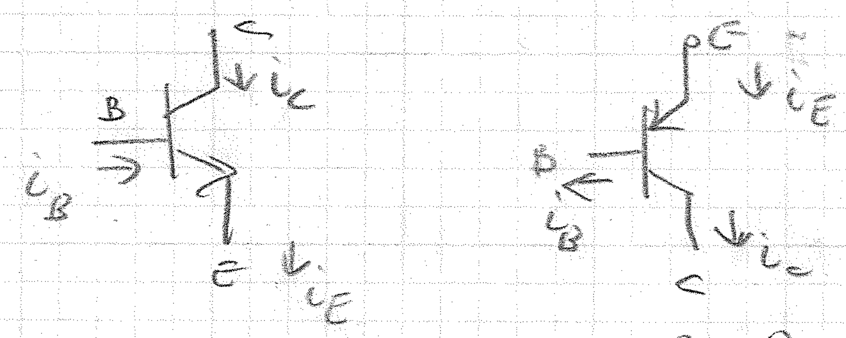
ECE 476 Spring 2017

LECTURE # 1

Review

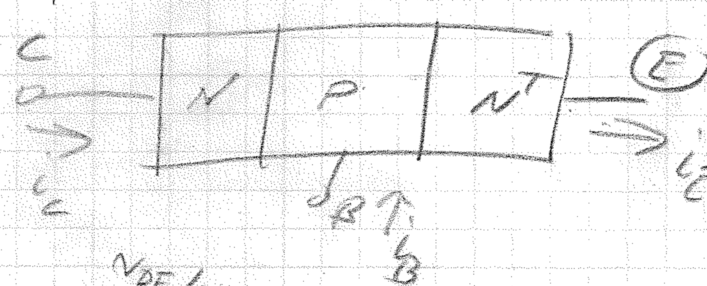


NPN PNP

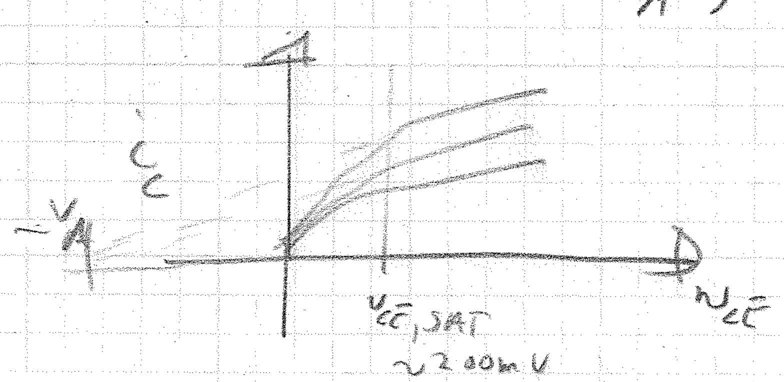
$i_E = i_B + i_C$ ALWAYS

$i_C = \beta i_B$ if ACTIVE — BE forward bias, BC reverse bias

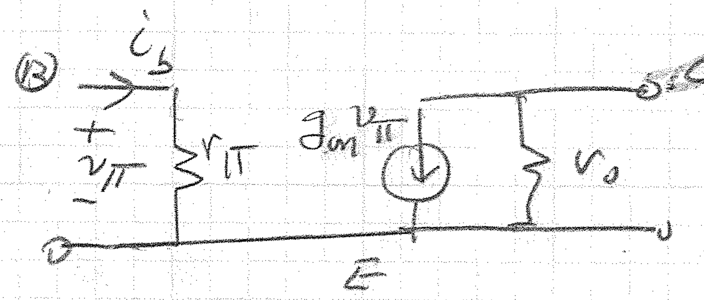
$\alpha = \frac{\beta}{\beta+1}$ $\beta = \frac{\alpha}{1-\alpha}$ $i_C = \alpha i_E$



$$i_C = I_S e^{v_{BE}/v_T} \left(1 + \frac{v_{CE}}{V_A}\right)$$



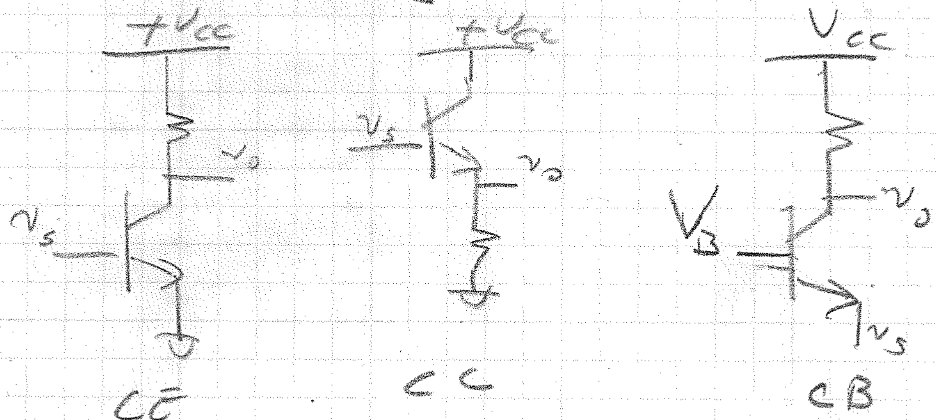
Small-signal hybrid-pi model



$g_m v_\pi = \beta i_b$

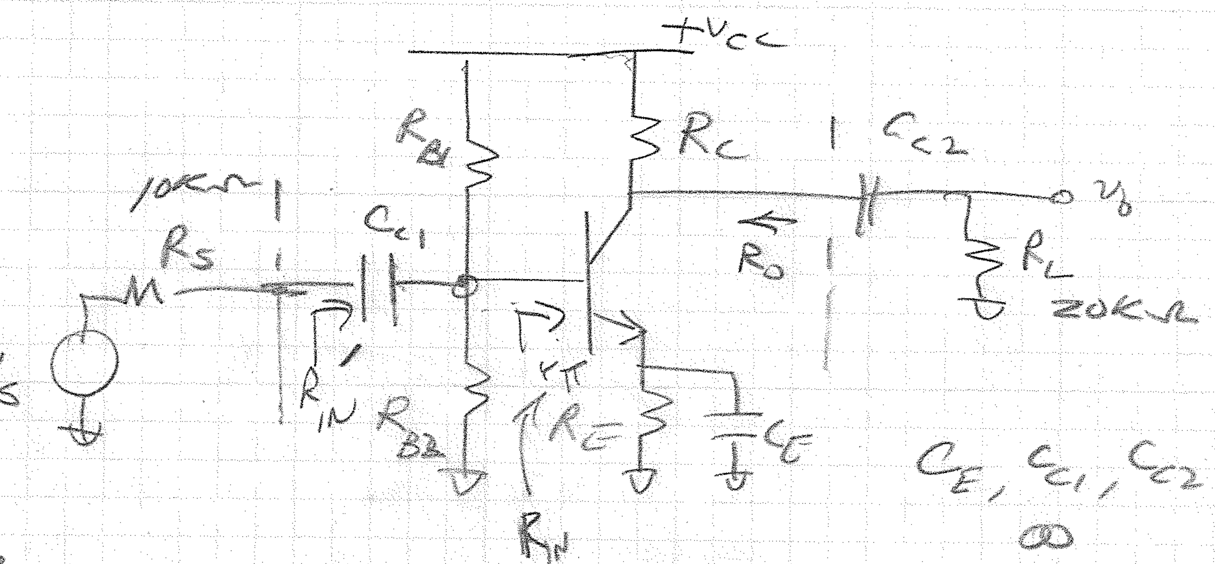
$$g_m = \frac{I_C}{V_T}, \quad r_\pi = \frac{\beta}{g_m}, \quad r_o = \frac{V_A}{I_C}$$

Amplifier topologies



Start with CE amplifier ③

See (Sect 7.5.2) page 470



Let's do exercise on p 471

Want $I_E = 0.5\,mA$, $V_C = 6V$.

Want current through R_{B2} of $50\mu A$ & $V_B = 5V$

Assume $V_{CC} = 15V$ $\beta = 100$, $V_{BE} = 0.7V$, $V_A = 100V$

Find R_C, R_E, R_{B1}, R_{B2} & A_V

Solve DC problem first!

$$I_C = \left(\frac{\beta}{\beta+1}\right) I_E = \left(\frac{100}{101}\right)(0.5\,mA) = 495\,\mu A$$

If $V_C = 6V$ then $R_C = \frac{V_{CC} - V_C}{I_C} = \frac{15-6}{0.495\,mA}$

$R_C = 18.2K \sim \boxed{18K\Omega} \leftarrow R_C$

If $I_E = 0.5\,mA$ then $I_B = \frac{500\,\mu A}{\underbrace{101}_{\beta+1}} = 4.95\,\mu A$

If $V_B = 5V$ & $I_{B2} = 50\,\mu A$, then

$$R_{B2} = \frac{5V}{50\,\mu A} = \boxed{100K\Omega} \leftarrow R_{B2}$$

Can also easily find R_E

$$V_E = V_B - V_{BE(ON)} = 5V - 0.7V = 4.3V$$

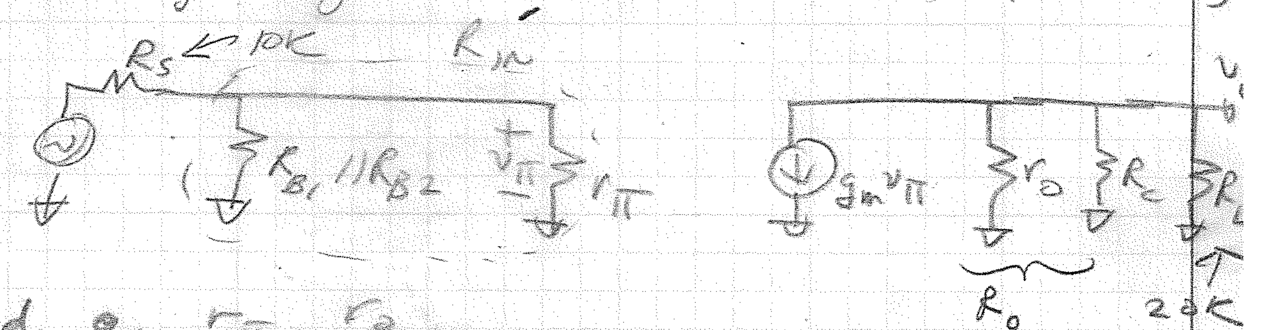
$$R_E = \frac{V_E}{I_E} = \frac{4.3V}{0.5mA} = \boxed{8.6K\Omega} \leftarrow R_E \checkmark$$

Still need to find R_{B1}!

$$I_{B1} = I_{B2} + I_B = .50mA + 4.95\mu A$$

$$R_{B1} = \frac{15 - 5V}{54.95\mu A} = \boxed{182K\Omega} \leftarrow R_{B1} \checkmark$$

Small-signal equivalent (Assume $R_S = 10K$, $R_L = 20K$)



Find g_m, r_π, r_o

$$g_m = \frac{I_C}{V_T} = \frac{495\mu A}{25.9mV} = 19.1\,m\mho \qquad \left(\text{I use } V_T = 25.9mV\right)$$

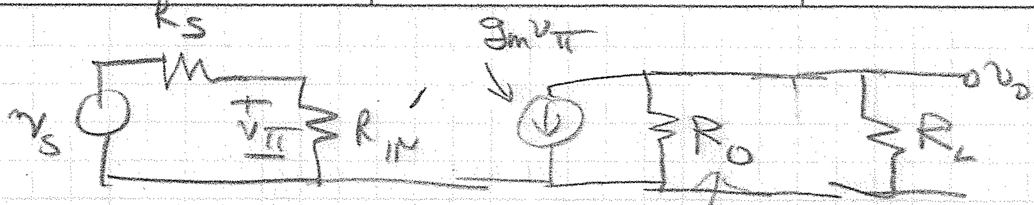
$$r_\pi = \frac{\beta}{g_m} = \frac{100}{19.1m\mho} \simeq 5.2K\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{100V}{0.495mA} = 202K\Omega$$

$$R_{IN}' = R_{B1} \| R_{B2} \| r_\pi = 4.8K\Omega \approx r_\pi$$

$$R_O = r_o \| R_C = 16.5K\Omega$$

$$R_{IN} = r_\pi$$



19.1m℧ $\quad r_o \| R_C \quad$ 4.8K

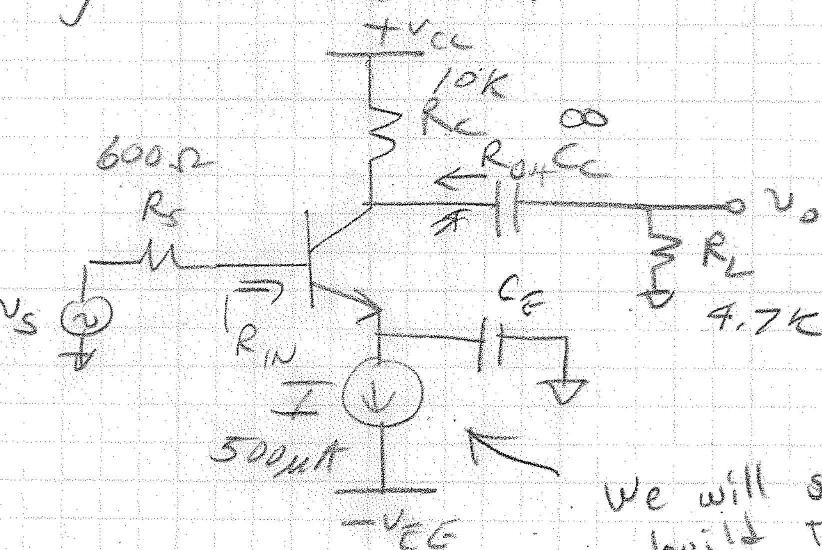
$$v_O = -g_m(R_o \| R_L)\left[\frac{R_{IN}'}{R_S + R_{IN}'}\right]$$

16.5K $\quad$ 20K $\quad$ 10K $\quad$ 4.8K

$$\underbrace{\frac{v_o}{v_S}}_{A_V} = -(172.7)(0.324) = \underbrace{-55.9 \ \frac{V}{V}}_{\sim 35 \text{ dB of gain!}}$$

$$\frac{v_o}{v_i} = -g_m(R_o \| R_L)$$

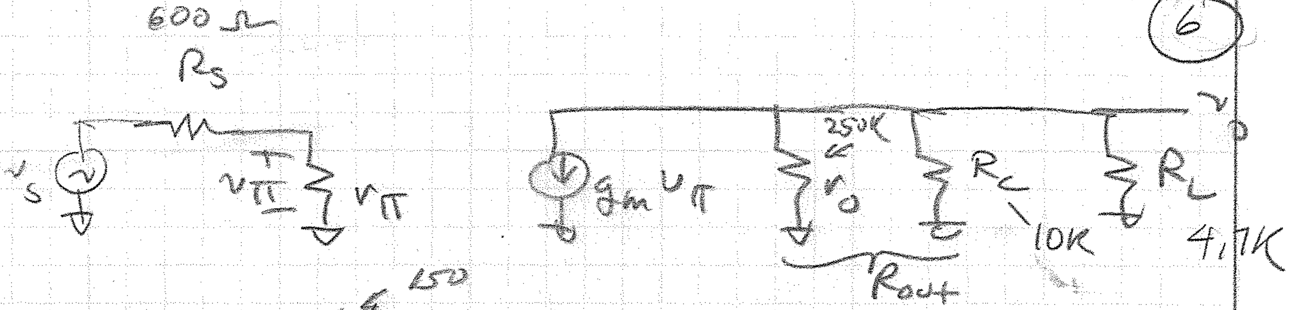
Let's try another one.



We will see how to build this in later chapter.

Assume $\beta = 150$, $V_A = 125V$

Find $A_v = \frac{v_o}{v_s}$, R_{IN}, R_{out}



$$g_m = \frac{I_C}{V_T} = \frac{\left(\frac{\overset{150}{\beta}}{\beta+1}\right)(I_E)}{V_T} \cong \frac{I_E}{V_T} = \frac{0.5\,mA}{25.9\,mV}$$

$$g_m = 19.3\,m℧$$

$$r_\pi = R_{IN} = \frac{\beta}{g_m} = \frac{150}{19.3\,m℧} \cong 7.8\,K\Omega$$

$$r_o = \frac{V_A}{I_C} \approx \frac{V_A}{I_E} = \frac{125\,V}{0.5\,mA} = 250\,K\Omega$$

$$R_O = r_o \,||\, R_C \cong 9.6\,K\Omega$$

$$A_V = -\left[\frac{R_{IN}}{R_{IN}+R_S}\right]\left[g_m\,(R_{out}\,||\,R_L)\right]$$

$$= -\left[\frac{7.8\,K\Omega}{7.8\,K\Omega + 0.6\,K}\right]\left[19.3\,m℧\,(3.2\,K\Omega)\right]$$

$$\cong -(0.9286)(61.8) = \underbrace{-57.4}_{\sim 35\,dB}$$

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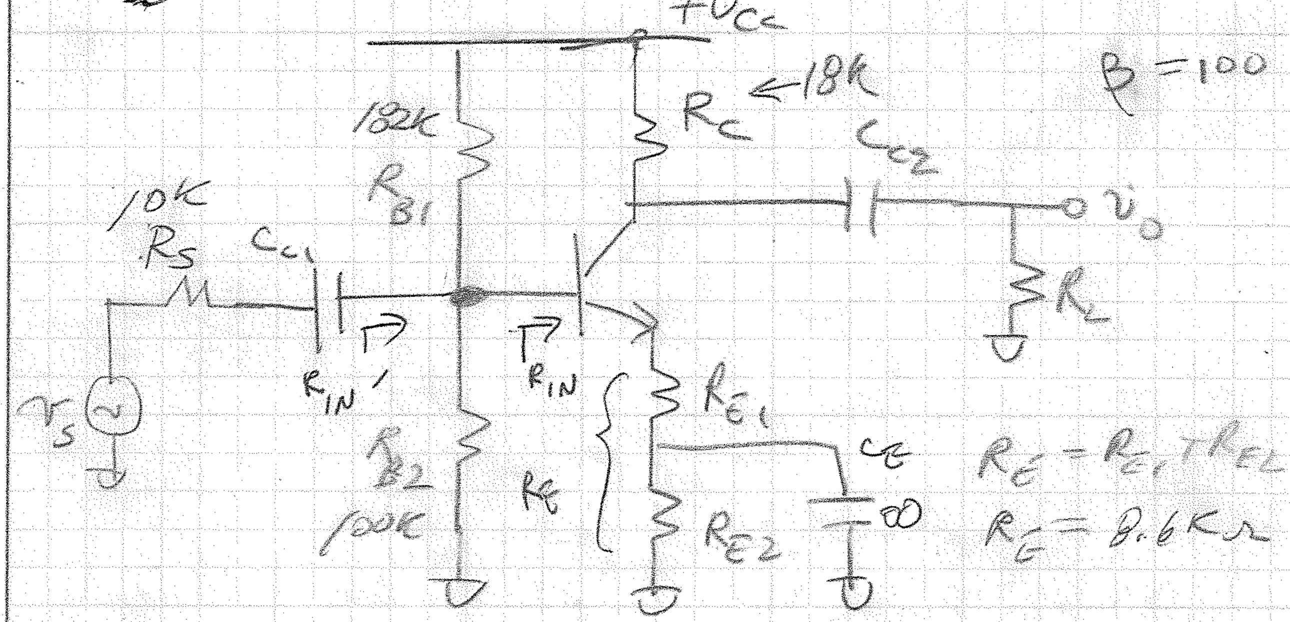
LECTURE #2

7.5.3 CE amp with Emitter Degeneration



Note we have replaced R_E with two resistors R_{E1} & R_{E2} where $R_E = R_{E1} + R_{E2}$

We do this to

1) Desensitize the gain, A_V. (Make it mostly depend on R_C/R_{E1})

2) Increase the input impedance (w/o Bias res). It was r_π & now its $r_\pi + (\beta+1)R_E$

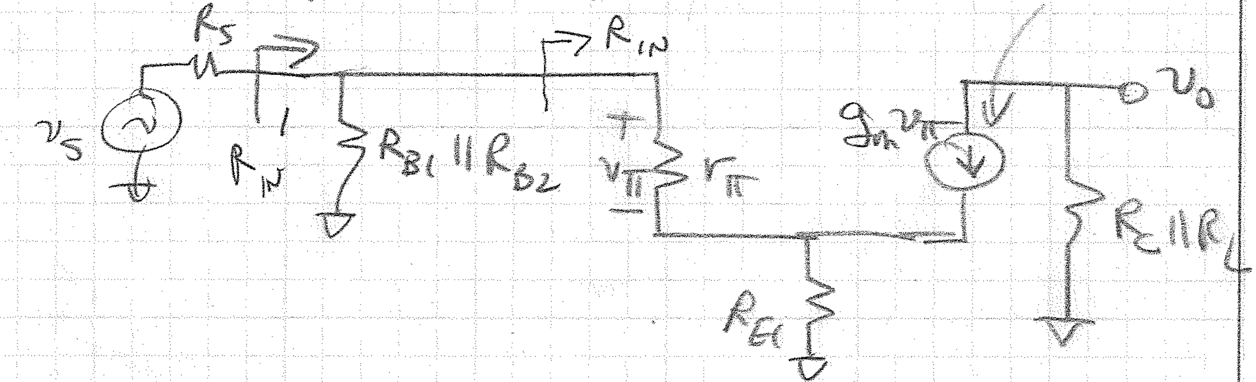
Cost we incur is lower A_V than if we had used a "strict" CE amp.

page 473

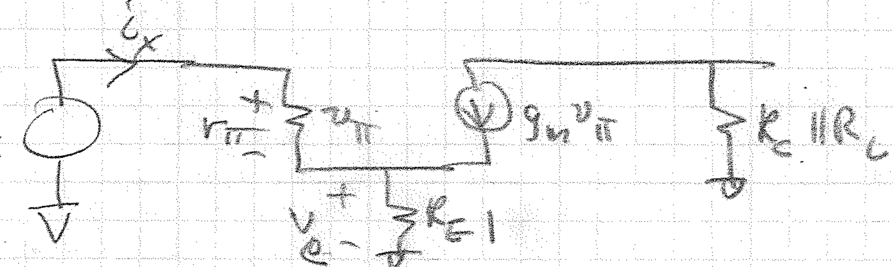
Exercise 7.4.2 ask that we raise R_{IN}' to 10 K and re-calculate A_V.

If $R_E = R_{E1} + R_{E2}$ then DC solution is the same.

Small-signal analysis. assume r_o ∞



Find R_{IN} then $R_{IN}' = R_{B1} \| R_{B2} \| R_{IN}$



$R_{IN} \equiv \frac{v_x}{i_x}$ = input resistance of a CE amp with emitter degen.

$v_x = v_e + v_\pi$ where $v_\pi = i_x r_\pi$

$v_x = v_e + i_x r_\pi$

But $v_e = (i_x + g_m v_\pi) R_{E1}$ KCL

So $v_x = [i_x R_{E1} + g_m R_{E1} (i_x r_\pi)] + i_x r_\pi$

$v_x = i_x R_{E1} + \beta R_{E1} i_x + i_x r_\pi$

3

$$\boxed{R_{IN} \equiv \frac{v_x}{i_x} = r_\pi + (\beta+1)R_{E1}}$$ * Very important

Want $G'_{IN} = \frac{1}{R'_{IN}} = \frac{1}{10K} = G_{B1} + G_{B2} + G_{IN}$

$$G_{IN} = 100\mu℧ - \frac{1}{100K} - \frac{1}{182K} = 84.5\mu℧$$

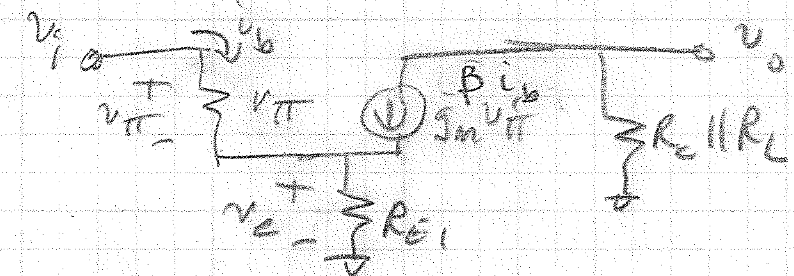
So $R_{IN} = 11.8K\Omega$ but $r_\pi = 5.2K\Omega$
and $\beta = 100$

$$R_{E1} = \frac{11.8K - \overset{r_\pi}{5.2K}}{\underbrace{(101)}_{\beta+1}} = \boxed{65\Omega}$$

So $R_{E2} = R'_E - R_{E1} = 8.2K - 65\Omega \approx \boxed{8.14K\Omega}$

Let's find the voltage gain of a CE amp with emitter degeneration:

Neglect r_o



$R_{out} \cong R_c$

Find v_o/v_i.

$$v_o = -(g_m v_\pi)(R_c \| R_L)$$

But $v_i = v_e + v_\pi \Rightarrow v_\pi = v_i - v_e$ (KVL)

where $v_e = (\beta+1)\, i_b R_{E1}$ & $i_b = \frac{v_\pi}{r_\pi} = g_\pi v_\pi$

so $v_\pi = v_i - (\beta+1)\, g_\pi R_{E1} v_\pi$

Thus $$v_\pi = \frac{v_i}{1 + (\beta+1) g_\pi R_{E1}}$$

So we conclude

$$v_o = -g_m (R_c \| R_L) v_\pi$$

$$\frac{v_o}{v_i} = \frac{-g_m (R_c \| R_L)}{1 + (\beta + 1) g_\pi R_{E1}}$$

But $\beta + 1 \approx \beta$ and $\beta g_\pi = g_m$!

So $$\boxed{\frac{v_o}{v_i} \approx -\frac{g_m (R_c \| R_L)}{1 + g_m R_{E1}}}$$ * Important

So gain $1 + g_m R_{E1}$ times smaller than what he had for CE amp!

Also note if $g_m R_{E1} \gg 1 \Rightarrow R_{E1} \gg \frac{1}{g_m}$

then $$\frac{v_o}{v_i} \cong -\frac{(R_c \| R_L)}{R_{E1}}$$ and the gain

is just the ratio of resistor values!

(THIS IS OFTEN VERY DESIRABLE!)

For the entire circuit we get

$$\frac{v_o}{v_s} = \underbrace{\left[\frac{R_{IN}'}{R_{IN}' + R_s}\right]}_{1/2 \text{ since } R_{IN} = R_S = 10K} \left[\frac{-g_m (\overset{18K}{R_c} \| \overset{20K}{R_L})}{1 + g_m R_{E1}}\right]$$

$g_m = 19.1 m℧$

$g_m R_{E1} = 11.24$

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$$\frac{v_o}{v_S} = -\frac{1}{2}\left(\underbrace{\frac{180.9}{1+1.24}}_{80.8}\right) \approx -40.4$$

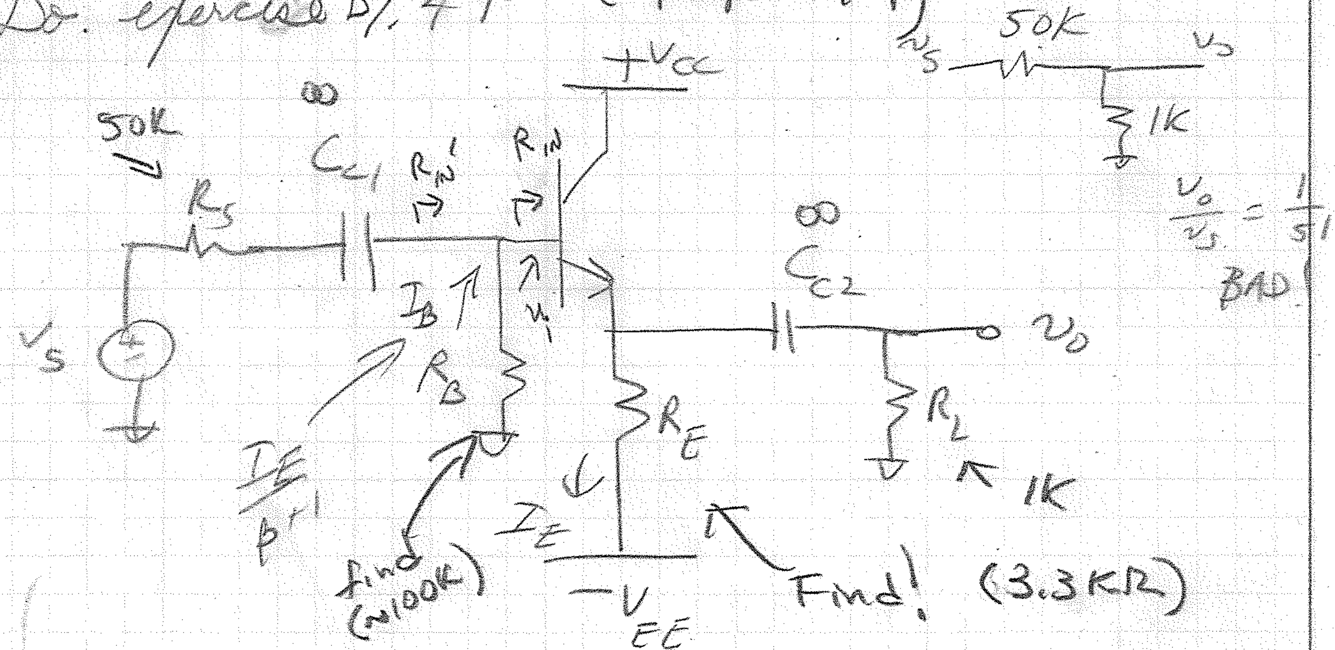
was 0.324 — 80.8 ← was 172.7 — 32dB

With CE amp had 35 dB. Now we have 32dB.

7.5.5 CC Amplifier (Emitter Follower)

Voltage gain ≈ 1 but we get current and hence power gain! We use this as a buffer!

Do exercise D7.44 (page 477)



Want I_E = 1mA. Allow a DC voltage drop across R_B of 1V. The available power supplies are ±5V, $\beta = 100$, $V_{BE(ON)} = 0.7V$ and $V_A = 100V$.

Also R_S = 50K & R_L = 1K

Specify values for R_B & R_E !

So $V_B = -1V \Rightarrow V_E = -1V - 0.7 = -1.7V$

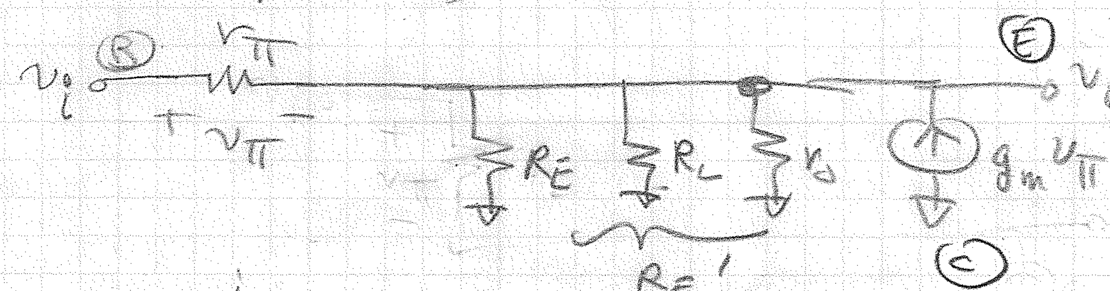
Hence $\underset{\text{want 1mA}}{I_E} = \frac{-1.7-(-5)}{R_E} \Rightarrow R_E = \frac{3.3V}{1mA} = 3.3K$

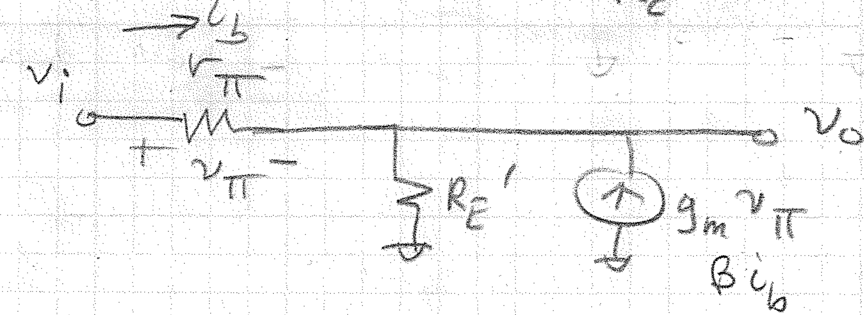
$$R_B = \frac{0 - V_B}{I_B} = \frac{-(\beta+1)(V_B)}{I_E} = \frac{-(101)(-1)}{1mA}$$

$I_B \leftarrow I_E/_{\beta+1}$

$R_B = 101k \approx 100\,k\Omega$

Small-signal equivalent





Find $\frac{v_o}{v_i}$:

$v_o = (\beta+1)\, i_b\, R_E'$ but $i_b = g_\pi v_\pi$

$v_o = (\beta+1)\, g_\pi R_E' v_\pi = i_e R_E'$ ($\beta+1\, i_b \leftarrow g_\pi v_\pi$)

But $v_i - v_\pi = v_o \Rightarrow v_\pi = v_i - v_o$

Hence $v_o = (\beta+1)g_{\pi} R_E'(v_i - v_o)$ (17)

$$\frac{v_o}{v_i} = \frac{(\beta+1)g_{\pi}' R_E'}{1+(\beta+1)g_{\pi}R_E'} \cong \frac{g_m R_E'}{1+g_m R_E'}$$

$$\boxed{\frac{v_o}{v_i} \approx \frac{g_m R_E'}{1+g_m R_E'}}$$ * Important

Can also easily show that

$$R_{IN} = r_{\pi} + (\beta+1)R_E'$$

and that $R_{out} = r_o \| R_E \| \left[\dfrac{r_{\pi} + \overset{R_B \| R_S}{R_{THS}}}{\beta+1}\right]$

$$g_m = \frac{I_C}{V_T} \approx \frac{\overset{1mA}{I_E}}{V_T} \quad \text{since } I_E = \left(\frac{\beta+1}{\beta}\right) I_C \quad \frac{101}{100}$$

$$g_m = \frac{1mA}{25.9mV} \cong 38.6 \text{ m}\mho$$

$$r_{\pi} = \frac{\beta}{g_m} = \frac{100}{38.6m\mho} \cong 2.6 k\Omega$$

$$r_o = \frac{V_A}{I_C} \approx \frac{V_A}{I_C} = \frac{100V}{1mA} = 100 k\Omega$$

$$R_E' = \underset{3.3K}{R_E} \| \underset{100K}{r_o} \| \underset{1K}{R_L} \cong 0.76K$$

$$R_{IN} = 2.6K + (101)(0.76K) \cong 79K\Omega$$

$$R_{IN}' = \underset{79K\Omega}{R_{IN}} \| \underset{100k}{R_B} \cong 44.1K\Omega$$

$$\frac{v_o}{v_s} = \left(\frac{v_i}{v_s}\right)\left(\frac{v_o}{v_i}\right)$$

$$\frac{v_o}{v_s} = \left(\frac{R_{IN}'}{R_{IN}'+R_S}\right)\left(\frac{g_m R_E'}{1+g_m R_E'}\right)$$

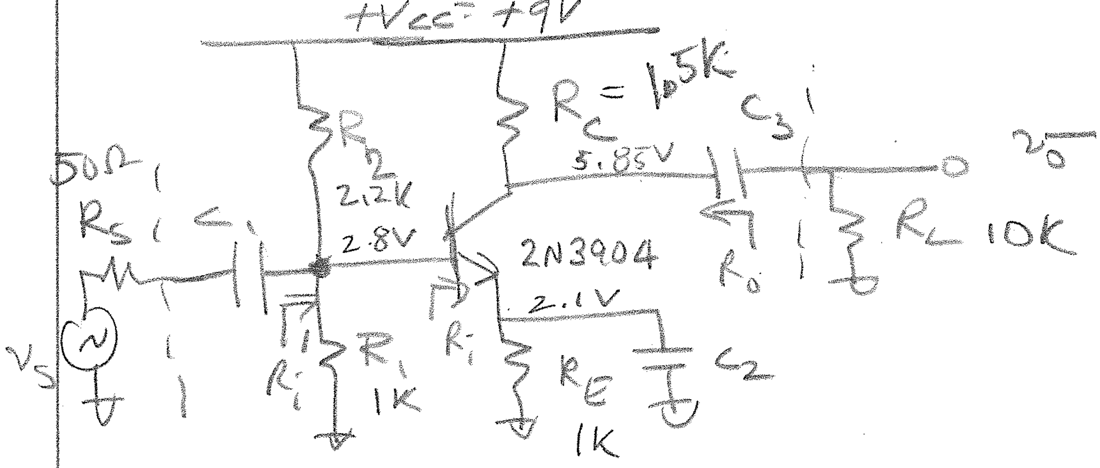
$$= \left[\frac{44.1\,k\Omega}{44.1k+50\,K}\right]\left[\frac{\overbrace{(38.6mU)(0.76k)}^{29.3}}{1+\underbrace{(38.6mU)(0.76k)}_{29.3}}\right] \quad -7.1\,dB$$

$$\cong (0.47)(0.967) = 0.454$$

$$R_{out} = 100k \parallel 3.3k \parallel \underbrace{\left[\frac{\overset{2.6k}{r_\pi} + \overset{33K}{(R_B \parallel R_S)}}{\beta+1}\right]}_{352\,\Omega} \approx 317\,\Omega$$

$$\sim 320\,\Omega$$

Analyze the following CE amp.



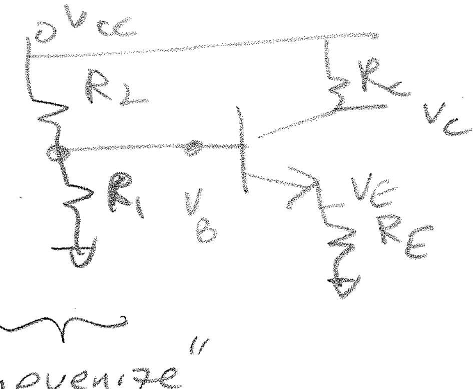
Assume $C_1, C_2, C_3 \to \infty$, $V_{BE(ON)} = 0.7$

$\beta = 150$

$V_A = 75V$

Find I_C, $A_V = \frac{v_o}{v_s}$, R_i, R_i', R_o

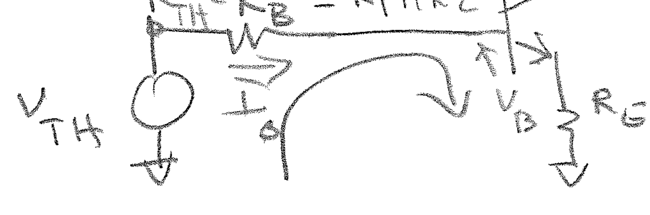
Solve DC problem first.



$$R_B = R_1 \| R_2 \cong 688\Omega$$

"Thevenize"

$$V_{TH} = \left(\frac{R_1}{R_1 + R_2}\right) V_{CC}$$



$$V_{TH} - I_B R_B - V_{BE} - \underbrace{(\beta+1) I_B}_{I_E} R_E = 0$$

$$\boxed{I_B = \frac{V_{TH} - V_{BE}}{R_B + (\beta+1)R_E}} \; *$$ (2)

In well designed amplifier we like

$V_{TH} \approx \frac{1}{3} V_{CC}$, I_C a few mA.

$R_B >> R_S$

$R_B << (\beta+1)R_E$ why?

If $(\beta+1)R_E >> R_B$

$$I_B \approx \frac{V_{TH} - V_{BE}}{(\beta+1)R_E} \Rightarrow I_C = \frac{\beta}{\beta+1}\left(\frac{V_{TH} - V_{BE}}{R_E}\right)$$

$$* \; \boxed{I_C \cong \frac{V_{TH} - V_{BE}}{R_E}} \text{ if } \beta >> 1$$

Note here

$$V_{TH} = \left(\frac{R_1}{R_1+R_2}\right)V_{CC} = \left(\frac{1K}{3.2K}\right)(9V) \cong 2.8V$$

$$V_B = V_{TH} - I_B R_B = V_{TH} - \left(\frac{R_B}{(\beta+1)R_E}\right)(V_{TH} - V_{BE})$$

$V_B \approx V_{TH}$ Note here $\frac{R_B}{(\beta+1)R_E} = \frac{(1K \parallel 2.2K)}{(151)(1K)}$

$$= 4.55 \times 10^{-3} \checkmark$$

$$V_E = V_B - V_{BE} \cong V_{TH} - V_{BE}$$

$$\cong 2.8V - 0.7V = 2.1V$$

$$I_E = \frac{2.1V}{1K} = 2.1mA \sim I_C \quad \left(I_C = \frac{\beta}{\beta+1} I_E\right)$$

$$I_B = \frac{I_E}{\beta+1} = \frac{2.1mA}{151} \cong 14\mu A$$

③

$V_C = V_{CC} - I_C R_C$

$V_C = 9V - (2.1\,mA)(1.5K)$

$V_C = 5.85V$

Now compute small-signal parameters

$g_m = \frac{I_C}{V_T} = \frac{2.1\,mA}{25.9\,mV} = 81.1\ m\mho$

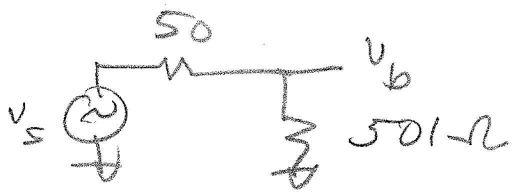
$r_o = \frac{V_A}{I_C} = \frac{75V}{2.1\,mA} = 35.7\,K\Omega$

$r_\pi = \beta/g_m = \frac{150}{81.1\,m\mho} = 1.85\,K\Omega$

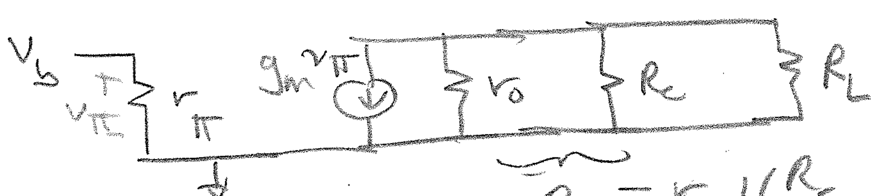
$A_V = \frac{v_o}{v_s} = \frac{v_b}{v_s} \cdot \frac{v_o}{v_b}$

Find $\frac{v_b}{v_s}$

$R_i' = \underset{\substack{\uparrow \\ 688\Omega}}{R_B} \parallel \underbrace{R_i}_{r_\pi = 1.85K}$



$\frac{v_b}{v_s} = \frac{501}{551} = 0.909 \leftarrow -0.8\,dB$



$v_\pi = v_b$

$\underbrace{R_o}_{} = r_o \parallel R_C \cong 1.4K$

$R_L' = R_L \parallel R_o \cong 1.25k$

4

$$\frac{v_o}{v_b} = -g_m R_L' = A$$

$$= -(81.1\,m\mho)(1.25k)$$

$$\cong -10^2 \sim 40\,dB$$

$$A_v = \frac{v_o}{v_s} = -(0.909)(10^2) = -92.9$$

$$\cong 39.4\,dB$$

$$R_o = R_c \parallel r_o$$

$$R_i = r_\pi$$

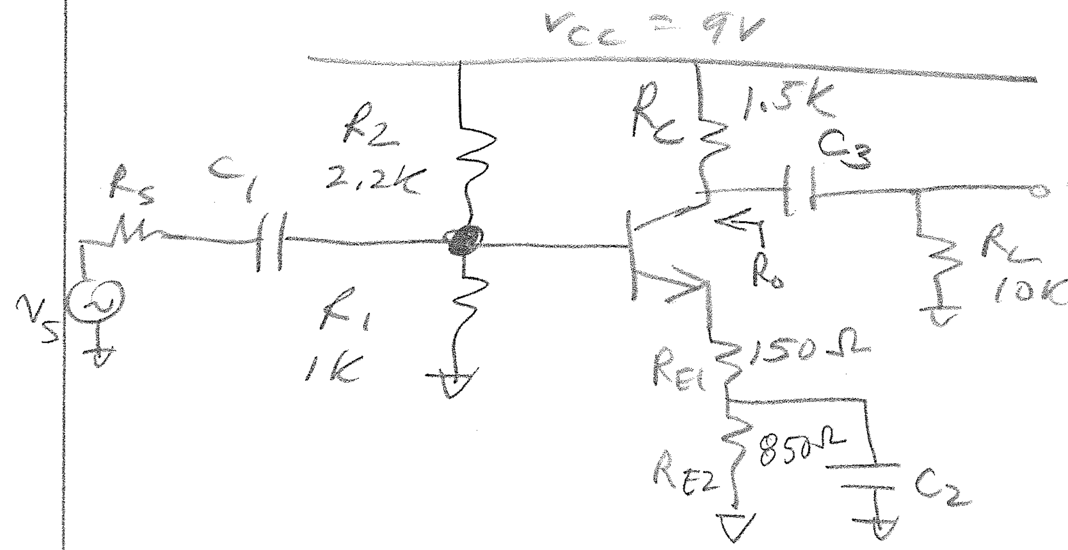
$$R_i' = r_\pi \parallel R_B$$

$$A = -g_m R_L'$$

$$A_v = \left[\frac{R_i'}{R_i' + R_s}\right] A$$

cheat sheet

Analyze the following CE amp
with emitter degeneration circuit



$R_o \cong R_C$

Note $R_{E1} + R_{E2} = R_E = 1K$

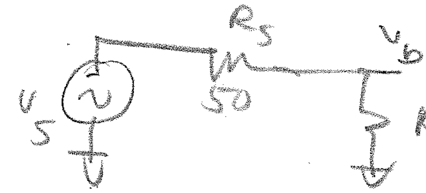
DC conditions same as in previous problem.

$I_C = 2.1\,mA$

$g_m = 81.1\,m℧$

$r_o = 35.7K\Omega$

$r_\pi = 1.85K\Omega$



$$R_i' = \underbrace{R_B}_{688\Omega} \parallel \overbrace{\left[\underbrace{r_\pi}_{1.85K} + \underbrace{(\beta+1)}_{151}\underbrace{R_{E1}}_{150\Omega}\right]}^{24.5K\Omega} = 669\Omega$$

$$\frac{v_b}{v_s} = \frac{669}{669+50} = 0.93$$

(2)

$$\frac{v_o}{v_b} = \frac{-g_m R_L'}{1+g_m R_{E1}} = \frac{-g_m (R_C \| R_L)}{1+\underbrace{(81.1mv)(150)}_{12.2}}$$

$$= \frac{-(81.1mv)(10K \| 1.5K)}{13.2}$$

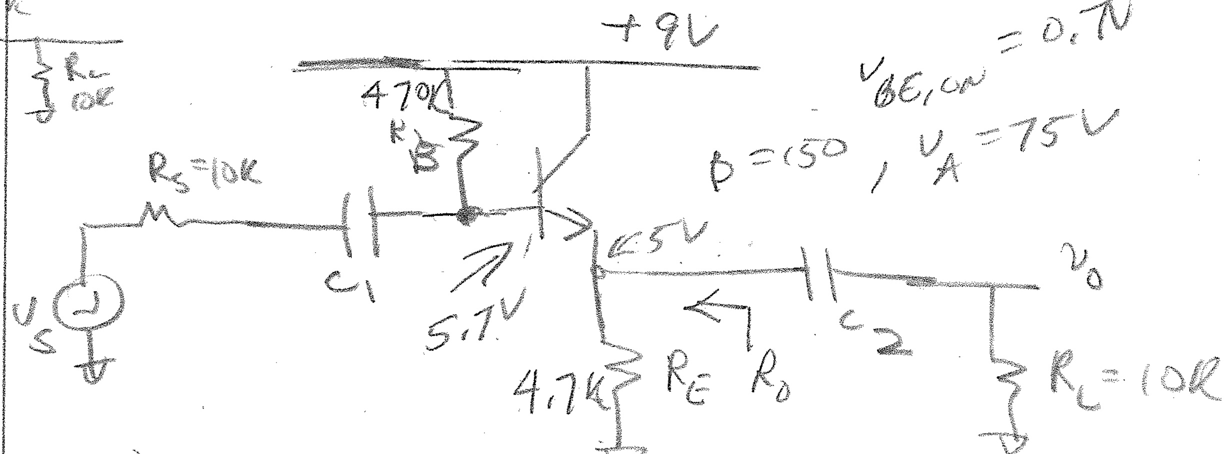
$$= \frac{-105.8}{13.2} \cong -8 \quad \text{Note} \quad \frac{1.5K}{150} = 10$$

$$\frac{-g_m R_L'}{1+g_m R_{E1}} \sim \frac{-R_L'}{R_{E1}} \cong -10$$

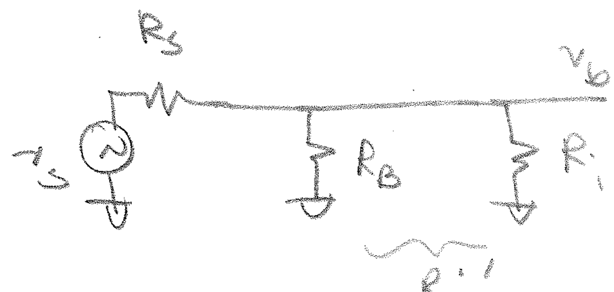
If supply voltage changes there will be a much smaller percentage change in the gain of the circuit.

7.5.5 CC Amplifier (A.K.A Emitter Follower)

Voltage gain ≈ 1 but we get current and hence power gain. We use this as a buffer



Like $1mA \le I_C \le 10mA$



$$\frac{v_b}{v_s} = \frac{R_i'}{R_i' + R_s}$$

Just like before

$R_i' = R_i \parallel R_B \qquad R_i = r_\pi + (\beta+1) R_E'$

$$R_E' = R_E \parallel r_o \parallel R_L$$

$$A_v = \underbrace{\left[\frac{R_i'}{R_i' + R_S}\right]}_{\frac{v_b}{v_s}} \underbrace{\left[\frac{g_m R_E'}{1 + g_m R_E'}\right]}_{\frac{v_o}{v_b}} = \frac{v_o}{v_s}$$

$$I_B = \frac{V_{CC} - V_{BE(on)}}{R_B + (\beta+1) R_E} = \frac{9 - 0.7}{470K + (151)(4.7k)}$$

2

$I_B \cong 7\mu A$

$v_B = \overset{9}{v_{cc}} - \overset{7\mu A}{I_B}\, \overset{470k}{R_B} = 5.7V$

$I_C \approx I_E = \frac{5V}{4.7k} \cong 1.06\, mA$

$g_m = \frac{I_C}{V_T} = \frac{1.06\, mA}{25.9\, mV} \cong 40.9\, m\mho$

$r_\pi = \frac{\beta}{g_m} = \frac{150}{40.9\, m\mho} \sim 3.7k$

$r_o = \frac{V_A}{I_C} = \frac{75V}{1.06\, mA} \cong 70.8\, K$

$R_E' = R_E \,\|\, r_o \,\|\, R_L$

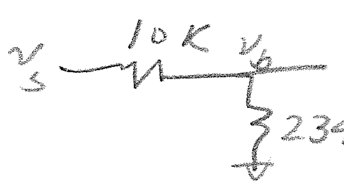
$\quad = 4.7k \,\|\, 70.8k \,\|\, 10k$

$R_E' \cong 3.1k$

$R_i = r_\pi + (\beta+1) R_E'$

$R_i = 3.7k + ((151)(3.1k) \cong 466k\Omega$

$R_i' = R_i \,\|\, R_B = 466k \,\|\, 470k = 234K$



$\Rightarrow \frac{v_o}{v_s} = \frac{234}{244} = 0.96$

$A = \frac{g_m R_E'}{1 + g_m R_E'} = \frac{(40.9\, m\mho)(3.1k)}{1 + (40.9\, m\mho)(3.1k)}$

$= \frac{126.79}{127.79} = 0.99 \Rightarrow A_v = 0.96(0.99) = 0.95$

$-0.4\, dB$

$$R_o = R_E \parallel r_o \parallel \left[\frac{r_\pi + R_S'}{\beta+1}\right] \quad R_S' = R_S \parallel R_B \sim R_S$$

③

$$R_o = 4.7k \parallel 70.8k \parallel \underbrace{\left[\frac{3.7k + 10k}{151}\right]}_{\sim 91\,\Omega}$$

$$\boxed{R_o \simeq 91\,\Omega}$$

Note if $R_L = 1K$ then

$$A = \frac{g_m R_E'}{1 + g_m R_E'} \simeq \frac{(40.9\,m\mho)(1K)}{1 + (40.9\,m\mho)(1K)}$$

$$A \cong 0.976$$

$$R_i' = r_\pi + (\beta+1) R_E'$$

$$R_i' \cong 3.7k + (151)(1K)$$

$$R_i' \simeq 154.7k$$

$$A_v \cong \left[\frac{154.7k}{154.7k + 10k}\right](0.976)$$

$$\cong 0.92 \qquad (-0.7\,dB)$$

Still quite good