Computational Methods in Optimal Control Lectures 9/10. Conference Overview and Future Directions

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Conference Themes and Future Directions

- Convergence analysis built around first-order optimality conditions
  1. Form a Lagrangian
  2. Differentiate with respect to each variable
  3. Apply to both continuous problem (Pontryagin) and discrete problem (KKT)
  4. Transformations may be needed for the discrete multipliers

- Close connection between optimality conditions and gradient calculations:
  1. The costate computed from the co-dynamics gave a formula for the gradient of the objective with respect to the control
  2. Reverse mode gradient computation
  3. In GPOPS2, gradient implemented by source transformation via operator overloading.
  4. Efficient implementation of the gradient calculation needed for fast solutions.

Conference Themes and Future Directions

- General approach introduced for convergence analysis.
  1. Insert the continuous solution into the discrete problem and obtain a bound for the norm of the residual.
  2. Establish a Lipschitz property for the linearization around the target solution of the first-order optimality conditions for the discrete problem
  3. The convergence domain depends on the size of the residual, the size of the Lipschitz constant, and how close the linearized problem at nearby points is to the linearization at the target solution.
  4. A proper choice for norms is needed.

- Discretization often leads to control problems involving big-data
  1. Dimension reduction techniques (e. g. do not discretize the control, only the state)
  2. Model reduction techniques
  3. Efficient utilization of parallel computing is important for achieving fast evaluation or approximation of the gradient

Conference Themes and Future Directions

- Directions for future research.
  1. Much needs to be done for nonsmooth problems.
  2. If the objective is linear, strong convexity assumptions often violated and current techniques for convergence analysis do not apply. This is a active area for the development of both theory and practical algorithms.
  3. Algorithms for singular control problems are very open
  4. Ways to deal with large data sets is an area of intense research. Solving problems quickly or in real time requires development of low dimensional models (hp techniques, model reduction) and the efficient use of parallel processors (gradient computations)
  5. Optimal control is being used in many applications
     1. Math Biology
     2. Medicine (cancer and disease treatments)
     3. Robotics (more than 1000 robots utilized in Tesla production)
     4. Automation
     5. Aerospace
     6. Automobiles

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- Directions for future research continued
  6. Shape and topology optimization problems
  7. Mechanical systems, fluids, elastic materials, . . .
  8. Many applications in the area of inverse problems (Determine initial temperature profile from measurement of terminal temperature problem, medical imaging)
  9. Developing research in the context of an application area can expand future employment opportunities.
  10. Model predictive control: Linearize the problem, determine an optimal control, use the control for a small time window, then re-linearize with new data and repeat. It is important to solve the problem quickly.
  11. Gauss and Radau collocation worked out (except for the 20,000 Yen Prize Problem). Open problems connected with Lobatto collocation.