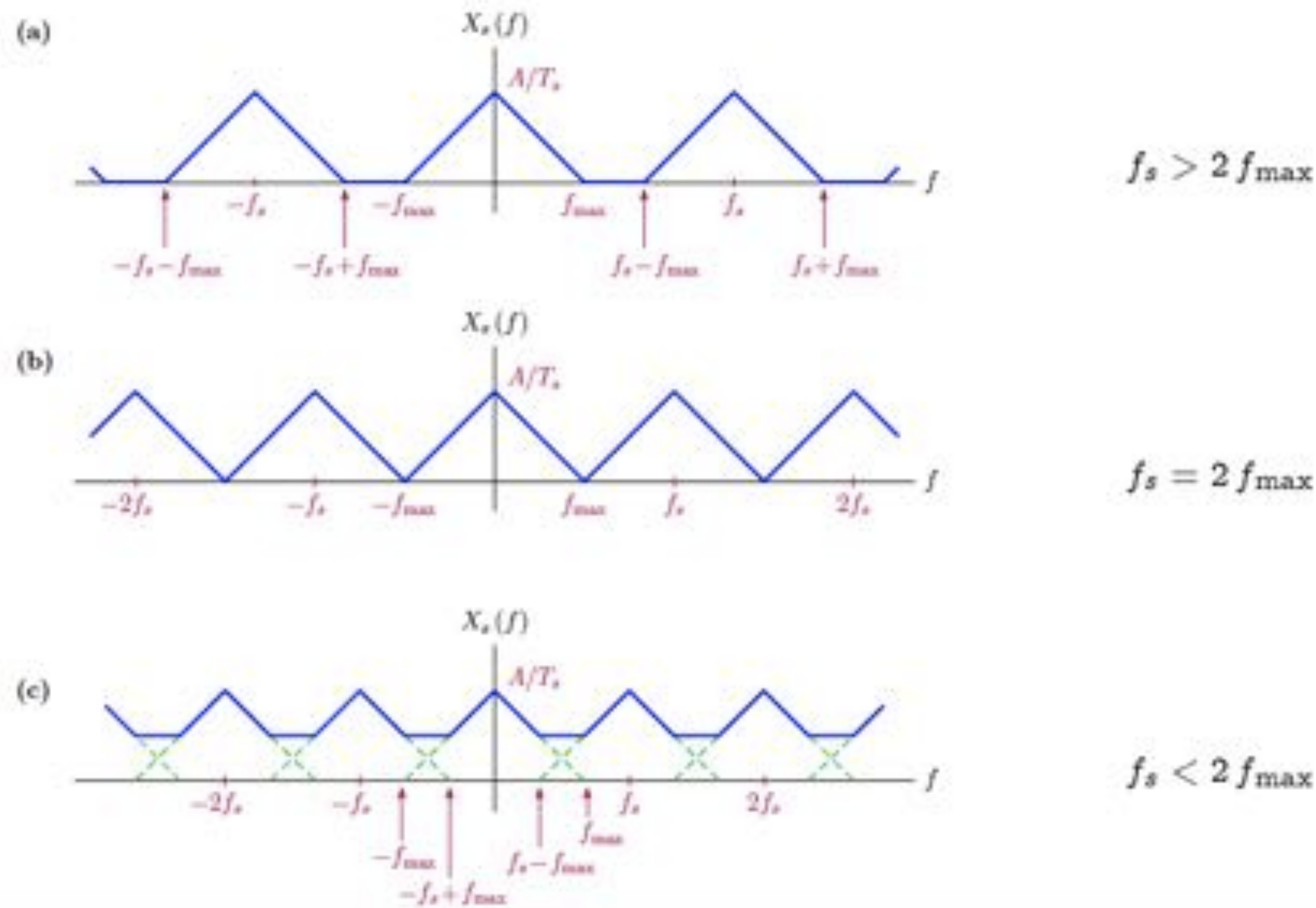
6.2 Sampling of a Continuous-Time Signal

Nyquist Sampling Criterion

For the impulse-sampled signal to form an accurate representation of the original signal, the sampling rate must be at least twice the highest frequency in the spectrum of the original signal

$$f_s \geq 2 f_{\max}$$

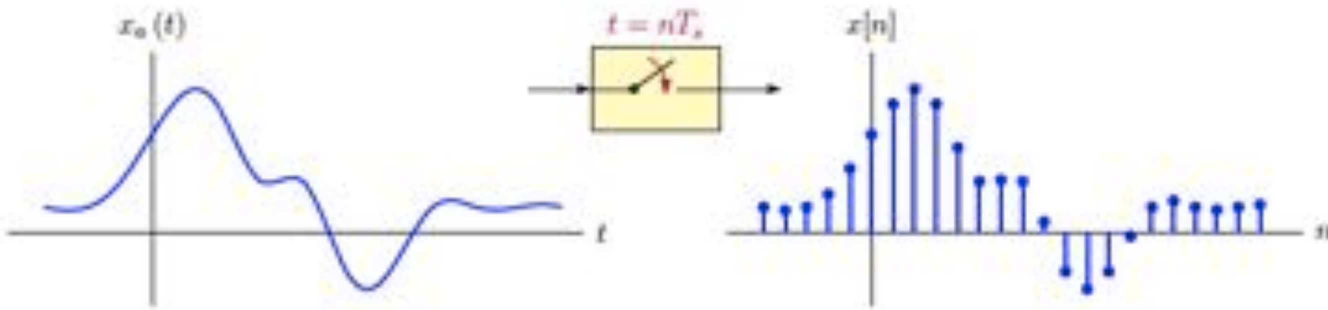
6.2 Sampling of a Continuous-Time Signal



6.2.3 DTFT of Sampled Signal

6.2.3 DTFT of Sampled Signal

$$x[n] = x_a(nT_s)$$



$$X_a(\omega) = \int_{-\infty}^{\infty} x_a(t)\, e^{-j\omega t}\, dt$$

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\Omega n}$$

$X(\Omega)$ is related to $X_a(\omega)$ by

$$X(\Omega) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X_a\left(\frac{\Omega - 2\pi k}{T_s}\right)$$

6.2.3 Practical Issues in Sampling

$X(\Omega)$ is related to $X_a(\omega)$ by

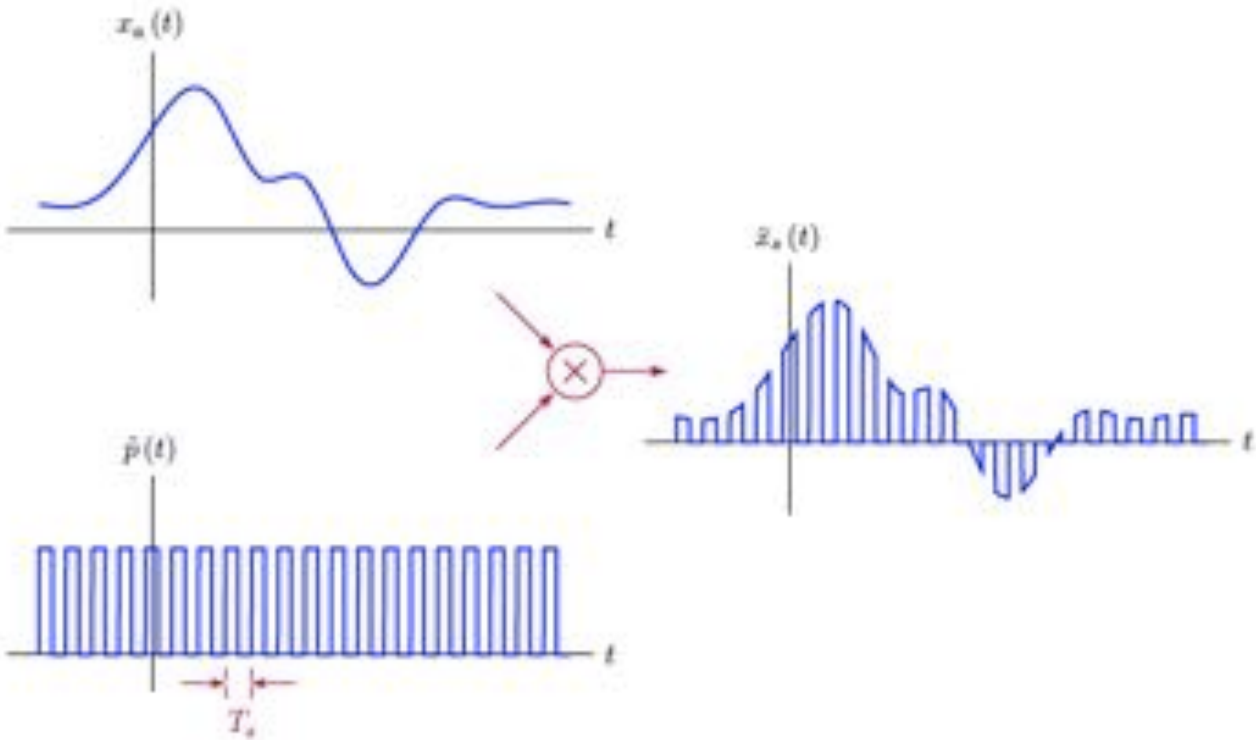
$$X(\Omega) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X_a\left(\frac{\Omega - 2\pi k}{T_s}\right)$$

An impulse train is used in the sampled continue-time signal. But impulse train is not practical available.

An alternative approach is using a periodic pulse train. But how would the use of pulses affect the method in recovering the original signal from its sampled version.

6.2.3 Practical Issues in Sampling

Natural Sampling

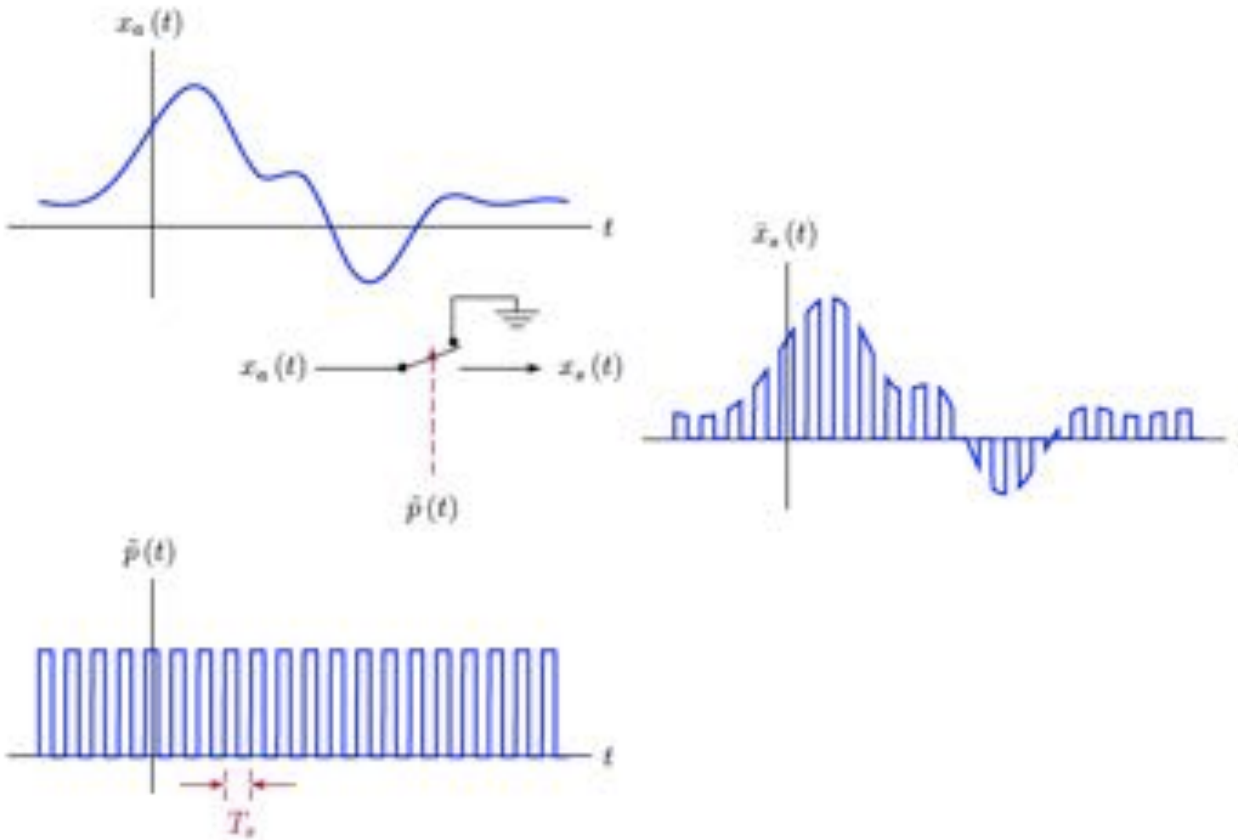


$$\bar{x}_s(t) = x_a(t)\,\tilde{p}(t)$$

$$= x_a(t) \sum_{n=-\infty}^{\infty} \Pi\left(\frac{t-nT_s}{dT_s}\right) \qquad d: \text{ Duty cycle}$$

6.2.3 Practical Issues in Sampling

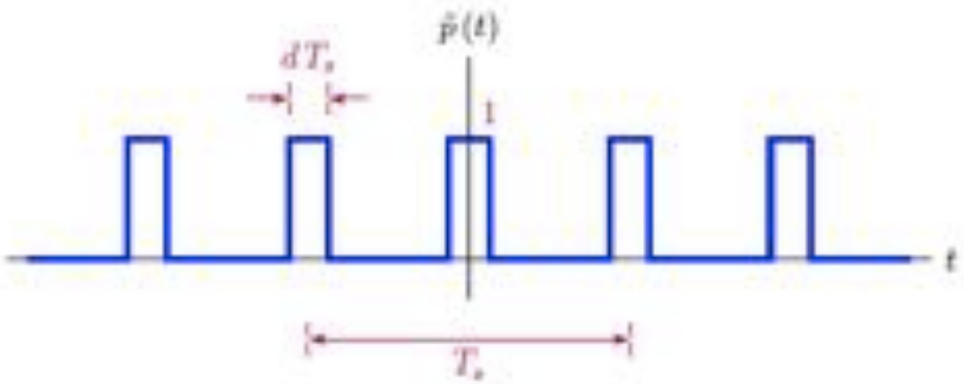
Natural Sampling



$$\tilde{x}_s(t) = \begin{cases} x_a(t) \;, & \tilde{p}(t) \neq 0 \\ 0; \;, & \tilde{p}(t) = 0 \end{cases}$$

6.2.3 Practical Issues in Sampling

Natural Sampling



$$\tilde{p}(t) = \sum_{n=-\infty}^{\infty} \Pi\left(\frac{t - nT_s}{dT_s}\right)$$

EFS representation:

$$\tilde{p}(t) = \sum_{k=-\infty}^{\infty} c_k \, e^{jk\omega_s t}$$

Coefficients:

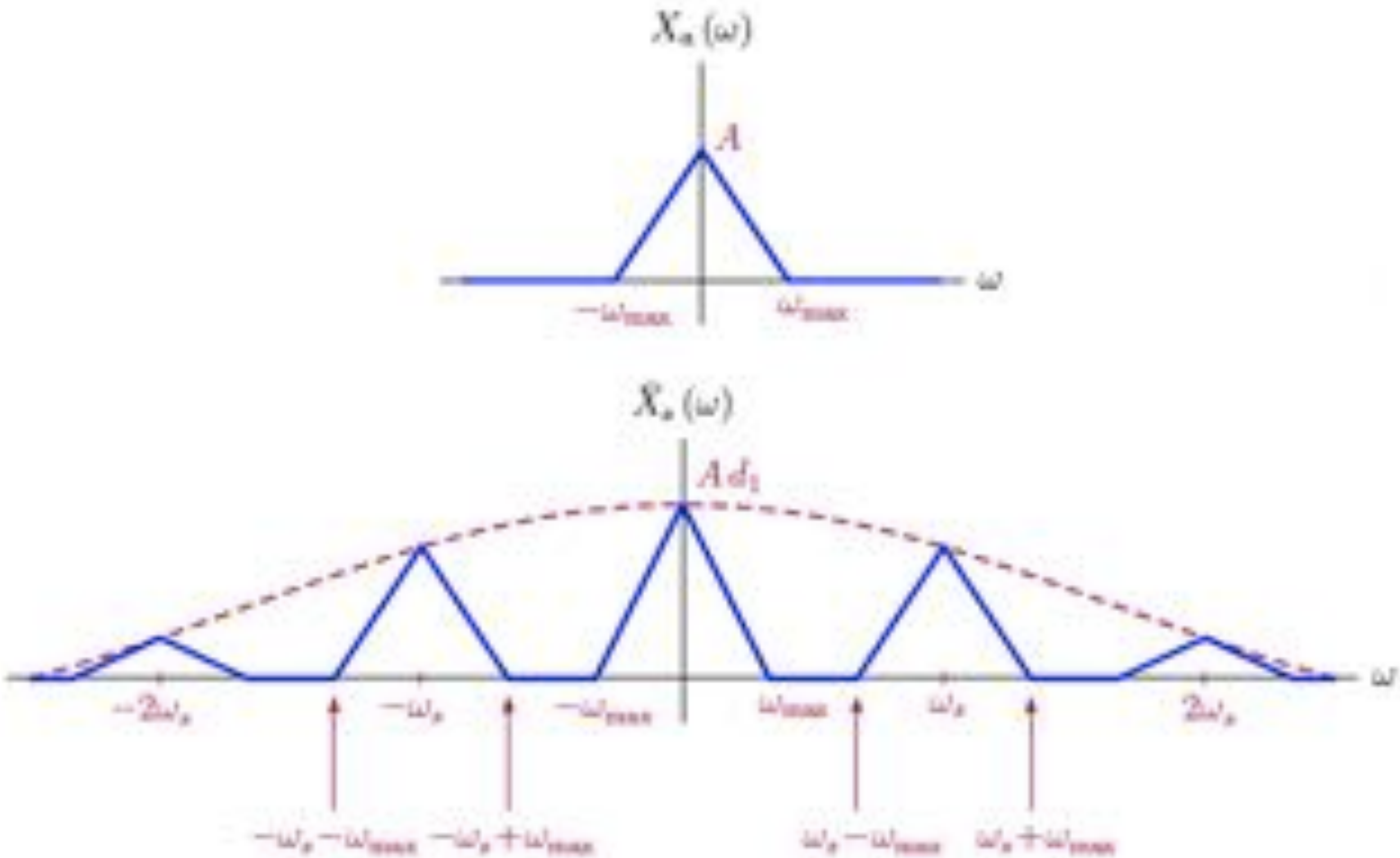
$$c_k = d \, \text{sinc}(kd)$$

Spectrum of naturally sampled signal

$$\bar{X}_s(\omega) = d \sum_{k=-\infty}^{\infty} \text{sinc}(kd) \; X_a(\omega - k\omega_s)$$

6.2.3 Practical Issues in Sampling

$$\bar{X}_s(\omega) = d \sum_{k=-\infty}^{\infty} \text{sinc}(kd)\; X_a(\omega - k\omega_s)$$



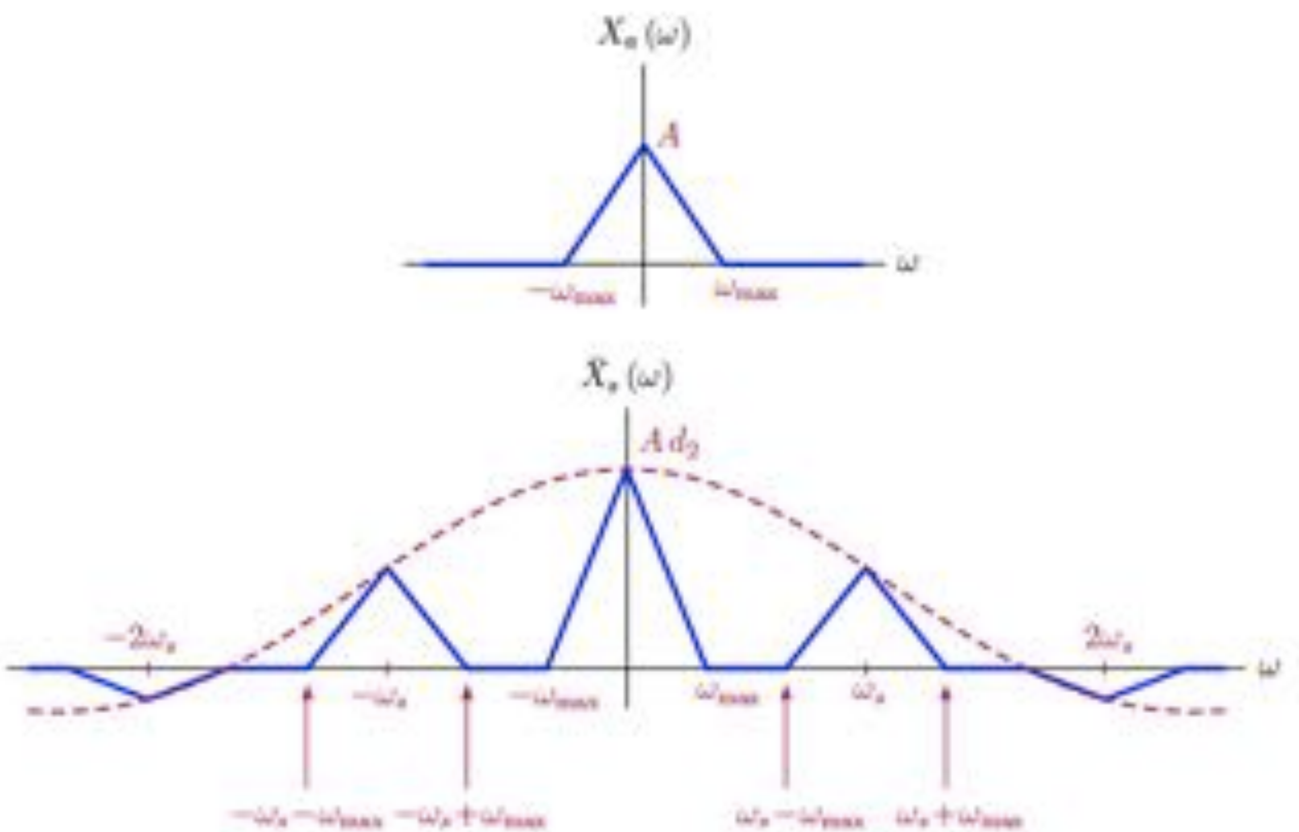
Duty cycle:

$d = d_1$

6.2.3 Practical Issues in Sampling

Natural Sampling

$$\bar{X}_s(\omega) = d \sum_{k=-\infty}^{\infty} \text{sinc}(kd)\, X_a(\omega - k\omega_s)$$



▸ MATLAB Exercise 6.4

▸ MATLAB Exercise 6.6

Duty cycle:

$d = d_2 > d_1$