Chapter 8

The EPR Paper

If property realism is true, then standard quantum mechanics must be incomplete, or so it would seem. For, if the state vector is about single systems, then it is obviously unable to give us the definite values of their observables, and therefore something that the theory should capture escapes it. If, on the other hand, the state vector is about ensembles (which, given strong property realism, must then be composed of elements with definite properties all around), then quantum mechanics is a theory like statistical mechanics: as the former does not deal with individual systems, so the latter does not deal with individual molecules in a gas.[1] Then, one should search for a deeper quantum theory tackling individual systems and playing for quantum mechanics the same

[1] The idea that the state vector gives information not about individual systems but sets of individual systems is rather appealing, not in small part because in practice quantum measurements *are* carried out on ensembles, for (barring the unusual case when the system's state is an eigenvector of the observable's operator) the only reproducible empirical results involve measurement averages (expected values) because identically prepared systems will normally provide different returns. Of course, one can adopt a type of ensemble interpretation that makes no realist assumptions about the dynamic properties of the particles, and with a bit of tinkering is therefore compatible with the view that when superposition is involved such properties come about only with measurement.

foundational role that Newtonian mechanics plays for statistical mechanics. Most of the founders of quantum mechanics held that the theory is complete. However, there were exceptions, most notably Schrödinger and Einstein. In this chapter, we look at the exchange between Einstein and Bohr, the main force behind the Copenhagen interpretation, the direct ancestor of the orthodox interpretation.

8.1 Bohr's Views

Bohr's very influential views, which formed the core of the Copenhagen interpretation, were developed in the late 1920's. In the view of some scholars they do not constitute a coherent whole, and they changed somewhat over time as well. In addition, the arguments Bohr used to support his views varied in order to address new objections. As a result, his authority has been invoked by neopositivists, idealists, and materialists (both dialectical and ordinary), just to mention a few.[2] Fortunately, we need not get into the issue of what exactly Bohr held and when, and we may content ourselves with considering Bohr's main views just before 1935.[3]

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[2] For a brief account of the interesting intertwining of science, philosophy, and political correctness (the real thing: Stalin!) that characterized the reception of Bohr's views in the USSR, see Graham, L., (1985).

[3] What follows is an account of Bohr's views in *Atomic Theory and the Description of Nature*, a collection of four essays and an introduction, including the famous 1927 Como Lecture, written between 1925 and 1929. The English version of the collection was published in 1934, and in his reply to the EPR paper (of which more later), Bohr explicitly refers his readers to it.

In spite of Bohr's original disagreement with Heisenberg about the latter's "derivation" and views of HUPI, the starting point of his line of argument is borrowed from Heisenberg. In Bohr's view, the most characteristic feature of the quantum world is the presence of the quantum of action *h*. The result is that the interaction between the measuring apparatus and the typical quantum system being measured cannot be made smaller than *h*. Given the smallness of the masses and energies involved, the measuring process, then, becomes problematical. If we try to determine the precise position of an electron we end up by altering its momentum uncontrollably. Every interaction between a quantum particle and a measurement apparatus contains an element of individuality (the result of uncontrollability) lacking in classical mechanics. HUP and HUPI are but the formal representation of this physical fact.

According to Bohr, at the level of classical mechanics, the spatio-temporal-causal account of phenomena is achieved in terms of the simultaneous determination of the position and momentum of the particle under study. Such determination is made possible by the fact that the interaction between the classical system and the measuring apparatus is controllable because *h* is totally negligible. Once the classical mechanical state (that is, the relevant position and momentum) of the system is known, it is possible (in principle) to determine the evolution of the system and the unique measurement returns associated with that state. Our ignorance, in such cases, merely arises because evolutionary determinism does not entail computational determinism and our observations are, at times, less than adequate. Still, in principle both limitations can be overcome.

However, things are different in the quantum world. The uncontrollability of the interaction between a quantum particle and the measuring apparatus entails the impossibility of any access to a nature-given separation, as it were, of the behavior of a quantum system and the measuring apparatus: the quantum object and the measuring apparatus have effectively become a whole. Hence, no spatio-temporal-casual account of a quantum particle as an independent reality is possible even in principle.

Later in his career, Bohr became fond of saying that as relativity shows that properties such as length or duration are ascribable only relative to a frame of reference, so quantum mechanics shows that physical properties are attributable only relative to an experimental setup. It may happen that the experimental set-ups for the precise determination of two properties are incompatible. For example, the precise determination of the position of a particle requires that the measuring instrument M be rigidly attached to the apparatus A used to fix the spatial reference frame. But the precise determination of momentum requires that M be loosely attached to A so that M's momentum, by hypothesis already known, may change as the result of the impact of the particle with M. Since momentum is conserved, the amount of change (the difference of the momentums before and after the impact) is the particle's momentum.[4] So, to measure position M must be rigidly attached to A, and to measure momentum it must be loosely attached to A. Hence, the two measurement set-ups are incompatible. Differently put, to know the particle's momentum we need to know M's momentum; however, HUPI tells us that a precise knowledge of M's momentum precludes a precise knowledge of M's position (M has moved uncontrollably). Hence, even if we find out where on M the particle hit, we cannot know where that is with respect to a fixed frame of reference.

[4] How this fits with Bohr's story about the uncontrollability of the interaction is, at best, unclear.

When the determination of two properties requires incompatible measurement apparatuses, the relevant observables are *complementary* in that a knowledge of them is both necessary to obtain an explanation (that is, an explanation based on classical concepts, the only one possible for us, as we shall see) of the phenomenon under study and in principle precluded. Indeed the quantum of action and HUPI "not only set a limit to the *extent* of the information obtainable by measurements, but they also set a limit to the *meaning* which we may attribute to such information" Bohr, N., (1934: 18).[5]

At this point, one might argue, as Schrödinger told Bohr in a May 5, 1928 letter, and Einstein eventually came to believe, that perhaps we must search for a radically new conceptual apparatus in order to deal with the micro-world (Bohr, N., (1984), v. 6: 47). The idea seems quite reasonable. After all, our physical concepts are ultimately

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[5] Already starting from the published version of his famous 1927 Como lecture, Bohr was prepared to use complementarity with relation to the particle-wave duality as well because while nothing can be both a particle and a wave, both a particle account and a wave account are required to explain quantum phenomena. However, references to the wave/particle dichotomy waned rather quickly. Bohr also thought that the notion of complementarity could be extended to other fields such as psychology (marred by the difficulty of separating subject from object) and biology (where the physico-chemical description of organisms in terms of their parts conflicts with the equally necessary but incompatible teleological account of the parts in terms of the whole). However, we need not go into this issue. Bohr was very attached to the notion of complementarity; the coat of arms for his induction into the Danish Elephant Order in 1947 depicts the Taoist Yin-Yiang symbol with the motto "Contraria sunt Complementa."

grounded in our (educated, to be sure) everyday perceptions of the world, that is, of a physical reality many orders of magnitude greater than the typical phenomena quantum theory is asked to explain. Hence, it would seem natural to hold that when it comes to the quantum world such ideas might be inadequate, and consequently should be drastically changed. In addition, scientific revolutions are accomplished by a change of conceptual apparatus. For example, the transition from Aristotelian to Galilean physics involved a radical conceptual change, and it would be preposterous to hold that Galileo should have expressed his views within the Aristotelian framework.[6] The same could be said for the relation between classical and quantum mechanics, and Bohr's prescription that classical language must be used is unjustified.

Bohr agreed with the diagnosis of the problem: the use of classical concepts appropriate in the macroscopic world, in which *h* is negligible, leads to complementarity in quantum accounts. However, he disagreed with the cure. For, he held that we cannot divest ourselves of classical concepts because they are inextricably intertwined with our very way of conceptualizing the world. They are nothing but a refinement of this basic conceptual apparatus ultimately dependent on our perceptual system and, as such, they are ineliminable. "The 'old' empirical concepts appear to me to be inseparably linked to the foundations of the human means of visualization" he replied to Schrödinger (ibidem, 48). In our theoretical and experimental interrogation of nature, therefore, we are bound

[6] That is, Galileo could express his views in a quasi-Aristotelian framework for polemical reasons, as in the *Dialogue On The Two Great Systems of the World*. However, when it comes to technical work, for example his analysis of motion in *Dialogues On Two New Sciences*, the language he uses it that of geometry and of Archimedes, not of Aristotle.

to use classical concepts that result in HUP and complementarity.[7] Bluntly put, even if the quantum world might have a claim of ontological priority over the macroscopic world, classical mechanics is epistemologically prior to quantum mechanics. This compels us to describe the measuring apparatus classically and not quantum-mechanically even if it is composed of atoms. Of course, what is a measuring device in one context may be the measured system in another, in which case it may be described quantum-mechanically, although the device measuring it must be described classically.

To be sure, the application of classical concepts to the quantum world is not clear-cut for, as we saw, classical observables are represented in quantum mechanics by Hermitian operators. Still, even in the deformed way required by the discontinuity that characterizes the microwold, such application is the best we can possibly do. Since we are macroscopic creatures, at the end our experiments must involve macroscopic and relatively stable instruments such as clocks, rods, and photographic plates that are to be described in terms of classical physics. Indeed, for Bohr the very communicability of

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[7] Einstein's reaction to Bohr's letter, showed to him by Schrödinger, was vintage Einstein: "The Heisenberg-Bohr soothing philosophy –or religion?- is so cleverly concocted that for the present it offers the believers a soft pillow of repose from which they are not so easily chased away. Let us therefore let them rest." (ibidem, 51). See his 1954 letter draft in Fine, A., (1986): 57. Heisenberg eventually came to hold Bohr's view; see Heisenberg, W., (1971): 56; 144-45. To what extent Newtonian concepts are just a refinement of our "natural" physical intuitions is hard to tell, as the "natural" intuitions of most people seem to be Aristotelian. Bohr's point is rather implausible, as Born did not fail to notice (Beller, M., (1999): 180-81).

experimental results presupposes that they are described in classical terms. The upshot of all this is that our very nature ties us to classical concepts and instruments. Quantum mechanics merely provides us with new sort of laws of applications for them. Aage Petersen, Bohr's assistant, reports that

> When asked whether the algorithm of quantum mechanics could be considered as somehow mirroring an underlying quantum world, Bohr would answer: "There is no quantum world. There is only an abstract quantum physical description. It is wrong to think that the task of physics is to find out how nature *is*. Physics concerns what we can *say* about nature" (Petersen, A., (1985): 305).

The point of this quasi-Kantian reply is not that quantum entities such as electrons do not exist independently of us since after all measurement returns are the result of their interaction with our instruments. Rather the idea Bohr wanted to impress on us is that any attempt to get at the nature of quantum things themselves, as it were, must be rejected. For us macroscopic creatures, there is no quantum world in itself because we can only conceptualize it in classical terms, and therefore in terms subject to HUPI.

Bohr's views constituted the core of the Copenhagen interpretation of quantum mechanics, a collection of occasionally inconsistent views gravitating around Bohr's Institute in Copenhagen and the direct ancestor of the orthodox interpretation.[8] The Copenhagen interpretation, and in particular the idea that the state vector contains a complete description of the quantum state it represents, came under serious attack in the

[8] For an analysis of some of the conflicting strands of the Copenhagen interpretation, see Beller, M., (1999): ch. 8.

late 1920's and in the 1930's, especially from Einstein, who had become convinced that quantum mechanics is essentially incomplete. We turn, therefore, to a very brief analysis of Einstein's views.

8.2 Einstein's Realism

Although early in his career Einstein seems to have adopted a rather strict empiricist position influenced by Mach, by the 1920's he became close to realism.[9] How close is unclear, as he never seems to have bothered to give us a complete and systematic account of his views on the matter. Still, it is relatively clear that he came to believe three claims. First, the task of science is to provide a description of the basic ontology, structure, and laws governing the world, which exists independently of our sense perceptions. Second, such description should, and can, be observer-independent. Third, the things constituting the basic furniture of the world exist in a causal (deterministic) network, and have definite physical characteristics which right theory corroborated by experience tries to reveal to us. The physical characteristics Einstein had in mind with respect to quantum objects seem to have been primarily their physical states and only secondarily, if at all, their properties (observables).

Whether he was prepared to assert the previous three claims with respect to the physical world itself (a piece of metaphysics, as it were), as Holton holds, or whether he viewed them as imposing meta-theoretical restrictions on physical theories in the sense that a satisfactory theory would not include among its fundamental laws the notion of probability or measurement, as Fine holds, is a matter of dispute and need not concern us

[9] For Einstein's early views, see J. Stachel, "Introduction to Einstein: The Formative Years", in Stachel, J., (2002): 121-39.

here. [10] Einstein was prepared to concede that one could not prove his views and that consequently one could only assume them, and that, moreover, opposing views that attributed reality only to what is observable or predictable are logically unobjectionable. Still, we can see why he came to dislike the Copenhagen and later the standard interpretation of quantum mechanics, with their emphasis on measurement and the role of the observer (an "epistemology-soaked orgy", as he colorfully told Schrödinger), and their probabilistic ontology.[11] In a famous set of exchanges with Bohr at the Solvay Conferences of 1927 and 1930, he argued for the incompleteness of quantum mechanics by trying to show that quantum particles have definite trajectories, that HUPI can be

[10] See, for example, Holton, G., (1988): ch. 7, and Fine, A., (1986): ch. 6 for contrasting views. Fine goes on to argue that the imposition of such meta-theoretical restrictions could be viewed as a kind of program ultimately justified, in Einstein's eyes, by the belief that is would produce better empirically adequate scientific theories than opposing programs.

[11] Letter of June 17, 1935. Einstein was also convinced that quantum mechanics was incompatible with Special Relativity. For an analysis of these issues, see Maudlin, T., (2002). Einstein's distaste for standard quantum mechanics did not wane; on July 5, 1952 he wrote D. Lipkin: "This theory (the present quantum theory) reminds me a little of the system of delusions of an exceedingly intelligent paranoic, concocted of incoherent elements of thought." (Quoted in Fine, A., (1986): 109, note 40.)

circumvented, and that the so called energy-time uncertainty can be violated. Bohr, however, succeeded in meeting his arguments. [12]

Einstein was also troubled by the possibility of entanglement between quantum particles and macroscopic objects, which even Copenhagen theorists would (or at least should) admit as having definite physical properties at all times. For example, a charge of gunpowder which may spontaneously combust because of some quantum occurrence will eventually be described as a superposition of exploded and non-exploded systems. As he told Schrödinger, "Through no art of interpretation can this ψ-function be turned into an adequate description of a real state of affairs; in reality, there is just no intermediary between exploded and non-exploded."[13] Of course, it would be possible to hold that the gunpowder is a "blend" of exploded and non-exploded up to the moment of observation, when the collapse of the state function occurs, but this view, as Einstein said a few years later in relation to an analogous example, would be "impracticable".[14] The supposition that the gunpowder is neither exploded nor non-exploded leads to the same unacceptable result. The only alternative, in Einstein's view, would be to hold that the

[12] For a brief account of them, see Greenstein, G., and Zajonc, A., (1997): 86-92. See also Bohr, N., (1949).

[13] Letter of August 8, 1935, in Fine, A., (1986): 78. Fine notes that this letter may have influenced Schrödinger's famous cat example, of which later. See also the letter to Schrödinger of August 9, 1939 in Przibram, K., (ed.) (1967): 35-36.

[14] Einstein, A., (1949): 670. Einstein also claimed that the orthodox view amounted to "the same thing as Berkeley's principle *esse est percipi* [to be is to be perceived]" (Ibidem, p. 669). In other words, in his view, the orthodox view resulted in idealism.

state function does not express the actual state of the powder, but merely our knowledge of it. The result would then be that "the laws of nature one can formulate do not apply to the change with time of something that exists, but rather to the time variation of the content of our legitimate expectations." [15] Adopting this, of course, would entail giving up the idea of science as a tool to understand the basic order of nature and sinking into epistemology.

Believing that quantum mechanics is incomplete, already in 1927 Einstein had proposed a variety of the ensemble interpretation, thinking that we need to look for a deeper theory. He told a correspondent in 1935: "In spite of the success of quantum mechanics, I do not believe that this method can offer a usable foundation of physics. I see in it something analogous to classical statistical mechanics, only with the difference that here we have not found the equations corresponding to those of classical mechanics."[16] Although his efforts in that direction were not successful, he never gave up his negative views. Rosenfeld recounts that in 1933 Einstein remarked to him that in quantum mechanics if two particles *a* and *b* interact and then become widely separated, a momentum measurement on *a* allows the immediate determination of *b*'s momentum, and similarly with position. This, however, is very paradoxical, for "how can the final

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[15] Letter to Schrödinger, August 9, 1939 in Przibram, K., (ed.) (1967): 35-36.

[16] Letter to Paul Langevin, 3 October 1935, quoted in Stachel, J., (2002): 395. Although Einstein never developed his variety of ensemble interpretation, what he had in mind were classical ensembles in which value determinism applies to each member, in contrast to the standard ensemble interpretation, which makes no such assumption. For some of the problems surrounding Einstein's attempt, see Whitaker, A., (1996): 213-16.

state of the second particle be influenced by a measurement performed on the first, after all interaction has ceased between them?"[17] In 1935, Einstein's idea provided the basic conceptualization of a paper (the so-called "EPR paper") that still occupies an important role in discussions about quantum mechanics. However, before we tackle it, it may be helpful to stake a step back.

8.3 A Cheap Approach

Suppose we are strong property realists. How could we show that, say, standard quantum mechanics is incomplete? If we could produce a physical situation such that the machinery of quantum mechanics would lead to the conclusion that incompatible observables simultaneously exist and have definite values, then we would show that the orthodox interpretation is inconsistent with the machinery of quantum mechanics at best, or that standard quantum mechanics is incomplete at worst. Here is a cheap try. The operators for the components of orbital angular momentum L do not commute. However, as we saw in our discussion of GUP, when the total angular momentum is zero, all the returns for angular momentum components are predicted to be (and are) zero all the times. Since the measurement returns are sharp, and are predicted to be sharp, it seems reasonable to conclude that the components of orbital angular momentum coexist and have definite values.

There are two objections to this move. One could claim that saying that a particle has zero orbital angular momentum is the same as saying that it has no property of angular momentum. However, a convincing retort is that in order to have value zero, the

[17] Rosenfeld, L., (1967): 127-28. On the prehistory of the EPR paper, see Jammer, M., (1974): 166-81.

property of orbital angular momentum must exists, much in the way in which a checking account must exist in order to have no money in it. The second objection, however, is inescapable. For one could simply note that in the case at hand the commutator of the two observables is zero and conclude that the argument attacks a straw man. Notice, however, that the objection is algebraic and as such radically different from Bohr's story about incompatible measurement apparatuses. In fact, here we have a case in which the need of incompatible measurement set-ups fails to result in HUPI at the theoretical level. This might require giving up part of Bohr's story but it hardly shows that quantum mechanics is incomplete or the orthodox interpretation problematic. A more serious attempt is required. To understand how such an attempt can be made, we need a bit more theory.

8.4 Entangled States

Suppose that particle 1 is described by $|\Psi_1\rangle$ in state space H_1 and particle 2 by $|\Psi_2\rangle$ in state space H_2. As we know, the composite system resides in $H = H_1 \otimes H_2$, and if $\{|e_1\rangle,...,|e_n\rangle\}$ is a basis for H_1 and $\{|d_1\rangle,...,|d_m\rangle\}$ a basis for H_2, then $\{|e_1\rangle \otimes |d_1\rangle, |e_1\rangle \otimes |d_2\rangle ..., |e_n\rangle \otimes |d_m\rangle\}$ is a basis for H so that any state vector $|\Psi\rangle$ describing the composite can be expressed as a linear combination of members of that basis. This may happen in two significantly different ways.

First, it may turn out that at all times $|\Psi\rangle = |\Psi_1\rangle \otimes |\Psi_2\rangle$, that is, the state vector of the composite system is the tensor product of the state vectors of the component systems, in which case the state vector is factorizable. For example, in the case of two spin-half particles it may happen that $|\Psi\rangle = |\uparrow_z^1\rangle \otimes |\uparrow_z^2\rangle$. Here particle 1 has an individual state vector, $|\uparrow_z^1\rangle$ and particle 2 another individual state vector, $|\uparrow_z^2\rangle$. Physically, this means that the two systems have not interacted, and are merely juxtaposed. For example, the two systems may have been prepared separately but be studied, for whatever reason, as one system. In practice, this means that the measurement returns from the two systems are independent of each other. In other words, the fact that a certain measurement return has been obtained for 1 has no relation whatsoever to the probability of obtaining any particular measurement return for 2.

Second, it may turn out that $|\Psi\rangle$ is not in general expressible as $|\Psi_1\rangle \otimes |\Psi_2\rangle$. Then, $|\Psi\rangle$ is said to be non-factorizable. In this case, according to standard quantum mechanics neither 1 nor 2 has an individual state vector: *only* the compound system 1+2 does. The two systems are then *entangled*, and the measurement returns for the two particles are not independent but correlated. Entanglement occurs when the two systems are not merely juxtaposed but they have interacted, for example by having a common origin or by coming in close proximity. As we shall see, entangled states play a very important role in discussions about quantum mechanics because if two systems become entangled, they remain entangled independently of their spatial separation as long as no measurement is performed. However, once measurement on either of the two particles is performed, the system will collapse into a non-entangled state. For example, consider a spin-half system in state $|\Psi\rangle = a|\uparrow_z^1\downarrow_z^2\rangle - b|\downarrow_z^1\uparrow_z^2\rangle$, and suppose that on measuring S_z^1 we get $\hbar/2$. Then the system collapses onto $|\Psi'\rangle = |\uparrow_z^1\downarrow_z^2\rangle$, which is not entangled. Hence, if systems 1 and 2 have become disentangled, and do not physically interact any longer, then further spin measurement returns on one are no longer correlated to spin measurement returns on the other.

A particular type of entangled system will be of special interest to us, namely that occurring between to spin-half particles (such as two electrons) when the total spin of the system is zero. Then, the two particles are in the *singlet configuration*,

$$\frac{1}{\sqrt{2}}\left(\left|\uparrow^{p}\downarrow^{e}\right\rangle - \left|\uparrow^{e}\downarrow^{p}\right\rangle\right).^{18} \qquad (8.4.1)$$

8.5 The EPR paper

In 1935, Einstein, Podolsky, and Rosen published a paper "Can Quantum-Mechanical Description of Physical Reality be Considered Complete?" in which they tried to show that the question should be answered negatively. After reasonably noting that we want a physical theory not only to be true but also complete, the paper, which has become known as the "EPR" paper after the initials of their names, makes three basic assumptions, two largely philosophical and one largely physical. The first philosophical assumption consists in providing a necessary condition for the completeness of a physical theory (NCC):

> Whatever the meaning of the term *complete,* the following requirement for a complete theory seems to be a necessary one: *every element of the physical reality must have a counterpart in the physical theory*." (Einstein, A., Podolsky, B., and Rosen, N., (1935) in Wheeler, J. A., and Zureck, W. H., (eds.) (1983): 138).

The paper does not explain what counts as an element of reality, although in the course of the argument it becomes clear that the values of physical properties and quantum states

[18] For the role of entanglement in the bizarre phenomenon of quantum teleportation, see appendix 3.

count as such. Leaving considerations of truth aside, NCC seems eminently reasonable because of its rather modest requirements. For example, it does not require that the theoretical counterpart of the element of physical reality render it with total accuracy.

Of course, NCC is of little use unless a criterion for being an element of reality is provided. The element of physical reality, we are told, cannot be determined by a priori philosophical considerations but by appealing to "the results of experiment and measurement." This leads to the second philosophical assumption (SER), which amounts to a sufficient condition for being an element of reality:

> If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of reality corresponding to this physical quantity (Ibidem, 138).

SER too is reasonable for it allows the theory itself to determine what counts as an element of reality, thereby eliminating the problem of judging the completeness of a theory in terms of another radically different theory. Furthermore, if a theory allows us successfully to predict with certainty the return of a certain measurement, it is plausible to claim that there is something in the world corresponding to what the theory says is being measured, for this seems part of the best explanation for the correct return of the measurement.

The physical assumption is the Principle of Locality (PL), the claim that if "two systems no longer interact, no real change can take place in the second system in consequence of anything that may be done to the first system" (Ibidem, 140). The key notion here is that of interaction. EPR seem to have considered this assumption a truism since, they immediately continued, "This is, of course, merely a statement of what is meant by the absence of interaction between the two systems." Two points should be noted. The paper did not explain what a real change amounts to, but in the course of the argument it becomes clear that the production of a new physical property, the alteration of its value, or the production of a new quantum state would constitute a real change. The paper also failed to explain what qualifies as a system. However, as before, the argument makes clear that even the widely separated compounds of an entangled system count as systems even if, according to standard quantum mechanics, they do not have a corresponding state vector.[19] Minimally, then, PL entails that if we have two systems (in the EPR paper sense), then their interaction (in terms of real change) can be made arbitrarily small, in fact so small as to become negligible, by increasing their separation. This seems quite reasonable, since all known forces diminish with distance.

Now, the authors claim, in quantum mechanics (that is, in the Copenhagen account, we should add) any two incompatible observables are subject to HUPI, and therefore the precise knowledge of one prevents that of the other. For example, the precise determination of momentum produces uncertainty in the determination of position. In addition, the "usual conclusion" drawn from this (that is, the conclusion drawn by Copenhagen and orthodox theorists) is that when the value of an observable *O*

[19] This is the Principle of Separability. It is not stated but it is obviously implied by the overall argument in that PL would not be applicable to the EPR's example without it.

is known, any other observable *Q* incompatible with *O* does not exist.[20] Hence, the paper continued, either quantum mechanics is incomplete, or incompatible observables cannot have simultaneous reality. With the help of SER, PL, and, in their view, uncontroversial quantum-mechanical manipulations, EPR argued that in some non-trivial cases incompatible observables can be shown to have simultaneous reality. Hence, they concluded (validly, but perhaps unsoundly) that quantum mechanics is incomplete.

There are two main problems with the EPR paper. The first, relatively minor, is that the specific quantum mechanical situation involving the incompatibles position/momentum that the paper sets up has been challenged. However, an analogous situation can be created by using incompatible spin components; indeed we shall do just that.[21] The second, and more difficult, problem is that the paper does not provide any clear, detailed argument for the conclusion that a system can have simultaneous definite incompatible properties. What follows is an interpretative restatement of the argument.

8.6 The Argument

Consider a system of two entangled spin-half particles *a* and *b* in the singlet configuration $\frac{1}{\sqrt{2}}\left(\left|\uparrow_z^a\downarrow_z^b\right\rangle-\left|\downarrow_z^a\uparrow_z^b\right\rangle\right)$. Let us assume also that the particles are allowed to

[20] Presumably, this is because the state of the system is not an eigenstate of *Q* and EE kicks in. However, as we saw, in some cases incompatible observables can share eigenvectors.

[21] In addition, we use spin because Bell's Inequality, which we shall study later, derives from Bohm's improvement (of which more shortly) on the EPR paper, and Bohm uses spin.

move sufficiently apart, so that they no longer interact. Now suppose that we measure S_z^a using $\hbar/2$ as a unit of measure, and obtain $+1$. Then, because of the collapse rule postulated by standard quantum mechanics (not to mention the conservation of total spin), S_z^b will certainly yield -1. Hence, SER and PL allow us to conclude that there is an element of reality corresponding to S_z^b, the *z*-component of *b*'s spin.

However, singlet states of total spin enjoy rotational invariance, which entails that when the singlet state vector is expanded in terms of the eigenvectors of S_x^a and S_x^b, one obtains $\frac{1}{\sqrt{2}}\left(\left|\uparrow_x^a\downarrow_x^b\right\rangle - \left|\downarrow_x^a\uparrow_x^b\right\rangle\right)$. Suppose now that instead of measuring S_z^a we measure S_x^a and obtain $S_x^a = 1$. Then, by the same reasoning as above, a measurement of S_x^b will certainly yield -1. Hence, SER and PL allow us to conclude that there is an element of reality corresponding to S_x^b, the *x*-component of *b*'s spin.

In addition, collapse plus EE imply that the measurement of $S_z^a = 1$ reduces *b*'s state to $\left|\downarrow_z^b\right\rangle$, and that of $S_x^a = 1$ reduces it to $\left|\downarrow_x^b\right\rangle$. However, PL prevents anything happening to *a*, including having its spin components measured, from altering or producing a new quantum state in *b*. Hence, $\left|\downarrow_z^b\right\rangle$ and $\left|\downarrow_x^b\right\rangle$ "belong" or "are assigned" to "the same reality." In short, *b* has incompatible observables with definite values and its quantum state is represented by their non-trivially different eigenvectors.

EPR were aware that one could try to resist their argument by restricting SER and claiming that two physical quantities can be considered "simultaneous elements of reality *only when they can be simultaneously measured or predicted.*" (Einstein, A., Podolsky, B., and Rosen, N., (1935) in "Wheeler, J. A., and Zureck, W. H., (1983): 141). However, they held, such a restriction would be unwarranted because it would make the reality of

S_z^b and S_x^b depend on measurements carried out on *a* which, PL tells us, does not disturb *b* at all. "No reasonable definition of reality could be expected to permit this," they claimed. The paper ended by stating their belief that a complete description of physical reality is possible.

What should one make of the argument, such as it is? It may be helpful to have a visual representation of the situation. Consider a very simple experiment. We flip a fair coin; if we get heads (H), we toss a die loaded so that the probability of getting an even number (E) is double that of getting an odd one (O); if we get tails (T), we toss a fair die. This is a finite stochastic process, and therefore we can represent it by using a diagram in the form of a tree.[22]

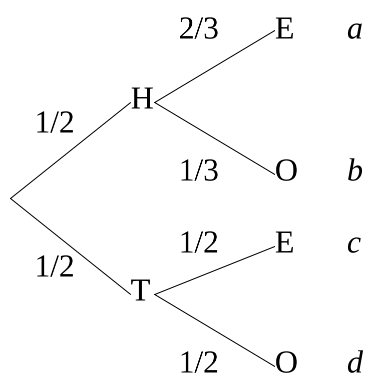

Figure 1

The tree represents all the possible outcomes of the experiment, with branch (*a*) expressing the sequence of events in which we get H with probability 1/2 and, given that we got H, E with probability 2/3; the other branches are interpreted similarly. We could

[22] A tree is a mathematical object, but we need not be more rigorous than necessary here, and therefore we just look at the diagram.

think of a branch as representing a possible world, a way in which the experiment could go.[23] The tree allows us to determine the probability of each possible world becoming the actual one. For example, the probability that (*a*) occurs is

$$\Pr(a) = \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}, \tag{8.6.1}$$

and similarly for the other branches.

A similar tree can be used to describe the EPR experiment. If we flip a coin to determine whether we measure S_z^a or S_x^a, the tree is given in figure 2.

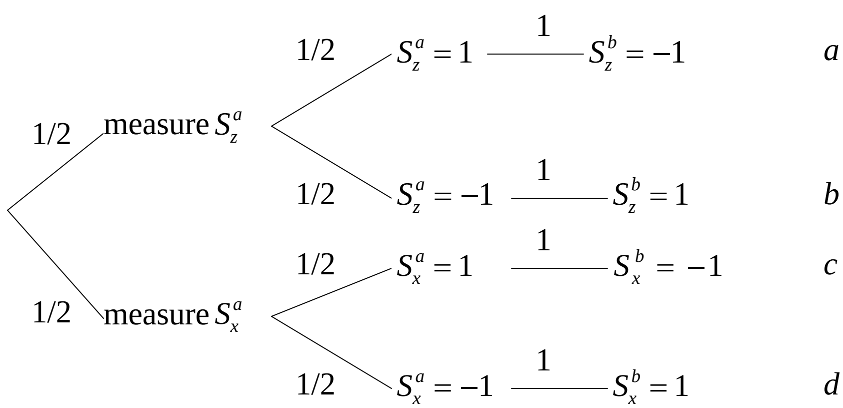

Figure 2

Armed with the diagram, let us try some possible readings of the argument. There is a reading of that obviously will not do. Here it is. Branch (a) says that if we decide to measure S_z^a and we obtain $S_z^a = 1$, then we can predict with certainty that $S_z^b = -1$, and consequently (here SER kicks in) that S_z^b has a definite value. Similarly, branch (b) says that if we decide to measure S_z^a and we obtain $S_z^a = -1$, then we can predict with

[23] Indeed, a branch, assuming that the experiment has been completed, could represent a counterfactual, a way the experiment could have gone but did not.

certainty that $S_z^b = 1$, and consequently (here SER kicks in) that S_z^b has a definite value. We can then infer that if we measure S_z^a, then S_z^b has a definite value. By an analogous reasoning on branches (c) and (d) we can infer that if we measure S_x^a then S_x^b has a definite value. So, if we measure both S_z^a and S_x^a, then both S_z^b and S_x^b have a definite value. The problem here is easy to spot: HUPI, which EPR assume, prohibits us from simultaneosuly knowing incompatible observables on *a*, which prevents us from knowing with certainty both S_z^b and S_x^b, which prevents the applicability of SER and the achievability of the conclusion that both have definite values.

There is another seemingly subtler rendition of the argument based on the idea that the conjunction of PL and the true statement "If we measure S_z^a, then S_z^b has a definite value" entails "S_z^b has a definite value." If it did, we could conclude that both S_z^b and S_x^b have definite values without performing any measurement on *a*. However, this will not do either. All that PL allows us to infer is that *if* S_z^b has a definite value, then it has that value independently of any measurement on *a*. But obviously "If we measure S_z^a, then S_z^b has a definite value" does not entail "S_z^b has a definite value." Of course, if we add "We measure S_z^a", we immediately obtain "S_z^b has a definite value", and then PL allows to infer"S_z^bwould have had a definite value even if we had not measured S_z^a." However, such a conclusion depends on the premise "We measure S_z^a". Without it, the argument cannot even start, and therefore we have failed to avoid the strictures of HUPI.

We might try a similar reading with respect to vector states and obtain "If we measure S_z^a and obtain 1, then *b*'s state is represented by $\left|\downarrow_z^b\right\rangle$" and "If we measure S_x^a

and obtain 1, then b's state is represented by $|\downarrow_x^b\rangle$." Then, we could add PL and try to obtain "*b* is represented both by $|\downarrow_z^b\rangle$ and $|\downarrow_x^b\rangle$," claiming that this is what EPR had in mind by their remark that $|\downarrow_z^b\rangle$ and $|\downarrow_x^b\rangle$ "belong" or "are assigned" to "the same reality." Finally, by adding EE we could conclude that *b* has definite simultaneous values for two spin components.[24] The problem here is the same as before: PL does not allow us to obtain "*b* is represented both by $|\downarrow_z^b\rangle$ and $|\downarrow_x^b\rangle$".

According to standard quantum mechanics, not only S_z^a and S_x^a cannot both be precisely measured simultaneously, they cannot both be precisely measured at all. One can measure either of the two only by bringing about the collapse of the system's state vector, and consequently the measurement of the other is applied to a different system that, for one thing, involves no entanglement. In other words, the measurement of, say, S_z^a reduces the state vector to either $|\uparrow_z^a\downarrow_z^b\rangle$ or $|\downarrow_z^a\uparrow_z^b\rangle$, and in either case *a* and *b* are not entangled any longer. This may be taken to suggest that the EPR paper makes essential use of counterfactuals: we actually measure S_z^a, but we must consider what would have occurred had we measured S_x^a.

As it happens, there is no agreement among philosophers and logicians regarding the proper treatment of counterfactuals. However, counterfactuals are used not only in science but in everyday life as well, and in handling them in the EPR context we need not get into the issues that make their full understanding very difficult. Once a branch in figures 1 and 2 has been selected as representing what actually happened, we could think

[24] This seems the view taken by Beller, M., (1999): 146, 153.

of the others as representing counterfactuals, ways in which the experiment could have gone but did not.

At this point, an orthodox theorist might claim that while the probabilities used in the coin-die tree are merely epistemic, that is, the result of our ignorance (the die and the coin, we may assume, behave according to both evolutionary and value determinism), the probabilities entering the EPR tree are ontological (for standard quantum mechanics the returns of quantum spin measurements do not agree with value determinism). This difference has an immediate consequence in the case of going back in time, as it were. For example, in the coin-die case, if (*a*) has actually occurred and we wonder what would happen if we were to go back in time to the moment when we got H, determinism guarantees that (*a*) would obtain again (that is, we would get E when tossing the die). By contrast, in the quantum case (Fig. 2), if (*a*) actually obtains and we wonder what would happen if we were to rerun the experiment from the moment we chose to measure S_z^a, since determinism does not apply we could not conclude that (*a*) would obtain again because there still would be a 50% (ontological) probability that the measurement return would be $S_z^a = -1$, which would inevitably lead to the actualization of (*b*) rather than (*a*). However, one can hardly see why this difference should be relevant to the EPR argument, which makes no use of such strange cases. Rather, the obvious problem for the counterfactual reading is that it allows only the counterfactual conclusion that, say, S_x^b would have such and such value if and only if thus and so had happened, which it did not.

Perhaps EPR were aware of all these problems when they claimed that no reasonable definition of reality could be expected to require that two physical quantities exist simultaneously only when they can be simultaneously measured or predicted. EPR might be right, but it is hard to see how this is relevant to their argument. The issue is not what a physical property requires in order to exist, or even what criteria must be satisfied before we can reasonably claim that a property exists, but what *their argument* requires in order to conclude that *b* has incompatible properties with definite values. And, as we saw, their argument requires that "We measure S_z^a" is true.

8.7 Bohr's Reply

"This onslaught came down upon us as a bolt from the blue" recounted Rosenfeld almost 30 years later. Pauli and Dirac expressed concern about the completeness of quantum mechanics. Bohr dropped everything else and "day after day, week after week," in a "state of exaltation" analyzed the EPR paper, eventually coming up with a reply (Rosenfeld, L., (1967)). The short of it is that Bohr persisted in the view that Copenhagen quantum mechanics is a complete theory, and that it is a mistake to consider the quantum mechanical system under study separately from the experimental apparatus used to study it. [25] The rest, however, is subject to different interpretations.

Bohr clearly thought that the account of complementarity he had developed had the resources to deal with the EPR paper. In fact, after referring the reader to his *Atomic Theory and the Description of Nature,* he proceeded to restate his views, using the usual references to the quantum of action, the individuality of quantum measurement, complementary properties and descriptions, and variations of his stock thought experiments allegedly showing the incompatible physical features designed to measure

[25] For an account of the early reactions to the EPR paper, see Jammer, M., (1974): ch. 6 and Beller, M., (1999): ch. 7.

position and momentum or the impossibility, due to HUPI, of knowing position and momentum precisely.

The core of his paper consisted in an analysis of a thought experiment designed to mirror the EPR example.[26] Consider a diaphragm D_1 suspended by weak springs from a fixed frame of reference R and with two parallel slits though which two non-interacting particles *a* and *b* pass at time *t*. As the particles go through the slits, they will be deflected, thus transferring momentum (that is, the vertical component of momentum) to D_1. Since we can know the change in D_1's momentum by measuring D_1's momentums before and after the particles' passage, we may discover that at time *t* the momentum of the two particles is $p_a + p_b$. We may also assume that the distance $x_b - x_a$ between the two particles at time *t* is known, being just the distance between the two slits. Curiously, however, although we know $x_b - x_a$, we cannot know either x_b or x_a. The reason is that HUPI tells us that since we know D_1's momentum at *t*, we cannot know its position, which entails that we cannot know where the slits are (where x_b and x_a are) with respect to R. Now, since momentum is conserved, in order to know p_a and p_b all we need to do is to measure one (for example, we could set up a second diaphragm D_2 behind D_1, and measure p_a) and obtain the other from $p_a + p_b$. So, we can know the precise momentum of each particle at time *t*. However, Bohr argued, we cannot know the corresponding positions. Since HUPI prevents us from obtaining them by looking at D_1, we must set up

[26] Here we follow Beller, M. and Fine, A., (1994). As usual with Bohr, there is wide disagreement on what his reply amounts to, especially because in this reading, the core of the paper is in a footnote. Quite sensibly, Bell claimed that he could not understand Bohr's reply (Bell, J. S., (1981): 155.

another diaphragm D_3, this one rigidly attached to the reference frame, behind D_1. But by the time we measure the position of *a* and *b* by using D_3, their position wave function will have spread out, with the result that the distance between the two particles will have become indefinite, thus preventing us from knowing x_b or x_a at time *t*.

Whatever the virtues of Bohr's analysis of his thought experiment, as Beller and Fine have argued, the basic problem is that it does not mirror the EPR situation. For while in the EPR situation a measurement on *a* is supposed neither to change nor create any physical property in *b*, the same cannot be said for Bohr's set up. For example, if we measure x_a by rigidly fixing D_1 to R, D_1's interaction with *b* will be physically different from what it would have been had we used D_1 to measure $p_a + p_b$, that is, had we not rigidly fixed it to R.

In his reply, Bohr focused on the notion of "element of reality," agreeing with EPR that the notion must be supplied by a "direct appeal to experiments and measurements." In effect, he attacked SER, claiming that it contains "an essential ambiguity when it is applied to the problems of quantum mechanics" which he located in the phrase "without in any way disturbing a system." He conceded that there is no "mechanical disturbance of the system under investigation during the last critical stage of the measuring procedure." However, he claimed,

> even at this stage there is essentially the question of *an influence on the very conditions which define the possible types of predictions regarding the future behavior of the system*. Since these conditions constitute an inherent element of the description of any phenomenon to which the term "physical reality" can be properly attached, we see that the argumentation

of the mentioned authors does not justify their conclusion that quantum-mechanical description is essentially incomplete" (Bohr, N., (1935), in Wheeler, J. A., and Zureck, W. H., (1983): 148).

Presumably, what he had in mind is that a quantum property is defined only within the context of a measurement set-up capable of measuring it. Since position and momentum require incompatible measurement set-ups, in a momentum measuring set up position is not so much unknowable as undefined. Reverting to our spin example, when we measure S_z^a we employ a SGZ device that makes S_x^a undefined and when we measure S_x^a we employ a SGX device that makes S_z^a undefined. Hence, at any one time we are dealing either with the particles in the singlet state plus a SGZ device or with the particles in the singlet state plus a SGX device, but not both. Consequently, contrary to the EPR paper, S_z^b and S_x^b are not aspects of the same element of reality (the spin of particle *b*). Rather, S_z^b is essentially associated with the use of a SGZ device on *a*, and S_x^b with the use of a SGX device on *a*.

The contextualization of quantum properties to their measuring devices was what Bohr chose to emphasize after 1935. The idea was not really new, being already contained in his 1934 book. However, the understanding of contextualization became more and more conceptual. As before, Bohr claimed that since in our investigation of the quantum world we are bound to use classical concepts developed in a macroscopic world in which the quantum of action is negligible, we cannot escape complementarity and HUPI, its formal manifestation. However, by now the focus was on the claim that talk of a quantum property without reference to the measurement apparatus is meaningless (a sort of meaning-contextualism), a different view from a contextualist position for which such talk is merely misguided because of the uncontrollable interaction between particle and instrument (a sort of casual contextualism).[27]

Of course, one could think of a being that need not use classical concepts. At the end of his life, Bohr recounted a conversation with Plank, a deeply religious man, who claimed that a God-like eye could certainly know the position and momentum of a quantum particle. Bohr's reply was that the issue is not what such an eye can see, but rather of what one means by "knowing" (Bohr, N., (1962) quoted in Favrholdt, D., (1994): 88). Even if there were such a thing as God's eye, we could not know what it would be like to have it; nor do we have any capacity to understand how to use classical concepts differently from how we use them, or to use concepts radically detached from everyday experience. In another Kantian twist, Bohr noted that knowledge, for us, can only be *human* knowledge. In this context, any appeal to a divine eye-view is meaningless.

Later in his career, perhaps because of increasing contacts with Soviet scientists, Bohr became concerned with rebuffing any sort of subjectivist interpretations of complementarity. He argued that the idea that physical attributes are created by the

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[27] Occasionally, Bohr seems to have come close to accepting the neopositivist view that a physical property is defined (that is, that it is sensible to talk about it) only if it can be measured or precisely predicted. However, as usual with Bohr's philosophical views, there is no agreement among scholars. For example, Folse, H. J., (1985) denies that Bohr adopted neopositivism, while Fine, A., (1986) claims that Bohr's answer to EPR is virtually textbook neopositivism. Faye, J., (1991) and Beller, M. and Fine, A., (1994) agree with Fine.

measurement is the result of a confusion, as is the view that the description of atomic phenomena makes an explicit reference to the individual observer, thus entailing that it is not perfectly objective (Bohr, N., (1958b); 3; 5). In a manuscript drafted around the same time, he expressed similar thoughts, directly criticizing Berkeley's "psychological individualism" perhaps in an effort to make clear that his views, contrary to Einstein's criticisms, did not amount to an acceptance of Berkeley's idealism (quoted in Favrholdt, D., (1994): 85).

8.8 Einstein's own Arguments

The EPR paper, for all its attempts at rigor, was neither clear nor rigorous. We know that Einstein did not actually write it; by and large, Podolski did. Privately Einstein was rather critical of it, claiming that the argument's main idea had been buried under too much formalism. In fact, his arguments for the incompleteness of Copenhagen quantum mechanics did not match that of the published paper. Rather than centering on incompatible observables, Einstein's arguments focused on the notion of quantum state, in effect trying to show that separability plus locality render quantum mechanics incomplete.

In a letter to Schrödinger written just after the publication of EPR, he argued as follows. (As before, we use the spin-half case). If we measure S_z^a we can at once infer that S_z^b has a definite value, and PL entails that S_z^b had that value before S_z^a was measured. However, before S_z^a was measured, according to standard quantum mechanics *b* did not have a state vector, and therefore standard quantum mechanics is incomplete.

In a short article in 1948, and later in a letter to his Swiss friend Michele Besso, Einstein produced a second argument. Quantum mechanics, he claimed, may be considered complete only if there is a one-to-one correspondence between state vectors and the real states of systems (Einstein, A. (1948), in Born, M., (ed.) (1971): 170-71).[28] Consider now a system made of two entangled subsystems *a* and *b* at a great distance from each other, so that they no longer interact. Then, because of the collapse rule adopted by standard quantum mechanics, depending on the type of measurement performed on *a*, $|\Psi_b\rangle$ (the state vector of the now un-entangled *b*) will assume various non-trivially different forms even if by hypothesis nothing that is done to *a* can affect *b*. Consequently, *b* is represented by more than one state vector, and therefore quantum mechanics is incomplete.[29] Note that this argument does not need the full force of PL in the EPR paper; all it requires is that a measurement on *a* not affect the physical state of *b*, thus eliminating the imprecision associated with EPR's reference to "real change." In addition, EPL, as we may call this weaken version of PL, entails that if *b* ends up in state $|\Psi_b\rangle$, then it was in that state even before measurement. Note also that such a conclusion conflicts with the standard quantum mechanical view that entangled subsystems do not have quantum states, the same result obtained in the previous argument.

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[28] Laudisia notes that this completeness requirement is identical to one put forth by von Neumannn (Laudisia, F., (1995)): 313.

[29] Letter to Michele Besso, 8 October, 1952, in Speziali, P., (ed.) (1972): 487-88; Einstein, A. (1948) in Born, M., (ed.) (1971): 170-71; see also Home, D., (1997): 365-66). Note that this argument is very close to a part of the original EPR argument.

8.9 Bohm's Simplification

In 1951, Bohm produced an argument, known as EPRB ("B" for Bohm) that, although in the same mold as the original EPR, is more powerful and drops the consideration of incompatible observables.[30] Consider again a system of two particles *a* and *b* in the singlet configuration and widely separated. Suppose now that we measure S_z^a and obtain $+1$. Then, we know that S_z^b must yield -1. PL allow us to infer that S_z^b had that value after *a* and *b* stopped interacting but before the measurement of S_z^a. But at that time the system was in the singlet configuration, which is certainly not an eigenstate of S_z^b (or, for what matters, of S_z^a). Hence, S_z^b fully existed even when the system was not in one of that observable's eigenstates, and therefore standard quantum mechanics, which adopts EE, could not determine it. This argument is quite forceful and, Bohm tells us, it won Einstein's approval (Bohm, D., (1986): 25).[31]

One can look at EPRB slightly differently. Suppose that we measure S_z^a and obtain $+1$. Then, a measurement of S_z^b must yield -1, and experience confirms the prediction. From the point of view of the property realist, there is nothing remarkable about this. Spin is conserved, and since the total spin is zero, the spins of the two particles must be equal in magnitude and opposite in sign. However, the situation is different for standard quantum mechanics, according to which before the measurement

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[30] He also used spin instead of position/momentum, which were used in the original EPR paper.

[31] This is hardly surprising, since the argument is almost identical to the one Einstein communicated to Schrödinger in 1935.

neither *a* nor *b* had spin up or spin down. It is the measurement of $S_z^a = +1$ that brings about the fact that *b* acquires the property $S_z^b = -1$, even if the distance separating *a* and *b* is such that no significant interaction between them seems possible. To make things worse, the propagation of the physical event presumably corresponding to the state vector collapse must be instantaneous. For if it were to take time to propagate from *a* (where, presumably it originates) to *b*, by sufficiently distancing the two particles it would be possible to measure S_z^b after *a* returned $S_z^a = +1$ but before the influence of the state collapse reached *b*, in which case *b* would return $S_z^b = +1$ half the times. Hence, for the standard interpretation to be correct the propagation of the physical event corresponding to the state vector's collapse must be non-local. At least on first inspection, it seems that if one refuses to abandon PL, one should conclude that standard quantum mechanics is incomplete. However, these are difficult issues, to which we shall return.