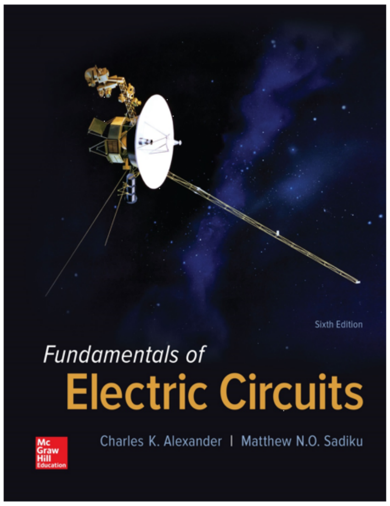


Chapter 11 - AC Power Analysis

- **This chapter will cover the concept of power in an AC circuit.**
- **The difference between instantaneous power and average power will be discussed.**
- **The difference between resistive and reactive power will be introduced.**
- **Other forms of averaged measurements will be covered**
- **Apparent power and complex power will also be covered.**

Ch 11, p1

Introduction

The concepts of voltage and current are clearly important in the analysis of circuits, but the concept of power analysis may be of even greater importance since:

- power is the most important quantity in the transmission of electrical energy from one point to another.
- every industrial and household electrical device requires the delivery of power to it in order to operate.

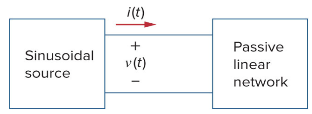
The most common form of electric power in the United States is 50- or 60-Hz AC power.

Ch 11, p2

11.2 Instantaneous and Average Power

Instantaneous power (in watts) is the power absorbed by a circuit element at any instant in time and is given by the expression: *p(t) = v(t)i(t)*.

Consider the generalized case of circuit elements under sinusoidal excitation where the voltage and current at the terminals of the circuit are given by the expressions:



$v(t) = V_m \cos(\omega t + \theta_v)$ and $i(t) = I_m \cos(\omega t + \theta_i)$

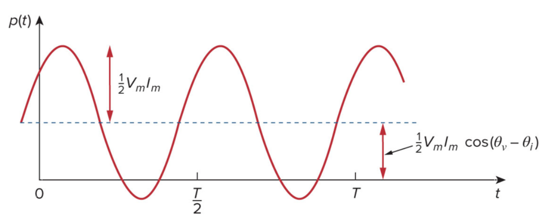
where V_m and I_m are the peak (amplitude) values while θ_v and θ_i are the phase angles of the voltage and current, respectively.

The power at the terminals then becomes: $p(t) = v(t)i(t) = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)$

This indicates that instantaneous power has two components:

- one is constant, depending on the phase difference between the voltage and current.
- the second is sinusoidal with a frequency twice that of the voltage and current.

$p(t) = v(t)i(t) = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)$ can be graphed as:



The positive values of power indicate times when power is being absorbed by the circuit.

The times the graph shows negative values of power indicate that power is being absorbed by the source and is possible with circuit elements like inductors or capacitors which can store and release energy.

Instantaneous power is very hard to measure since it is constantly changing, and the more common power measurement is that of average power.

Average power, in watts, is the average of the instantaneous power over one period.

$$P = \frac{1}{T}\int_0^T p(t)dt \longrightarrow P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$$

p(t) is in the time domain and, even though *P* is independent of time, to find instantaneous power both *v* and *i* must be in the time domain.

Average power can also be found when the voltage and current are expressed in the frequency domain:

$$P = \frac{1}{2}\text{Re}\left[\mathbf{VI}^*\right] = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$$

When the voltage and current are in phase ($\theta_v = \theta_i$), a purely resistive circuit or resistive load is implied and:

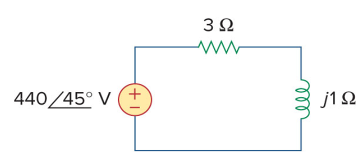
$$P = \frac{1}{2}V_m I_m = \frac{1}{2}I_m^2 R = \frac{1}{2}|\mathbf{I}|^2 R$$

When $\theta_v - \theta_i = \pm 90^o$, the circuit is purely reactive and: $P = \frac{1}{2}V_m I_m \cos(90^o) = 0$

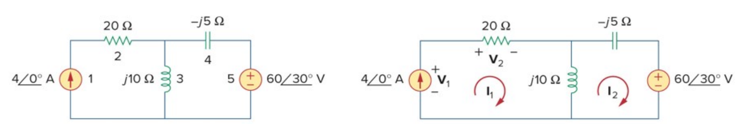
A resistive load (*R*) absorbs power at all times while a reactive load (*L* or *C*) absorbs zero average power.

Example and Practice Problems (Cont.)

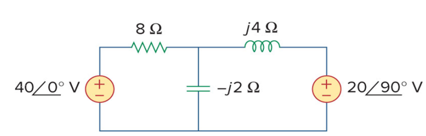
P11.3 Calculate the average power absorbed by the resistor and inductor, and find the average power supplied by the voltage source in this circuit.



E11.4 Determine the average power generated by each source and the average power absorbed by each passive element in the circuit shown below.

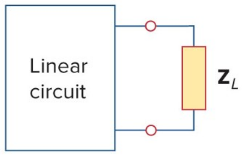


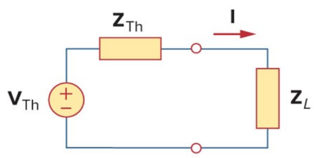
P11.4 Calculate the average power absorbed by each element in this circuit.



11.3 Maximum Average Power Transfer

It was previously shown that, for a DC supply circuit, maximum power was transferred to a load when the load resistance equaled the Thevenin resistance of the supply circuit as seen from the load.





For a sinusoidal, steady-state supply circuit, the maximum average power transfer occurs when the load impedance ($\mathbf{Z}_L$) is equal to the complex conjugate of the Thevenin impedance as seen from the load:

$$\mathbf{Z}_L = R_L + jX_L = R_{Th} - jX_{Th} = \mathbf{Z}^*_{Th}$$

This is known as the *maximum average power transfer theorem* for sinusoidal steady-state.

The maximum average power is given by: $P_{\max} = \dfrac{|V_{Th}|^2}{8R_{Th}}$

For maximum average power transfer to a purely resistive load, therefore, the load impedance (or resistance) is equal to the magnitude of the Thevenin impedance:

$$R_L = |Z_{Th}| = \sqrt{\left(R_{Th}^2 + X_{Th}^2\right)}$$

Example and Practice Problems

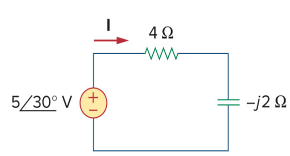
E11.1 Find the instantaneous power and average power for a passive linear network when $v(t) = 120 \cos(377t + 45^\circ)$ V and $i(t) = 10 \cos(377t - 10^\circ)$ A.

P11.1 Calculate the instantaneous and average power absorbed by a passive linear network when $v(t) = 330 \cos(10t + 20^\circ)$ V and $i(t) = 33 \sin(10t + 60^\circ)$ A

E11.2 Calculate the average power absorbed by an impedance $\mathbf{Z}$ = 30 - j70 ohms when a voltage of $\mathbf{V} = 120\angle 0^\circ$ is applied across it.

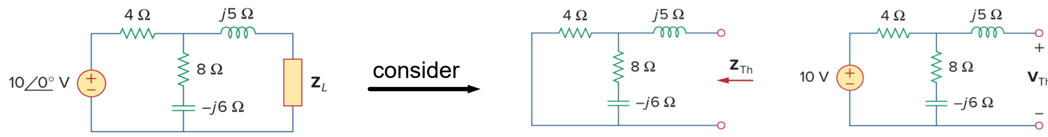
P11.2 Find the average power delivered to an impedance Z = 40$\angle$=-22° ohms when a current of $\mathbf{I} = 33\angle 30^\circ$ A flows through it.

E11.3 Find the average power supplied by the source and the average power absorbed the resistor in this circuit.

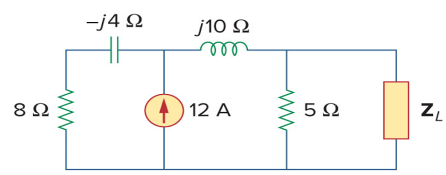


Example and Practice Problems

E11.5 Determine the load impedance ($\mathbf{Z}_L$) that maximizes the average power drawn from the circuit shown below, and find the maximum average power.

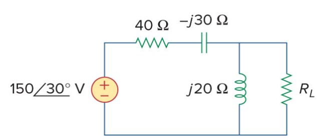


P11.5 Find the load impedance ($\mathbf{Z}_L$) that absorbs the maximum average power, and calculate that maximum average power, for this circuit

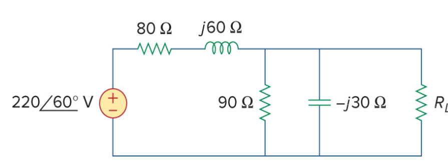


Example and Practice Problems (Cont.)

E11.6 Find the value of R_L that will absorb the maximum average power, and calculate that power, for the circuit shown below.



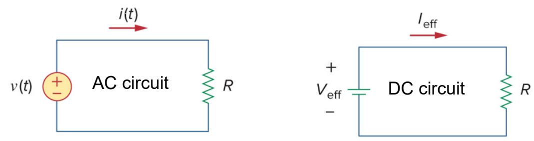
P11.6 Resistor R_L is adjusted until it absorbs the maximum average power. Calculate R_L and the maximum average power absorbed by it.



11.4 Effective or RMS Value

When a time-varying current source is delivering power to a resistive load, it is often necessary or useful to know how effective the source is in delivering power to that load.

The *effective value* of a periodic current is the DC current that delivers the same average power to a resistor as the periodic current.



The effective value of a periodic signal is its root mean square (rms) value:

$$I_{eff} = I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2 dt} \qquad V_{eff} = V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2 dt}$$

The average power can then be written:

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$$

$$P = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos(\theta_v - \theta_i)$$

$$P = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

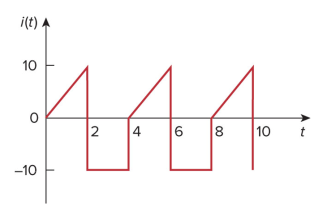
The average power absorbed by a resistor can be written:

$$P = I_{rms}^2 R = \frac{V_{rms}^2}{R}$$

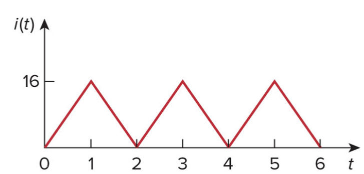
Ch 11, p11

Example and Practice Problems

E11.7 Determine the rms value of the current waveform shown and, if the current passes through a 2-ohm resistor, find the average power absorbed by the resistor.



P11.7 Find the rms value of the current waveform and, if the current flows through a 9-ohm resistor, calculate the average power absorbed by the resistor.



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11.5 Apparent Power and Power Factor

Average power delivered to a resistive load by a time-varying circuit, when written in terms of rms values, is: $P = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$

The product $V_{rms} I_{rms}$ is known as *apparent power* (*S*) and is modulated by the phase difference between the voltage and current [$\cos(\theta_v - \theta_i)$], which is referred to as the *power factor* (pf).

The apparent power is the product of the rms values of voltage and current, and is measured in volt-amperes (VA) to distinguish it from the average or real power, which is measured in watts.

The power factor is the cosine of the phase difference between voltage and current, and is also the cosine of the angle of the load impedance. The power factor is dimensionless since it is the ratio of the average power (*P*) to the apparent power ($V_{rms} I_{rms}$).

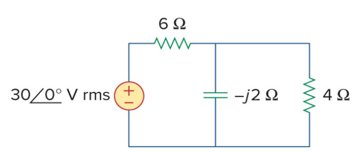
pf ranges between 0 and 1. For a purely resistive load, the voltage and current are in phase, so that $(\theta_v - \theta_i)$ =0 and pf = 1, which implies that the apparent power is equal to average power. For a purely reactive load, $(\theta_v - \theta_i) = \pm 90^o$ and pf =0. In this case, the average power is 0. Between these two extremes pf is said to be *leading* (current leads voltage, implying a capacitive load) or *lagging* (current lags voltage, implying an inductive load.

Example and Practice Problems

E11.9 A series-connected load draws a current *i(t)* = 4 cos (100πt + 10°) A while the applied voltage is *v(t) = 120 cos (100πt - 20°)* V. Find the apparent power, the power factor, and determine the element values that form the series-connected load.

P11.9 Find the power factor and the apparent power of a load whose impedance is **Z** = 60 + *j*40 ohms when the applied voltage is *v(t)* = 155.56 cos(377*t* + 10°) V.

E 11.10 Determine the power factor of the circuit shown below as seen by the source, and calculate the average power delivered by the source.



11.6 Complex Power

The term *complex power* refers to the total effect of parallel loads on power.

The complex power, **S**, absorbed by an AC load is the product of the rms voltage phasor and the complex conjugate of the rms curent phasor:

$$S = \frac{1}{2}VI^* = I_{rms}^2 Z = \frac{V_{rms}^2}{Z^*} = V_{rms} I_{rms}^*$$

As a complex quantity, its real part is the average or real power (*P*) and its imaginary part is reactive (or quadrature) power (*Q*).

The complex power expressed in rectangular form is: $S = I_{rms}^2(R + jX) = P + jQ$

where: $P = \mathrm{Re}(S) = I_{rms}^2 R$

$Q = \mathrm{Im}(S) = I_{rms}^2 X$

The real power is the only useful power. It is measured in watts.

The reactive power is a measure of the energy exchange between the source and the reactive load, and is measured in units of volt-ampere reactive (VAR)

Summarizing Power

Complex Power= $S = P + jQ = V_{rms}(I_{rms})^*$

$= |V_{rms}||I_{rms}|\angle(\theta_v - \theta_i)$

Apparent Power= $S = |S| = |V_{rms}||I_{rms}| = \sqrt{P^2 + Q^2}$

Real Power= $P = \mathrm{Re}(S) = S\sin(\theta_v - \theta_i)$

Reactive Power= $Q = \mathrm{Im}(S) = S\sin(\theta_v - \theta_i)$

Power Factor= $\frac{P}{S} = \cos(\theta_v - \theta_i)$

Example and Practice Problems

E11.11 The voltage across a load is $v(t) = 60 \cos(\omega t - 10^\circ)$ V and the current through the element in the direction of the voltage drop is $i(t) = 1.5 \cos(\omega t + 50^\circ)$ A. Find (a) the complex and apparent powers, (b) the real and reactive powers, and (c) the power factor and the load impedance.

E11.12 A load **Z** draws 12 kVA at a power factor of 0.856 lagging from a 120-V_{rms} sinusoidal source. Calculate (a) the average and reactive powers delivered to the load, (b) the peak current, and (c) the load impedance.

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11.7 Conservation of AC Power

The principle of conservation of power (energy) applies to AC circuits as well as to DC circuits.

In fact, all forms of AC power are conserved: instantaneous, real, reactive, and complex.

Regardless of how circuit elements are connected, the total complex power delivered is equal to the total complex power absorbed by the elements.

The complex, real, and reactive powers of the sources equal the respective sums of the complex, real, and reactive powers of the individual loads.

NOTE: this is not true of apparent power.

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11.8 Power Factor Correction

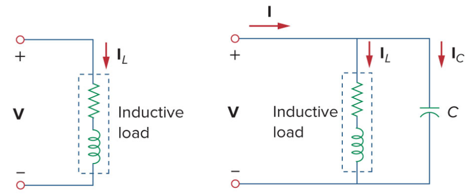
The process of increasing the power factor (pf) without altering the voltage or current to the original load is known as *power factor correction*.

Alternatively, power factor correction may be viewed as the addition of a reactive element (usually a capacitor) in parallel with the load in order to make the power factor closer to unity.

Most domestic and industrial loads, such as washing machines, air conditioners, and induction motors are inductive and have a low, lagging power factor.

The load cannot be changed, but the power factor can be increased without altering the voltage or current to the original load.

To mitigate the inductive aspect of the load, a capacitor is added in parallel with the load.



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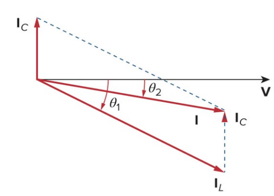
With the same supplied voltage, the current draw is less by adding the capacitor, which is a benefit since power companies charge more for larger currents because it leads to larger power losses.

Overall, the power factor correction benefits the power company and the consumer.

By choosing a suitable size for the capacitor, the power factor can be made to approach or become unity.

Adding a capacitor causes the phase angle between the voltage and current $(\theta_v - \theta_i)$ to become smaller, and the more it approaches 0° the more the pf approaches its maximum of 1.



The value of the shunt capacitance can be found from:

$$C = \frac{Q_c}{\omega V_{rms}^2} = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{rms}^2}$$

Be careful with what θ_1 and θ_2 represent.**

Note that the real power dissipated in the load is not affected by the shunt capacitor.

Although it is not as common, if a load is capacitive in nature, the same treatment with an inductor can be used.

** θ_1 is the power factor $[\cos(\theta_v - \theta_i)]$ for the circuit without the shunt capacitor.
θ_2 is the power factor $[\cos(\theta_v - \theta_i)]$ for the circuit with the shunt capacitor.

CH 11, p21

Example and Practice Problems

E11.15 When connected to a 120-V_{rms}, 60-Hz power line, a load absorbs 4 kW at a lagging power factor of 0.8. Find the value of capacitance necessary to raise the pf to 0.95.

P11.15 Find the value of parallel capacitance needed to correct a load of 140 kVAR at 0.85 lagging pf to unity pf. Assume that the load is suppled by a 220-V (rms), 60-Hz line.

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