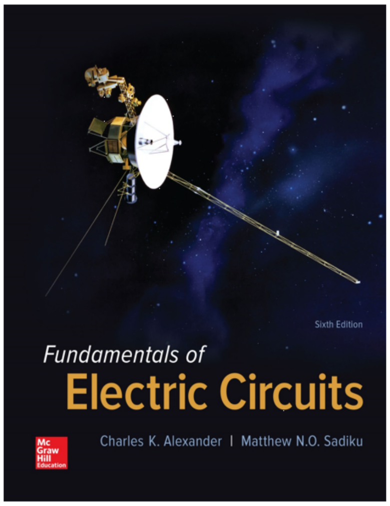


Chapter 4 - Circuit Theorems

- In this chapter, the concept of superposition will be introduced.
- Source transformation will also be covered.
- Thevenin and Norton's theorems will be covered.
- Examples of applications for these concepts will be presented.

4.1 Introduction

The use of Kirchhoff's laws and Ohm's law in the analysis of large, complex circuits can lead to tedious computations. To handle the analysis of such circuits, and to reduce some of the calculations required, engineers have developed some theorems to simplify the analysis of complex, linear circuits.

4.2 Linearity Property

A linear circuit is one whose output is linearly related (or directly proportional) to its input.

Phrased differently, a circuit is linear if it is both homogeneous and additive.

Homogeneity requires that if the input (aka excitation) is multiplied by a constant, then the output (aka response) is multiplied by the same constant.

A resistor is an example of a linear circuit element since the input (current) is related to the output (voltage) by a constant (R):

$$v = iR$$

If the current is increased by a constant k, then the voltage correspondingly increases by the same amount:

$$kiR = kv$$

Being additive requires that the response to a sum of inputs is the sum of the responses to each input applied separately.

Using the current-voltage relationship of a resistor as an example:

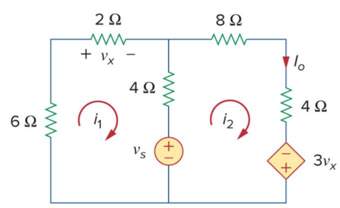
$v_1 = i_1 R$ $\quad$ $v_2 = i_2 R$ $\quad$ $v_3 = i_3 R$

$v = (i_1 + i_2 + i_3)R = i_1 R + i_2 R + i_3 R = v_1 + v_2 + v_3$

Ch 4, p3

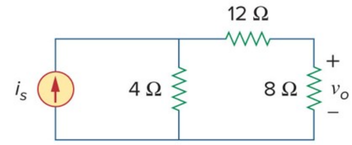
Example and Practice Problems

E4.1 Find I_0 when v_S = (a) 12 V and (b) 24 V.

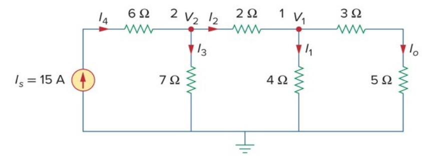


Example and Practice Problems (Cont.)

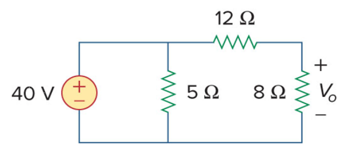
P4.1 Find v_0 when i_s = (a) 30 A and (b) 45 A.



E4.2 Assume I_0 = 1 A and use linearity to find the actual value of I_0 in the following circuit.



P4.2 Assuming that V_O = 1 V, use linearity to calculate the actual value of V_O in this circuit.



Ch 4, p4

4.3 Superposition

If there are two or more independent sources in a circuit, there are two ways to solve for the circuit parameters:

– Nodal or mesh analysis

– Use superposition

The ***superposition principle*** states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

In applying this principle, two things must be kept in mind:

1. One independent source is considered at a time while all other independent sources are turned off. This implies that the voltage sources to be turned off are replaced with 0 V (or a short circuit) and every current source that is turned off is replaced by 0 A (or an open circuit).
2. Dependent sources are left intact because they are controlled by circuit variables.

Ch 4, p5

Steps to applying the Superposition Principle

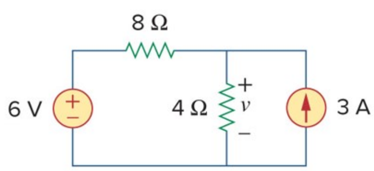
1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using the techniques covered in Chapters 2 and 3.
2. Repeat step 1 for each of the other independent sources.
3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

This sequence of steps suggests one major disadvantage to use of superposition - more work may very well be required since analysis of multiple, simple (or simpler) circuits may be required in order to complete the analysis of a single, complex circuit.

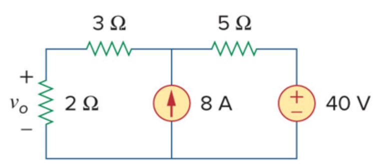
It's also important to remember that superposition is based on linearity. As a result, it is not applicable to the determination of power due to each source since, for example, the power absorbed by a resistor depends on the square of the voltage or current. If the power is needed, the current through or voltage across the element has to be found first using superpostion.

Example and Practice Problems

E4.3 Use superposition to find *v* in this circuit:

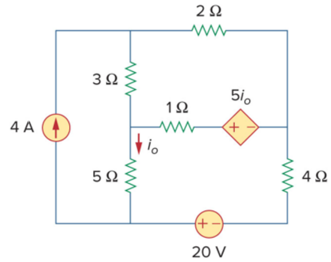


P4.3 Use superposition to find v_0 in this circuit:

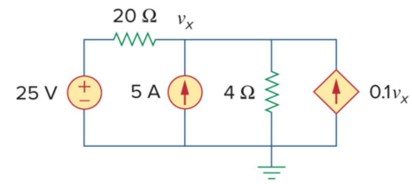


Example and Practice Problems (Cont.)

E4.4 Find I_0 in the circuit shown below using superposition.



P4.4 Use superposition to find v_x in this circuit:

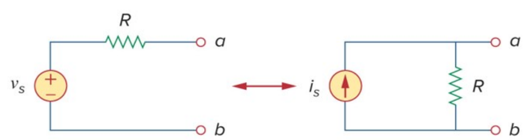


4.4 Source Transformation

Much like the delta-wye transformation, it is possible to transform a source from one form to another, and this can be useful for simplifying circuits.

The principle behind all of these transformations is equivalence.

A ***source transformation*** is the process of replacing a voltage source v_s in series with a resistor *R* by a current source i_s in parallel with a resistor *R*, or vice versa.

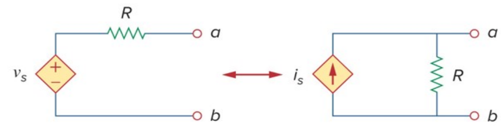


These two circuits are equivalent since they have the same voltage-current relationship at terminals *a-b*.

- If the sources are turned off the resistance at the terminals are both R.
- If the terminals are short circuited, the currents need to be the same.
  - From this we get the following requirement: $v_s = i_sR$ or $i_s = v_s/R$

Source transformation also applies to dependent sources provided that special care is taken with the dependent variable.

The same relationship between the voltage and current holds here:



where care is taken to be sure that $v_s = i_s R$ or $i_s = v_s/R$

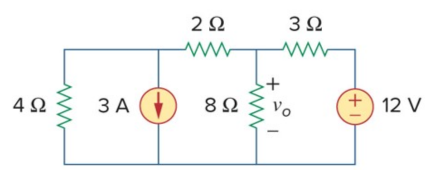
Source transformation "rules"

- The arrow of the current source points in the same direction as the positive terminal of the voltage source.
- Source transformation is not possible when R=0 for an ideal voltage source. For a realistic source, $R \neq 0$.
- For an ideal current source $R = \infty$, which also prevents the use of source transformation.

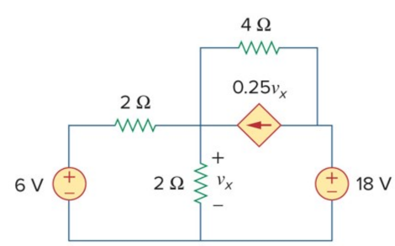
Ch 4, p10

Example and Practice Problems

E4.6 Use source transformation to find v_0:



E4.7 Find v_x using source transformation.



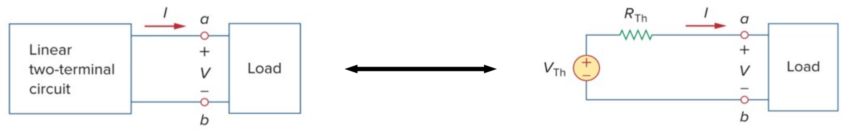
Ch 4, p11

4.5 Thevenin's Theorem

In practice, it's not at all unusual for one or more elements in a circuit to be variable (often referred to as the *load*) while the other elements are fixed. [For example, think of a household outlet with various appliances (i.e., variable elements/loads) connected to it.]

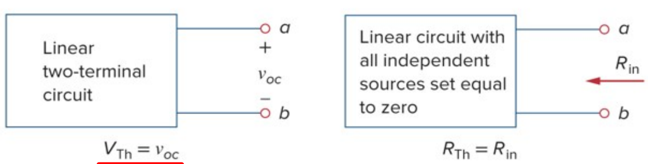
The analysis of the complete circuit would have to be repeated each time the variable elements/loads changed. To avoid this problem, Thevenin's theorem provides a technique by which the fixed part of the circuit is replaced by an equivalent (and much simpler) circuit.

Thevenin's theorem states that a linear, two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source (V_{Th}) in series with a resistor (R_{Th}) where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.



Two circuits are said to be equivalent if they have the same voltage-current relation at their terminals.

For two circuits to be equivalent, therefore, the following conditions must be met:



There are two possible types of circuits to be replaced by the Thevenin equivalent resistance:

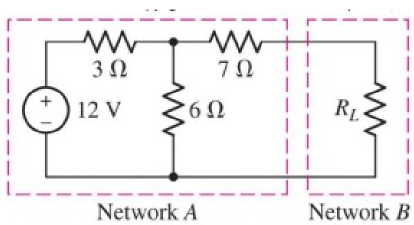
Type 1: Contains only independent sources

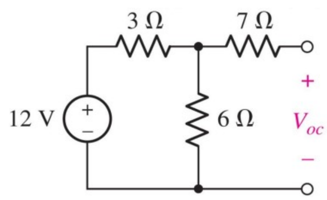
Type 2: Contains dependent sources

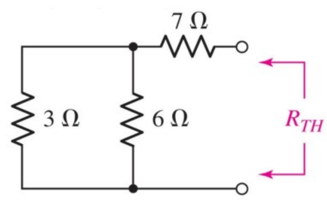
Type 1: Only Independent Sources

– Find V_{oc} at terminals A & B

– Turn off sources and solve for R_{Th}







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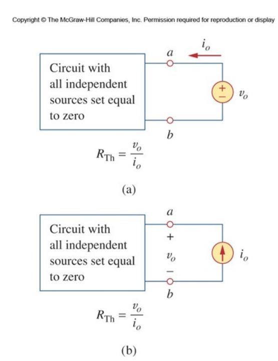
Type 2: Both independent and dependent sources

-- Turn off the independent sources

-- Apply a voltage source (v_o) [or current source (i_o)] at the terminals and find the resulting current (i_o) [or voltage (v_o)].



Ch 4, p15

Thevenin's theorem is very powerful in circuit analysis since:

a large circuit may be replaced by a single independent voltage source and a single resistor.

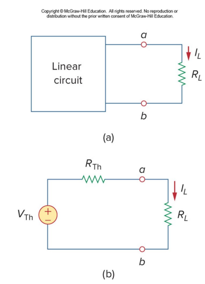
the equivalent circuit behaves externally exactly the same as the original circuit.

It is possible for the result of this analysis to end up with a negative resistance, which implies the circuit is supplying power and is reasonable with dependent sources.

Note that in the end, the Thevenin equivalent makes working with variable loads much easier.

Load current can be calculated with a voltage source and two series resistors

Load voltages use the voltage divider rule.

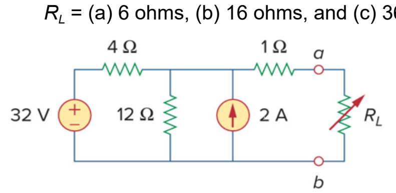


$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

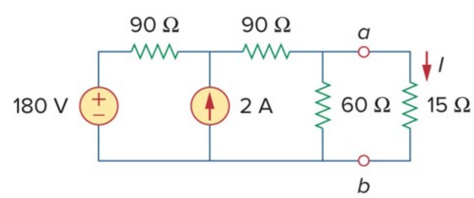
$$V_L = R_L I_L = \frac{R_L}{R_{Th} + R_L} V_{Th}$$

Example and Sample Problems

E4.8 Find the Thevenin equivalent circuit to the left of terminals a-b, and then find the current through R_L = (a) 6 ohms, (b) 16 ohms, and (c) 36 ohms.

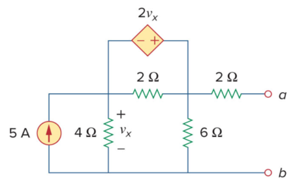


P4.8 Find the Thevenin equivalent circuit to the left of terminals a-b, and then find *I*.

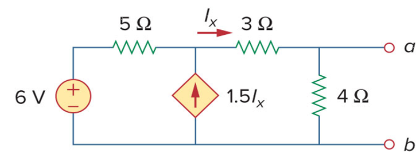


Example and Sample Problems (Cont.)

E4.9 Find the Thevenin equivalent circuit to the left of terminals a-b.



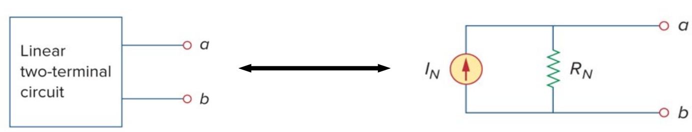
P4.9 Find the Thevenin equivalent circuit to the left of the terminals in the circuit shown below.



4.6 Norton's Theorem

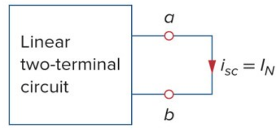
Similar to Thevenin's theorem, Norton's theorem states that a linear two terminal circuit may be replaced with an equivalent circuit containing a resistor and a current source.

Norton's theorem states that a linear, two-terminal circuit can be replaced by an equivalent circuit consisting of a current source (I_N) in parallel with a resistor (R_N) where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.



It turns out that the Thevenin and Norton resistances are equal: $R_N = R_{Th}$

The Norton current I_N is found by short circuiting the circuit's terminals and measuring the resulting current.



There is a close relationship between Norton's and Thevenin's theorems, and the Norton current and Thevein voltage are related to each other as follows:

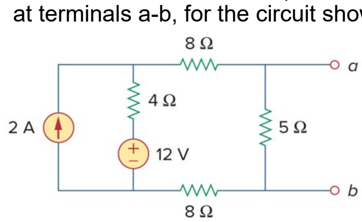
$$I_N = V_{Th}/R_{Th}$$

With V_{Th}, I_N, and R_{Th} (= R_N) related, finding the Thevenin or Norton equivalent circuit requires that we find:

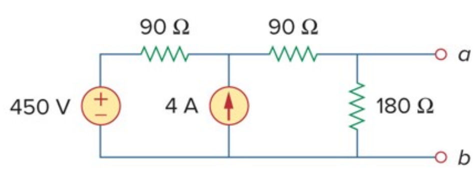
- the open-circuit voltage across terminals a and b.
- the short-circuit current at terminals a and b.
- the equivalent or input resistance at terminals a and b when all independent sources are turned of

Example and Sample Problems

E4.11 Find the Norton equivalent circuit, at terminals a-b, for the circuit shown below.

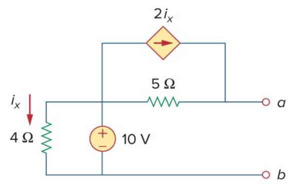


P4.11 Find the Norton equivalent circuit for this circuit at terminals a-b:

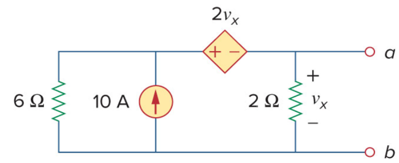


Example and Sample Problems (Cont.)

E4.12 Using Norton's theorem, find I_N and R_N of the circuit shown below at terminals a-b:



P4.12 Find the Norton equivalent circuit at terminals a & b of the circuit shown below:



4.8 Maximum Power Transfer

In many applications, a circuit is designed to power a load and, among those applications, there are many cases where it is desirable to maximize the power transferred to the load.

Unlike an ideal source, internal resistance will restrict the conditions where maximum power is transferred.

The Thevenin equivalent is useful in finding the maximum power a linear circuit can deliver to a load since, for a given circuit, V_{Th} and R_{Th} are fixed.

The *maximum power theorem* states that maximum power is transferred to a load when the load resistance equals the Thevenin resistance as seen from the load (i.e., $R_L = R_{Th}$).

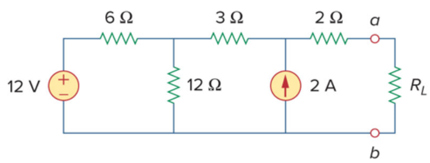
The source and load are said to be matched when $R_L = R_{Th}$

The power transferred is given by: $p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$

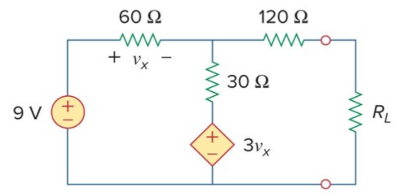
When the source and load are matched, this equation becomes: $p_{max} = \frac{V_{Th}^2}{4R_{Th}}$

Example and Sample Problems

E4.13 Find the value of R_L for maximum power transfer then find the maximum power.



P4.13 Determine the value of R_L that will draw the maximum power from the circuit shown below and then calculate the maximum power.



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