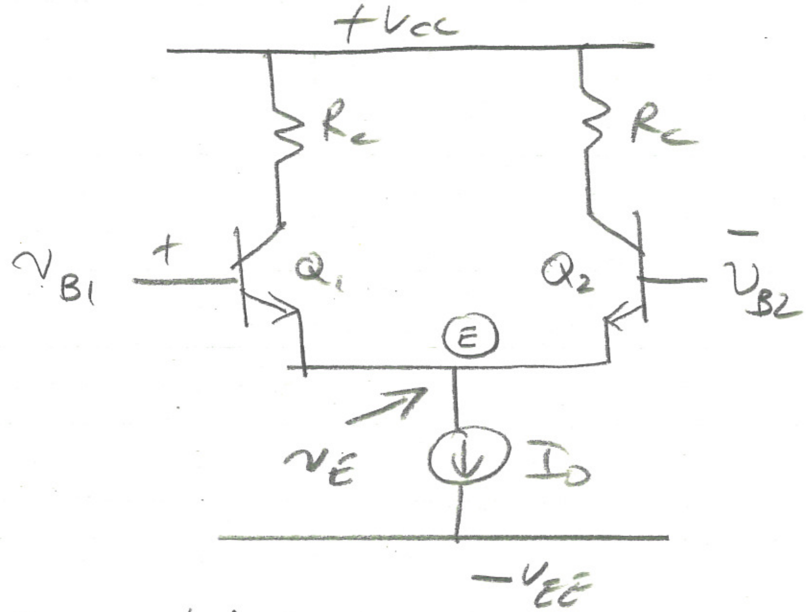
ECE 426 Ch 9 Lecture Notes

Large signal behavior of differential amplifier.



Q_1, Q_2 matched

$V_A = \infty, \quad \beta = \infty$

$$i_{E1} + i_{E2} = I_0 \quad \leftarrow \text{KCL at node } E$$

But $i_{E1} = I_{ES} e^{v_{BE1}/V_T}$ (1), $i_{E2} = I_{ES} e^{v_{BE2}/V_T}$ (2)

Take eqn (1) and divide by (2)

$$\frac{i_{E1}}{i_{E2}} = e^{\frac{v_{BE1} - v_{BE2}}{V_T}}$$

But $v_{BE1} - v_{BE2} = v_{B1} - v_E - (v_{B2} - v_E) = v_{B1} - v_{B2}$

We define $v_{ID} = v_{B1} - v_{B2}$

$$\frac{i_{E1}}{i_{E2}} = e^{v_{ID}/V_T} \quad \text{or} \quad \frac{i_{E2}}{i_{E1}} = e^{-v_{ID}/V_T}$$

KCL at node E

$$i_{E1} + i_{E2} = I_0 \Rightarrow i_{E1}\left(1 + \frac{i_{E2}}{i_{E1}}\right) = I_0$$

& $$i_{E2}\left(1 + \frac{i_{E1}}{i_{E2}}\right) = I_0$$

So $$i_{E1}\left(1 + e^{-v_{ID}/V_T}\right) = I_0$$

$$i_{E2}\left(1 + e^{+v_{ID}/V_T}\right) = I_0$$

For $\beta = \infty$ $\quad i_E = i_C$

$$i_{C1} = \frac{I_0}{1 + e^{-v_{ID}/V_T}} \; ; \; i_{C2} = \frac{I_0}{1 + e^{+v_{ID}/V_T}}$$

See handout

Note $v_{ID} = +100\,mV$ then

$i_{C1} \simeq I_0 \qquad i_{C2} \simeq 0$

$0.98\,I_0 \qquad\qquad 0.02\,I_0$

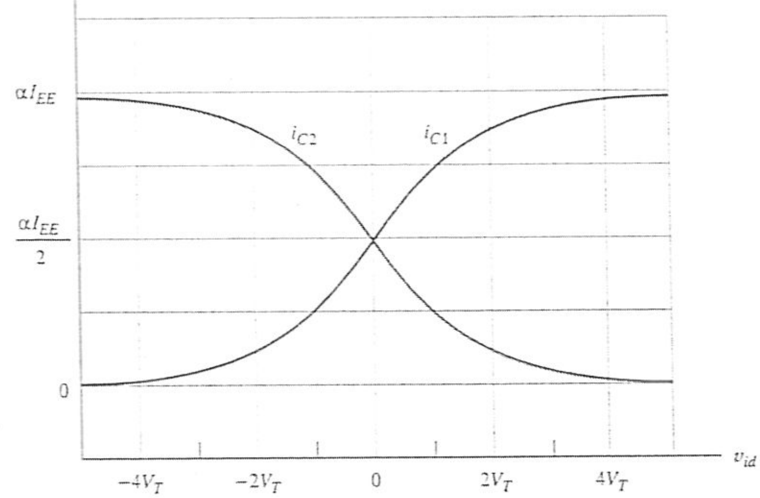


Figure 7.25 Collector currents versus differential input voltage.

If we use Equations (7.44) and (7.45) to substitute for the currents in Equation (7.26), we can eventually reduce the result to

$$v_{od} = \alpha I_{EE} R_C \tanh\left(\frac{-v_{id}}{2V_T}\right) \tag{7.46}$$

in which tanh is the **hyperbolic tangent** function. By definition, the hyperbolic tangent is

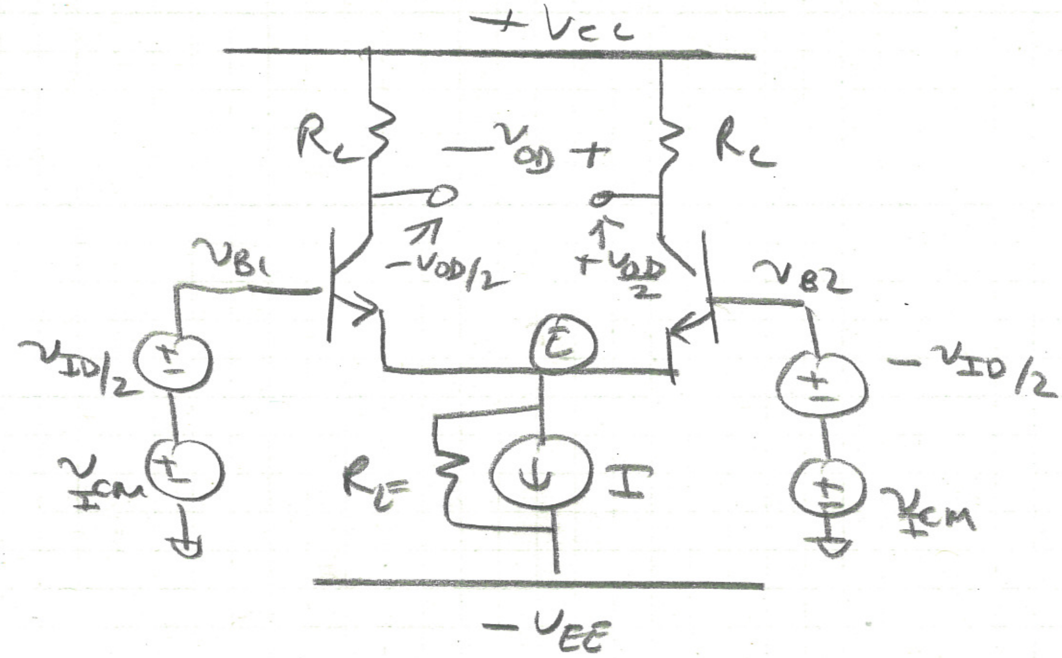
$$\tanh(x) = \frac{\exp(x) - \exp(-x)}{\exp(x) + \exp(-x)} \tag{7.47}$$

A plot of the transfer characteristic given by Equation (7.46) is illustrated in Figure 7.26. The curvature of the transfer characteristic shows that the differential amplifier can distort a signal if the amplitude is too large. For $|v_{id}| < V_T$, the characteristic is fairly straight, and the distortion is not excessive. Notice that if the magnitude of the input voltage exceeds $4V_T \cong 100\,\text{mV}$, the output will be severely clipped. (Sometimes we use the circuit as a waveshaper to convert a sinusoidal input into a square wave. It is quite effective for this purpose if the peak input is one volt or more.)

Emitter Degeneration

Sometimes it is advantageous to add emitter degeneration resistors R_{EF} to the circuit, as shown in Figure 7.27. (*Degeneration* is the term for negative feedback, which we consider in detail in Chapter 9.) We will see that these resistors have the disadvantage of reducing the differential voltage gain of the circuit. However, two reasons for including the resistors are to increase input impedance and to reduce distortion due to the nonlinearity of the BJTs. (These are also the reasons for using an unbypassed emitter resistor in a common-emitter amplifier.)

Small-signal analysis of BJT differential pair.

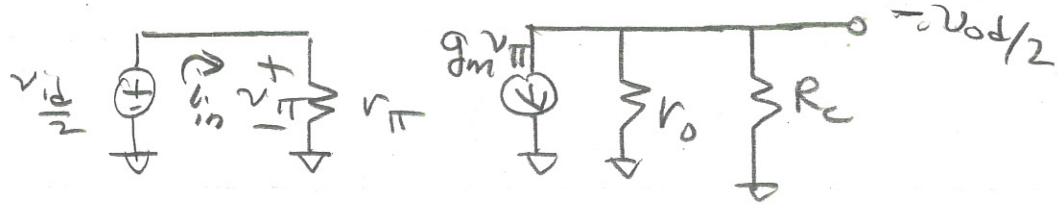


$$v_{ID} = v_{B1} - v_{B2} = (v_{ICM} + v_{ID}/2) - (v_{ICM} - v_{ID}/2)$$
$$= v_{ID} \checkmark$$

$$v_{CM} = \frac{v_{B1} + v_{B2}}{2} = \frac{(v_{ID}/2 + v_{ICM}) + (-\frac{v_{ID}}{2} + v_{ICM})}{2}$$
$$= v_{CM} \checkmark$$

If there are no signal changes in the common-mode signal but if we do allow for small-signal changes in the differential signal, then node (E) becomes a virtual ground for differential signals

We only have to analyze the half-ckt (4)
shown below.



This is a CE amp

$$\frac{-v_{od}/2}{v_{id}/2} = -g_m(R_C \| r_o) \simeq -g_m R_C \quad \text{if } r_o \text{ large!}$$

$$\boxed{A_{dm} \equiv \frac{v_{od}}{v_{id}} \simeq +g_m R_C}$$

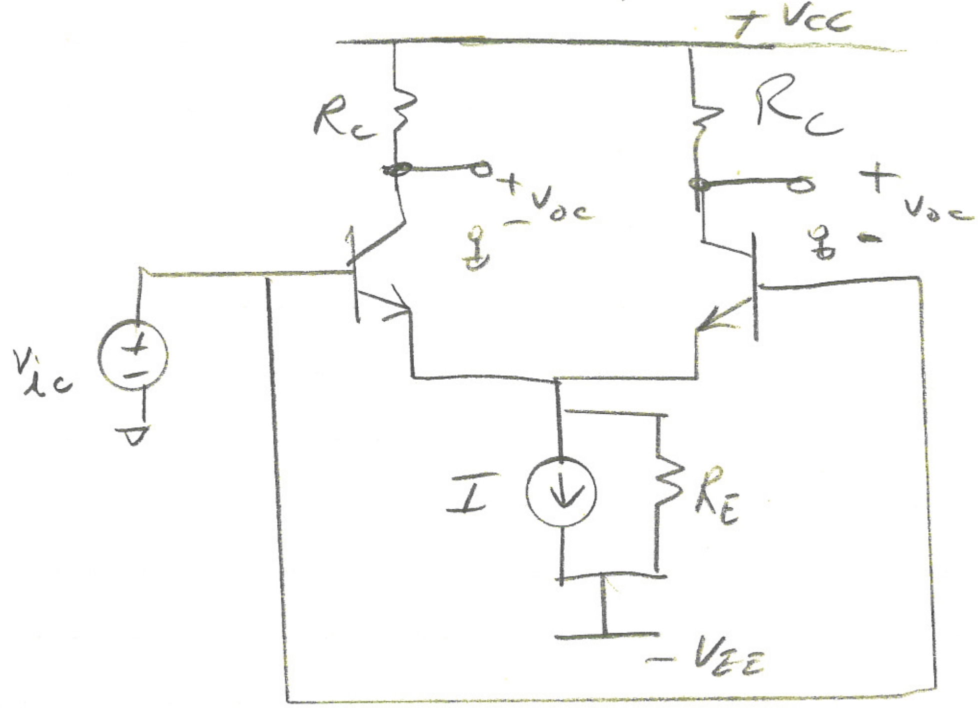
differential mode gain!

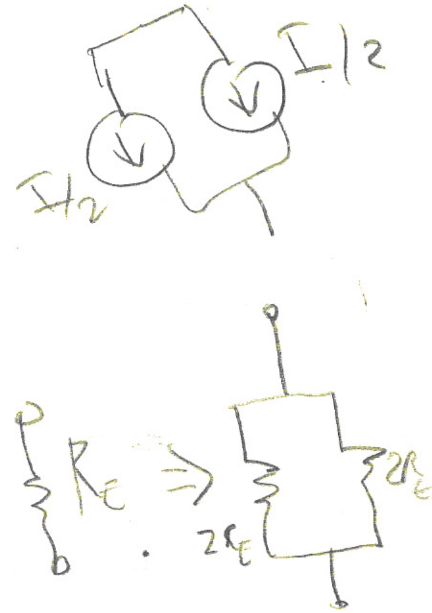
$$i_{in} = \frac{\left(\frac{v_{id}}{2}\right)}{r_\pi} \Rightarrow \frac{v_{id}}{i_n} = 2r_\pi$$

So $\boxed{R_{IN} = \frac{v_{id}}{i_{in}} = 2r_\pi}$

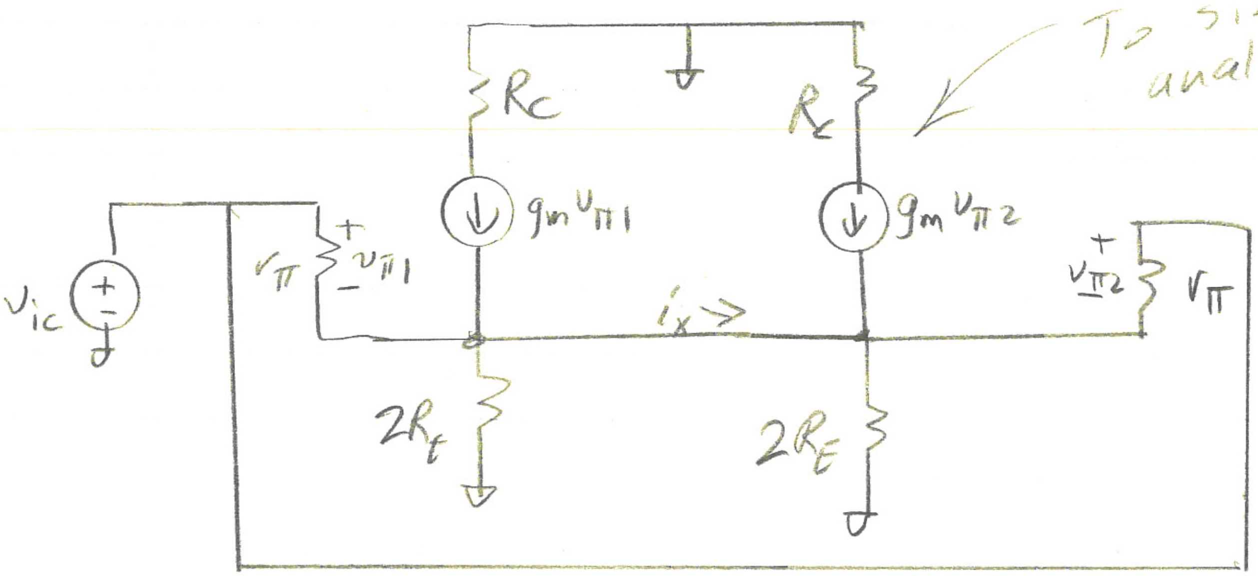
Differential input resistance.

Emitter-coupled pair w/ pure common mode input

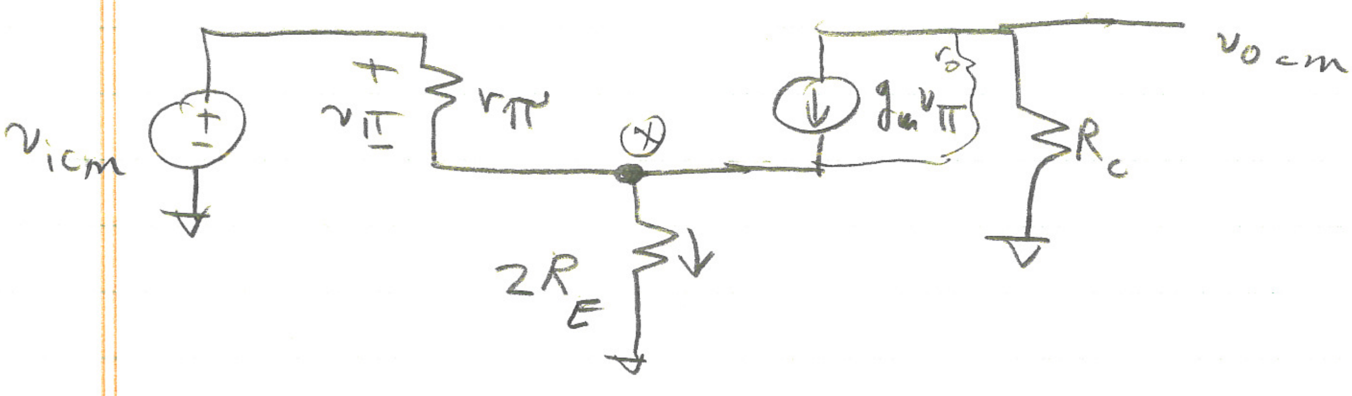




Small-signal equivalent ckt @ pure common-mode input.



To simplify analysis we'll assume ∞ output resistance which we neglected it w/ the diff mode ckt at end anyhow



$$v_{ocm} = -(g_m v_\pi) R_c$$

KCL at x

But $$\frac{v_{icm} - v_\pi}{2R_E} = g_m v_\pi + g_\pi v_\pi$$

$$v_{icm} - v_\pi \approx g_m v_\pi (2R_E)$$

$$v_{icm} \approx v_\pi (1 + g_m 2R_E)$$

so $$\boxed{v_{ocm} \cong \frac{-g_m R_c}{(1+2g_m R_E)}} \approx -\frac{R_c}{2R_E}$$

if $2g_m R_E \gg 1$

With r_o included

$$\frac{v_x}{2R_E} = g_\pi v_\pi + g_m v_\pi + g_o(v_{ocm} - v_x)$$

$$v_x = v_{icm} - v_\pi$$

$$\frac{v_{ocm}}{R_c} + (v_{ocm} - v_x) g_o + g_m v_\pi = 0$$

$$\frac{v_{ocm}}{v_{icm}} = \frac{-g_m R_c + 2R_c R_E g_o g_\pi}{1 + 2g_m R_E + \left[R_c g_o + 2R_E g_o + 2R_E g_\pi + 2R_c R_E g_o g_\pi\right]}$$

CMRR (Common-Mode Rejection Ratio)

Conventionally defined as the magnitude of the ratio of differential to common-mode gain

$$CMRR \equiv \left| \frac{A_{dm}}{A_{cm}} \right|$$

So for differential pair

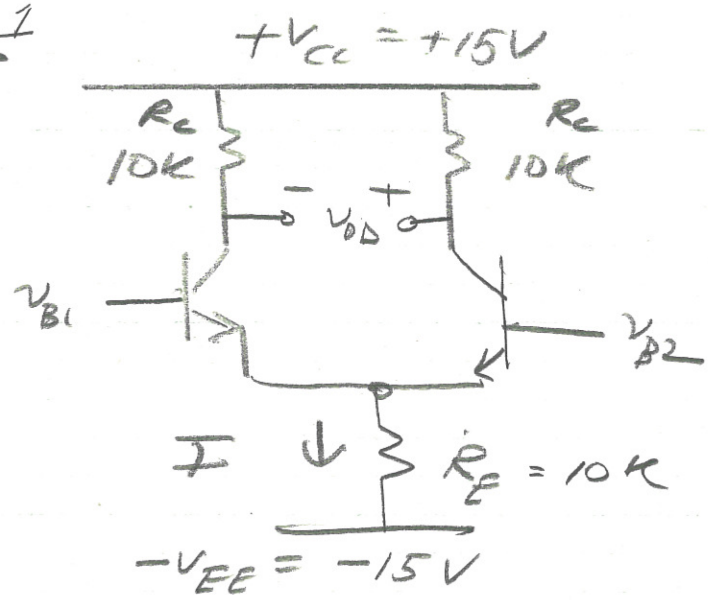
$$CMRR = \left| \frac{-g_m R_C}{\left(\frac{-g_m R_C}{1 + 2 g_m R_E} \right)} \right|$$

$$\boxed{CMRR = |1 + 2 g_m R_E|}$$

Express in dB

$$\boxed{CMRR_{dB} = 20 \log_{10} |1 + 2 g_m R_E|}$$

Example 1



Let $v_{B1} - v_{B2} = v_{ID}$. Assume $v_{CM} = 0$

Also assume $V_{BE(ON)} = 0.7V$ and $\beta = 100$ and $V_A = \infty$.

Find I. $I = \dfrac{-0.7 - (-15V)}{10K\Omega} = 1.43mA$

What is $A_{dm} = \dfrac{v_{OD}}{v_{ID}}$

$$A_{dm} = g_m R_c$$

$$g_m \cong \frac{(I/2)}{V_T} = \frac{\left(\frac{1.43mA}{2}\right)}{25.9mV} = 27.6\ m\mho$$

$$A_{dm} = (27.6m\mho)(10k) = 276$$

What is A_{cm}?

$$A_{cm} = -\frac{g_m R_c}{1 + 2g_m R_E} = \frac{(27.6m\mho)(10K)}{1 + 2(27.6m\mho)(10K)}$$

$$A_{cm} = -0.5$$

What is CMRR?

$$CMRR = 1 + 2 g_m R_E = 1 + 2(27.6\,m\mho)(10k)$$

$$CMRR = 553$$

Express in dB.

$$20 \log_{10}(553) = 54.9\,dB$$

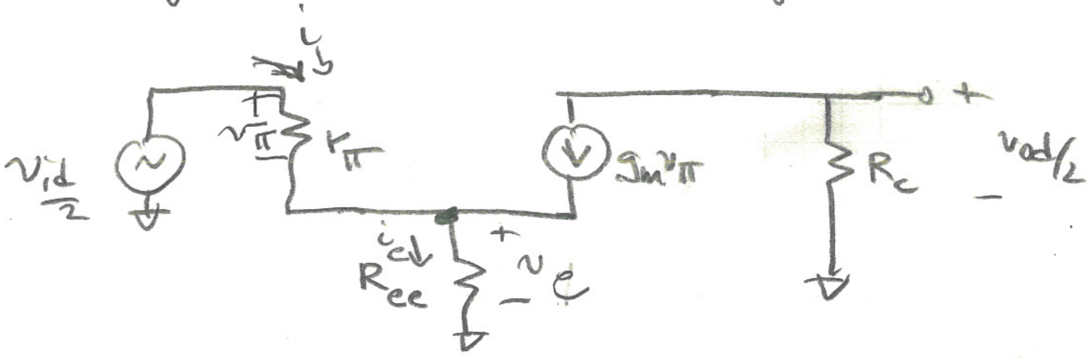
What is the differential input resistance

$$\frac{\left(\frac{v_{id}}{2}\right)}{i_{IN}} = r_\pi \Rightarrow \frac{v_{id}}{i_{IN}} = R_{id} = 2 r_\pi$$

$$R_{id} = 2\left(\frac{\beta}{g_m}\right) = 2\left(\frac{100}{27.6\,m\mho}\right) = 7.2\,K\Omega$$

$$v_O = A_{cm} v_{cm} + A_{dm} v_{id}$$

What if I have emitter degeneration? $r_o \to \infty$ (11)



$$v_{od}/2 = -g_m v_\pi^{?} R_c$$

$$v_e + v_\pi = v_{id}/2 \Rightarrow v_\pi = v_{id}/2 - v_e$$

But $v_e = R_{ee} i_e = (\beta+1) i_b = (\beta+1)\left(\frac{v_\pi}{r_\pi}\right) R_{ee}$

So $v_\pi = v_{id}/2 - (\beta+1)\frac{v_\pi}{r_\pi} R_{ee}$

$$v_\pi\left(1 + \frac{\beta+1}{r_\pi} R_{ee}\right) = \frac{v_{id}}{2}$$

$$\frac{\beta}{r_\pi} = g_m \approx \frac{\beta+1}{r_\pi}$$

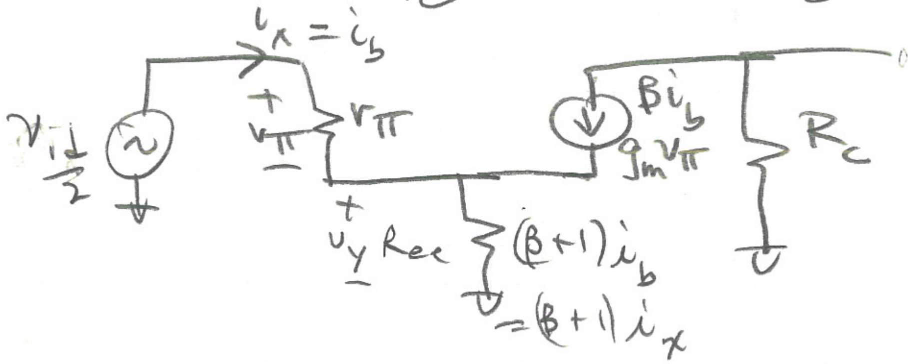
So $\boxed{v_\pi \approx \frac{\left(\frac{v_{id}}{2}\right)}{1 + g_m R_{ee}}}$

$$\frac{v_{od}}{2} = -g_m v_\pi R_c = -g_m\left[\frac{\frac{v_{id}}{2}}{1 + g_m R_{ee}}\right]$$

Hence $\boxed{A_d = \frac{v_{od}}{v_{id}} \approx -\left[\frac{g_m}{1 + g_m R_{ee}}\right] R_c} \approx -\frac{R_c}{R_{ee}}$

if $g_m R_{ee} \gg 1$

What is R_{id}? $\qquad R_{id} \equiv \frac{v_{id}}{i_b}$



$$\frac{v_{id}}{2} = v_\pi + v_y = i_b r_\pi + (\beta+1) R_e i_b$$

$$\boxed{\frac{v_{id}}{i_b} \equiv 2\left\{ r_\pi + (\beta+1) R_{ee} \right\}}$$ Very important!

$$R_{id} = 2\left[r_\pi + (\beta+1) R_{ee} \right]$$

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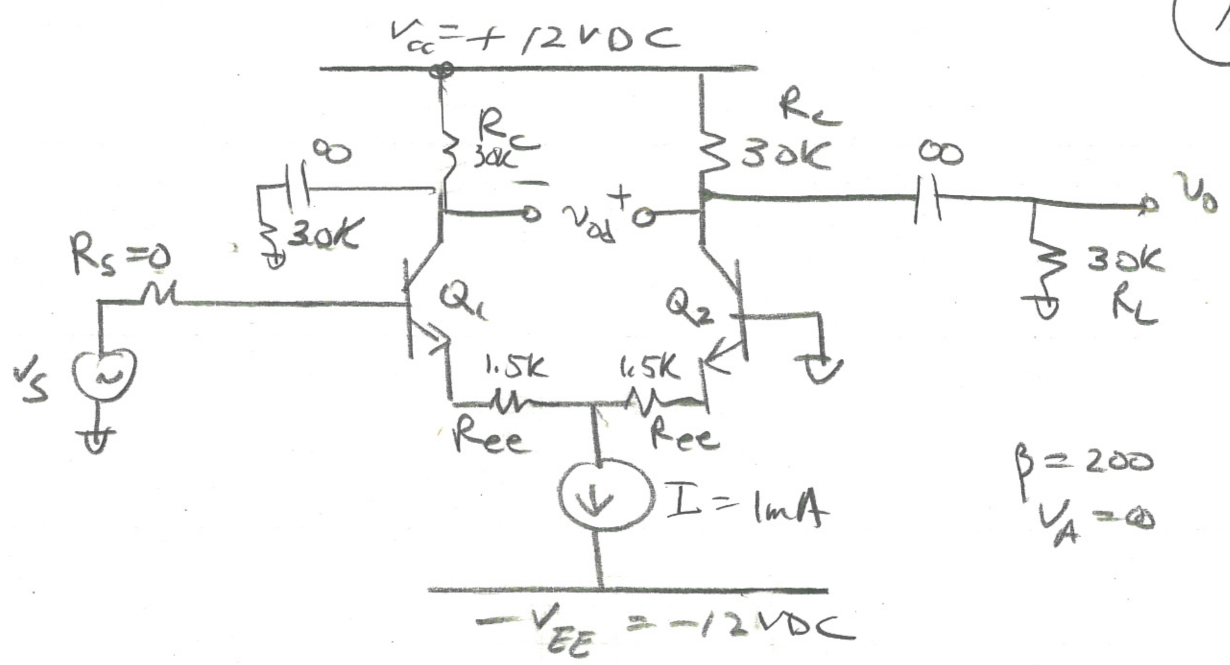

$\beta = 200$
$V_A = \infty$

(a) Find g_m, r_π (b) Find R_{id} (c) Find $\frac{v_o}{v_S}$

$I_C = 0.5$ mA

$g_m = \frac{I_C}{V_T} = \frac{0.5\,mA}{25.9\,mV} = 19.3\,m℧$

$r_\pi = \beta / g_m = \frac{200}{19.3\,m℧} \cong 10.4\,K\Omega$

$R_{id} = 2\left[r_\pi + (\beta+1) R_{ee}\right]$

$R_{id} = 2\left[10.4K + (201)(1.5K)\right] \cong 624\,K\Omega$

$\frac{v_o}{v_S} = \frac{1}{2} \frac{g_m R_C'}{1 + g_m R_{ee}} \qquad R_C' = R_C \parallel R_L$

$= \frac{1}{2}\left[\frac{(19.3\,m℧)(15k)}{1 + (19.3\,m℧)(1.5k)}\right]$

$= \frac{1}{2}\left[\frac{289.5}{29.95}\right]$

$= 4.8 \leftarrow 13.7$ dB