MATH466/462 Project 1. *Due in class on Monday, Jan 27, 2020*

`Instruction: your project report should include necessary mathematical justification, description, and details of your algorithms/conclusions. Your MATLAB codes and generated outputs may be attached in the end of the report. Please make sure you addressed all the questions in each problem. Both the report and codes will be graded. Each group member shoud contribute to both.`

Problem A (10 pts): Bubble Sorting Algorithm

1. Study the algorithm online, check e.g. `https://www.studytonight.com/data-structures/bubble-sort`

2. Implement the Bubble Sorting algorithm in MATLAB as the following function :

```
function y=BBsort(x)
%Input: x is a vector of real numbers
%Output: y is the sorted (in increasing order) vector of x

```

3. Test your code with the following script. What is the approximate slope of the line? may use `polyfit`.

```
dim=1e6*2.^(1:8); %test dimensions, start with smaller dimensions
cpu=zeros(length(dim),1); %for store CPU times
for k=1:length(dim)
  tic %start timing
  x=rand(dim(k),1); %random vector
  y=BBsort(x); %call your sorting algorithm
  cpu(k)=toc; %measure CPU time in seconds
end
loglog(dim,cpu,'-o') %plot CPU time growth, in log-log scale

```

4. (Bonus) Compare with MATLAB's `sort`. How many times slower is your code? Try faster algorithms:

   Quick Sorting: `https://www.studytonight.com/data-structures/quick-sort`

   Merge Sorting: `https://www.studytonight.com/data-structures/merge-sort`.

Problem B (10 pts): Control Round-off Errors

We want to numerically compute a sequence of definite integrals (with $n = 0, 1, 2, \cdots, 30$ and $a = 10$):

$$y_n = \int_0^1 \frac{x^n}{x+a} dx.$$

1. Mathematically show the following recurrence relation holds:

$$y_n = \frac{1}{n} - a y_{n-1}, \qquad y_0 = \ln\left(\frac{1+a}{a}\right). \tag{1}$$

2. Mathematically estimate the lower and upper bounds for y_n (by choosing $x = 0$ or $x = 1$ in denominator).

3. Write a code according to (1) to compute the sequence $\{y_n\}_{n=1}^{30}$. What you observe and why?

4. Solve for y_{n-1} from the recurrence (1) to get a backward recurrence relatioin. Write another code based on this new recurrence to compute the sequence $\{y_n\}_{n=1}^{30}$ backward. Take the lower bound for y_{30} as the starting (initial) value. Again, what your observe and why?

5. Compare your numerical results with the integral values directly computed by MATLAB function `integral` or `quad`. Using either a Table or Figure to show the difference in integral values and CPU times?

6. (Bonus) Rerun your codes with $a = 0.5$ and $a = 1$, respectively? What you observe and explain why?