Rules for Making Bode Plots

Term	Magnitude	Phase
Constant: K	$20\cdot\log_{10}(\lvert K\rvert)$	K>0: 0° K<0: ±180°
Real Pole: $\frac{1}{\frac{s}{\omega_0}+1}$	• Low freq. asymptote at 0 dB • High freq. asymptote at -20 dB/dec • Connect asymptotic lines at ω_0,	• Low freq. asymptote at 0°. • High freq. asymptote at -90°. • Connect with straight line from $0.1\cdot\omega_0$ to $10\cdot\omega_0$.
Real Zero*: $\frac{s}{\omega_0}+1$	• Low freq. asymptote at 0 dB • High freq. asymptote at +20 dB/dec. • Connect asymptotic lines at ω_0.	• Low freq. asymptote at 0°. • High freq. asymptote at +90°. • Connect with line from $0.1\cdot\omega_0$ to $10\cdot\omega_0$.
Pole at Origin: $\frac{1}{s}$	• -20 dB/dec; through 0 dB at ω=1.	• -90° for all ω.
Zero at Origin*: s	• +20 dB/dec; through 0 dB at ω=1.	• +90° for all ω.
Underdamped Poles: $\frac{1}{\left(\frac{s}{\omega_0}\right)^2+2\zeta\left(\frac{s}{\omega_0}\right)+1}$	• Low freq. asymptote at 0 dB. • High freq. asymptote at -40 dB/dec. • Connect asymptotic lines at ω_0. • Draw peak† at freq. $\omega_r=\omega_0\sqrt{1-2\zeta^2}$ with amplitude $H\ j\omega_r\ = -20\cdot\log_{10}\ 2\zeta\sqrt{1-\zeta^2}$	• Low freq. asymptote at 0°. • High freq. asymptote at -180°. • Connect with straight line from ‡ $\omega=\omega_0\frac{\log_{10}\left(\frac{2}{\zeta}\right)}{2}$ to $\omega=\omega_0\frac{2}{\log_{10}\left(\frac{2}{\zeta}\right)}$
Underdamped Zeros*: $\left(\frac{s}{\omega_0}\right)^2+2\zeta\left(\frac{s}{\omega_0}\right)+1$	• Draw low freq. asymptote at 0 dB. • Draw high freq. asymptote at +40 dB/dec. • Connect asymptotic lines at ω_0. • Draw dip† at freq. $\omega_r=\frac{\omega_0}{\sqrt{1-2\zeta^2}}$ with amplitude $H\ j\omega_r\ = +20\cdot\log_{10}\ 2\zeta\sqrt{1-\zeta^2}$.	• Low freq. asymptote at 0°. • Draw high freq. asymptote at +180°. • Connect with a straight line from ‡ $\omega=\omega_0\frac{\log_{10}\left(\frac{2}{\zeta}\right)}{2}$ to $\omega=\omega_0\frac{2}{\log_{10}\left(\frac{2}{\zeta}\right)}$

Notes:

* Rules for drawing zeros create the mirror image (around 0 dB, or 0°) of those for a pole with the same ω_0.

† For underdamped poles and zeros peak exists only for $0<\zeta<0.707=\frac{1}{\sqrt{2}}$ and peak freq. is typically very near ω_0.

‡ For underdamped poles and zeros If $\zeta<0.02$ draw phase vertically from 0 to -180 degrees at ω_0

For n^{th} order pole or zero make asymptotes, peaks and slopes n times higher than shown (i.e., second order asymptote is -40 dB/dec, and phase goes from 0 to –180°). Don't change frequencies, only plot values and slopes.

Quick Reference for Making Bode Plots

If starting with a transfer function of the form (some of the coefficients b_i, a_i may be zero).

$$H(s)=C\frac{s^n+\cdots+b_1s+b_0}{s^m+\cdots+a_1s+a_0}$$

Factor polynomial into real factors and complex conjugate pairs (p can be positive, negative, or zero; p is zero if a_0 and b_0 are both non-zero).

$$H(s)=C\cdot s^p\,\frac{\left(s+\omega_{z1}\right)\left(s+\omega_{z2}\right)\cdots\left(s^2+2\zeta_{z1}\omega_{0z1}s+\omega_{0z1}^2\right)\left(s^2+2\zeta_{z2}\omega_{0z2}s+\omega_{0z2}^2\right)\cdots}{\left(s+\omega_{p1}\right)\left(s+\omega_{p2}\right)\cdots\left(s^2+2\zeta_{p1}\omega_{0p1}s+\omega_{0p1}^2\right)\left(s^2+2\zeta_{p2}\omega_{0p2}s+\omega_{0p2}^2\right)\cdots}$$

Put polynomial into standard form for Bode Plots.

$$H(s)=C\frac{\omega_{z1}\omega_{z2}\cdots\omega_{0z1}^2\omega_{0z2}^2\cdots}{\omega_{p1}\omega_{p2}\cdots\omega_{0p1}^2\omega_{0p2}^2\cdots}\cdot s^p\,\frac{\left(\frac{s}{\omega_{z1}}+1\right)\left(\frac{s}{\omega_{z2}}+1\right)\cdots\left(\left(\frac{s}{\omega_{0z1}}\right)^2+2\zeta_{z1}\left(\frac{s}{\omega_{0z1}}\right)+1\right)\left(\left(\frac{s}{\omega_{0z2}}\right)^2+2\zeta_{z2}\left(\frac{s}{\omega_{0z2}}\right)+1\right)\cdots}{\left(\frac{s}{\omega_{p1}}+1\right)\left(\frac{s}{\omega_{p2}}+1\right)\cdots\left(\left(\frac{s}{\omega_{0p1}}\right)^2+2\zeta_{p1}\left(\frac{s}{\omega_{0p1}}\right)+1\right)\left(\left(\frac{s}{\omega_{0p2}}\right)^2+2\zeta_{p2}\left(\frac{s}{\omega_{0p2}}\right)+1\right)\cdots}$$

$$=K\cdot s^p\,\frac{\left(\frac{s}{\omega_{z1}}+1\right)\left(\frac{s}{\omega_{z2}}+\right)\cdots\left(\left(\frac{s}{\omega_{0z1}}\right)^2+2\zeta_{z1}\left(\frac{s}{\omega_{0z1}}\right)+1\right)\left(\left(\frac{s}{\omega_{0z2}}\right)^2+2\zeta_{z2}\left(\frac{s}{\omega_{0z2}}\right)+1\right)\cdots}{\left(\frac{s}{\omega_{p1}}+1\right)\left(\frac{s}{\omega_{p2}}+1\right)\cdots\left(\left(\frac{s}{\omega_{0p1}}\right)^2+2\zeta_{p1}\left(\frac{s}{\omega_{0p1}}\right)+1\right)\left(\left(\frac{s}{\omega_{0p2}}\right)^2+2\zeta_{p2}\left(\frac{s}{\omega_{0p2}}\right)+1\right)\cdots}$$

Take the terms (constant, real poles and zeros, origin poles and zeros, complex poles and zeros) one by one and plot magnitude and phase according to rules on previous page. Add up resulting plots.

Comments? Questions? Suggestions? Corrections?

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Matlab Tools for Bode Plots

```
>> n=[1 11 10];                    %A numerator polynomial (arbitrary)
>> d=[1 10 10000 0];               %Denominator polynomial (arbitrary)
>> sys=tf(n,d)
Transfer function:
    s^2 + 11 s + 10
----------------------
s^3 + 10 s^2 + 10000 s

>> damp(d)                         %Find roots of den. If complex, show zeta, wn.
        Eigenvalue              Damping     Freq. (rad/s)
  0.00e+000                   -1.00e+000     0.00e+000
 -5.00e+000 + 9.99e+001i       5.00e-002     1.00e+002
 -5.00e+000 - 9.99e+001i       5.00e-002     1.00e+002

>> damp(n)                         %Repeat for numerator
Eigenvalue       Damping     Freq. (rad/s)
 -1.00e+000      1.00e+000      1.00e+000
 -1.00e+001      1.00e+000      1.00e+001

>> %Use Matlab to find frequency response (hard way).
>> w=logspace(-2,4);               %omega goes from 0.01 to 10000;
>> fr=freqresp(sys,w);
>> subplot(211);  semilogx(w,20*log10(abs(fr(:))));  title('Mag response, dB')
>> subplot(212);  semilogx(w,angle(fr(:))*180/pi);   title('Phase resp, degrees')

>> %Let Matlab do all of the work
>> bode(sys)

>> %Find Freq Resp at one freq.     %Hard way
>> fr=polyval(n,j*10)./polyval(d,j*10)
fr =   0.0011 + 0.0010i

>> %Find Freq Resp at one freq.     %Easy way
>> fr=freqresp(sys,10)
fr =   0.0011 + 0.0009i

>> abs(fr)
ans =    0.0014

>> angle(fr)*180/pi   %Convert to degrees
ans =   38.7107


>> %You can even find impulse and step response from transfer function.
>> step(sys)
>> impulse(sys)
```


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```
>> [n,d]=tfdata(sys,'v')            %Get numerator and denominator.
n =
     0     1    11    10
d =
           1          10       10000           0

>> [z,p,k]=zpkdata(sys,'v')         %Get poles and zeros
z =
   -10
    -1
p =
         0
  -5.0000 +99.8749i
  -5.0000 -99.8749i
k =
     1


>> %Matlab program to show individual terms of Bode Plot.
>> %Code is available at
>> %   http://www.swarthmore.edu/NatSci/echeeve1/Ref/Bode/BodePlotGui.html
>> BodePlotGui(sys)
```