Chapter 14

Individuals

In this and the remaining chapters, we address a few traditional philosophical issues in the light of quantum mechanics. The problems raised are complex, and for this reason the discussion is not only more introductory than usual but also often centered on the orthodox interpretation. However, it should be sufficient to give a good idea of both the issues and the difficulties involved. As before, new quantum mechanical machinery will be introduced as needed.

14.1 Indistinguishable Particles

Rather misleadingly, physicists qualify as *identical* any two particles that have the same static properties, that is, the same mass, the same electrical charge, the same spin number, and so on. In other words, all the particles belonging to the same species are identical. For example, all electrons are identical, and so are all protons, while no proton is identical to any electron. Two particles that are not identical are always distinguishable in principle because they can always be differentiated by at least a static property.

However, the situation is different when it comes to identical particles. Here is the standard account of the situation. Imagine two identical particles such as two electrons that are in the same spin state. How could we tell them apart? The obvious answer is that their spatial locations are different. However, if the two electrons are not in eigenstates of position, according to the orthodox interpretation, they have no position, and therefore nothing to distinguish them. One could look at their wave functions but, even leaving aside the fact that the two wave functions exist in configuration space, if these overlap and we observe a particle in the overlap area, as it were, then we cannot tell which of the two electrons we have observed. In short, the two electrons are not only identical, but also in principle *indistinguishable*: there is no experiment by which we can tell which is which. This fact, the orthodox story goes, must be reflected in the quantum mechanical formalism, as the theory is, primarily, about observations.

If two particles cannot be distinguished by any physical measurement, then any operator representing a physical quantity, when applied to the system made up of them, must remain unchanged if we switch their labels. At this point, we are faced with a problem: if the two particles are indistinguishable, on what ground do we label them? The answer is that strictly speaking we do not. Indistinguishable particles are numerically distinct. For example, if two electrons in the same spin state have overlapping wave functions, we still have two electrons, not one. Hence, if we are dealing with *n* particles, the system's representation involves n labels $\mathbf{r}_1,...,\mathbf{r}_n$. We do not know which label refers to which particle, and in the formalism this fact is expressed by showing what must be the case if labels are completely interchangeable. In other words, we exchange (physical) indistinguishability for (notational) label interchangeability.

To consider the switching of labels systematically, we introduce the exchange operator $\hat{P}$ which, given any function or operator *f*, switches its arguments:

$$\hat{P}f(\mathbf{r}_1,\mathbf{r}_2) = f(\mathbf{r}_2,\mathbf{r}_1). \tag{14.1.1}$$

Obviously,

$$\hat{P}^2 f(\mathbf{r}_1,\mathbf{r}_2) = \hat{P}\hat{P}f(\mathbf{r}_1,\mathbf{r}_2) = \hat{P}f(\mathbf{r}_2,\mathbf{r}_1) = f(\mathbf{r}_1,\mathbf{r}_2), \tag{14.1.2}$$

which entails that $\hat{P}^2$ is a unit matrix (one with ones in the main diagonal and zeros everywhere else), since it leaves its argument unchanged. This is possible only if $\hat{P}$ is a matrix with all –1 or all +1 in the main diagonal and zeros everywhere else, which entails that for every eigenvalue λ_i, $\lambda_i = \pm 1$. It can also be shown that $\hat{P}$ is Hermitian.

Now, since $\mathbf{r}_1$ and $\mathbf{r}_2$ are interchangeable, the Hamiltonian operator will treat them identically, and therefore

$$\hat{H}(\mathbf{r}_1,\mathbf{r}_2) = \hat{H}(\mathbf{r}_2,\mathbf{r}_1). \tag{14.1.3}$$

Hence,

$$\hat{P}\hat{H}(\mathbf{r}_1,\mathbf{r}_2) = \hat{H}(\mathbf{r}_2,\mathbf{r}_1) = \hat{H}(\mathbf{r}_1,\mathbf{r}_2), \tag{14.1.4}$$

and in addition,

$$\hat{P}\hat{H}(\mathbf{r}_1,\mathbf{r}_2)\psi(\mathbf{r}_1,\mathbf{r}_2) = \hat{H}(\mathbf{r}_2,\mathbf{r}_1)\psi(\mathbf{r}_2,\mathbf{r}_1) = \hat{H}(\mathbf{r}_1,\mathbf{r}_2)\hat{P}\psi(\mathbf{r}_1,\mathbf{r}_2). \tag{14.1.5}$$

Consequently,

$$(\hat{P}\hat{H}(\mathbf{r}_1,\mathbf{r}_2) - \hat{H}(\mathbf{r}_1,\mathbf{r}_2)\hat{P})\psi(\mathbf{r}_1,\mathbf{r}_2) = 0, \tag{14.1.6}$$

which entails that

$$(\hat{P}\hat{H}(\mathbf{r}_1,\mathbf{r}_2) - \hat{H}(\mathbf{r}_1,\mathbf{r}_2)\hat{P}) = 0, \tag{14.1.7}$$

that is,

$$\left[\hat{P},\hat{H}\right] = 0. \tag{14.1.8}$$

In short, the exchange and the Hamiltonian operators commute, and therefore share some eigenfunctions constituting a basis. Consequently, (14.1.8) entails that there are solutions of the (time independent) Schrödinger equation that are eigenfuctions of the exchange operator.[1] In other words, if $\psi(\mathbf{r}_1,\mathbf{r}_2)$ is the state function for two particles at $\mathbf{r}_1$ and $\mathbf{r}_2$, there are at least some solutions such that

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[1] The time independent Schrödinger equation is obtained by TDSE when the Hamiltonian (the energy operator) is independent of time, in other words, when the system's energy is

$$\hat{P}\psi(\mathbf{r}_1,\mathbf{r}_2) = \lambda\psi(\mathbf{r}_1,\mathbf{r}_2), \qquad (14.1.9)$$

so that

$$\hat{P}\psi(\mathbf{r}_1,\mathbf{r}_2) = \pm\psi(\mathbf{r}_1,\mathbf{r}_2). \qquad (14.1.10)$$

Hence, either

$$\psi(\mathbf{r}_2,\mathbf{r}_1) = \psi(\mathbf{r}_1,\mathbf{r}_2), \qquad (14.1.11)$$

or

$$\psi(\mathbf{r}_2,\mathbf{r}_1) = -\psi(\mathbf{r}_1,\mathbf{r}_2). \qquad (14.1.12)$$

We now introduce a new quantum principle, the Symmetrization Postulate (SP) that we add to the list of quantum mechanical principles:

All the physically acceptable solutions to TISE for two indistinguishable particles must satisfy either (14.1.11) or (14.1.12).

Particles that satisfy (14.1.11), a symmetry requirement, are *bosons*; those satisfying (14.1.12), an anti-symmetric requirement, are *fermions*. It can be shown in relativistic quantum mechanics, and experience confirms, that all particles with an integer spin

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constant, in which case the system is said to be conservative. Then, one uses $-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x) + V(x)\psi(x) = E\psi(x)$, or $\hat{H}\psi_i = E_i\psi_i$, an eigenvalue equation, where E is an eigenvalue of energy. The general solution to TDSE becomes

$$\Psi(x,t) = \sum_i c_i\psi_i(x)e^{\frac{-iE_i t}{\hbar}}.$$

quantum number are bosons and all particles with a half-integer spin quantum number are fermions.[2]

At this point one might object that the rationale for embarking on a derivation of SP was provided by the orthodox interpretation, and therefore SP is only as good as the orthodox story. However, this objection is misplaced. For, aside from the fact that unsound arguments can have true conclusions, it turns out that without SP one gets the wrong statistics. In particular, SP is behind the famous Pauli Exclusion Principle, without which one cannot explain the behavior of any atom more complex that hydrogen (which is the simplest of them all, one proton "orbited" by one electron). In short, SP is part of the quantum mechanical formalism. In fact, that the standard interpretation naturally leads to SP militates in its favor.

SP has remarkably bizarre consequences. The state of a system composed of two non-interacting particles is *factorizable*, that is, it is just the product of the states of the two particles:

$$\psi(\mathbf{r}_1,\mathbf{r}_2) = \chi(\mathbf{r}_1)\varphi(\mathbf{r}_2). \tag{14.1.13}$$

If we switch the two particles, we obtain

$$\psi(\mathbf{r}_2,\mathbf{r}_1) = \chi(\mathbf{r}_2)\varphi(\mathbf{r}_1). \tag{14.1.14}$$

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[2] So, for example, electrons, protons, neutrons, neutrinos (all spin 1/2), and deltas (spin 3/2) are fermions, while pions (spin 0), deuterons (spin 1), and gravitons (spin 2) are bosons. In addition, if the number of fermions in a composite particle is odd, then it is a fermion; otherwise, it is a boson. For example, the Hydrogen atom is made of two fermions, a proton and an electron. Hence, it is a boson. By contrast, Deuterium, an isotope of Hydrogen with an extra neutron, is a fermion.

If the two particles are distinguishable, then (14.1.13) and (14.1.14) express *two* states. To be sure, if the two particles have the same mass, then the two states have the same value, but they are still two.

However, if the two particles are indistinguishable bosons or fermions, then things are different because both (14.1.13) and (14.1.14) must obey SP, that is, satisfy (14.1.11) or (14.1.12). Consequently, (14.1.13) takes the form

$$\psi(\mathbf{r}_1,\mathbf{r}_2) = \chi(\mathbf{r}_1)\varphi(\mathbf{r}_2) \pm \chi(\mathbf{r}_2)\varphi(\mathbf{r}_1) \qquad (14.1.15)$$

and (14.1.14) takes the form

$$\psi(\mathbf{r}_2,\mathbf{r}_1) = \chi(\mathbf{r}_2)\varphi(\mathbf{r}_1) \pm \chi(\mathbf{r}_1)\varphi(\mathbf{r}_2), \qquad (14.1.16)$$

both of which treat the two particles identically by leaving undecided which particle is in which state. When (14.1.15) and (14.1.16) satisfy (14.1.11), they are identical, and therefore express the same physical state. When they satisfy (14.1.12), they differ by the factor –1. However, two state functions differing by a mere factor express the *very same* physical state. In short, (14.1.15) and (14.1.16) express not two physical states but *one*. Consequently, (14.1.13) and (14.1.14) express the very same physical state as well.

If the two particles are indistinguishable fermions, then (14.1.13) and (14.1.14) must satisfy the anti-symmetry requirement (14.1.12), and therefore

$$\psi_-(\mathbf{r}_1,\mathbf{r}_2) = \chi(\mathbf{r}_1)\varphi(\mathbf{r}_2) - \varphi(\mathbf{r}_1)\chi(\mathbf{r}_2). \qquad (14.1.17)$$

If $\chi = \varphi$, then $\psi_-(\mathbf{r}_1,\mathbf{r}_2) = 0$, and the state function vanishes, which entails that it cannot be normalized and therefore does not represent a physical state. Hence, it must be the case that $\chi \neq \varphi$. The point can be generalized into the *Pauli Exclusion Principle:* no two indistinguishable fermions can be in the same state.[3] If the two particles are indistinguishable bosons, then

$$\psi_+(\mathbf{r}_1,\mathbf{r}_2) = \chi(\mathbf{r}_1)\varphi(\mathbf{r}_2) + \varphi(\mathbf{r}_1)\chi(\mathbf{r}_2), \tag{14.1.18}$$

which satisfies (14.1.11). In this case, nothing remarkable happens.

It can be shown that SP entails that on average two indistinguishable bosons (fermions) are closer together (farther apart) than two distinguishable particles in the same states. In other words, all indistinguishable fermions behave as if there were a repulsive force among them, and all indistinguishable bosons as if there were an attractive force among them.[4] This, of course, generates a problem: if the wave functions of all identical bosons must satisfy the symmetrization requirement, then no boson can be studied without first considering its relation to all the other identical bosons in the

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[3] More precisely, each single particle state may be used only once in constructing products whose linear combination makes up the state function of the compounded system. Note that the principle does not apply to bosons. Pauli came up with the principle in 1925, a few months before the development of the Heisenberg-Schrödinger version of quantum mechanics, in order to account for the electronic structure of atoms within Bohr's theory.

[4] These are the so-called exchange forces. Typically, they are not taken to be real forces but manifestations of geometrical properties of the wave functions. For a discussion of the disadvantages of this notion (for example, two electrons in a singlet state, which is anti-symmetric, behave like two spinless bosons, that is, they 'attract' each other), see (Mullin, W. J., and Blaylock, G., (2003)).

universe. The same is true for identical fermions, linked by the anti-symmetrization requirement.

Fortunately, however, things are not as bad as they seem. The quantity by which on average two indistinguishable bosons (fermions) are closer together (farther apart) than two distinguishable particles depends on how much their two wave functions overlap. Hence, if the two particles are sufficiently far apart, their wave functions do not overlap (or just about), and consequently they can be treated as if they were distinguishable. For example, in an atom, the electrons occupying the same shell (that is, having the same energy and therefore occupying the same region of space around the nucleus) are sufficiently close for indistinguishability to appear. More generally, in atoms and molecules SP must be taken into account. However, as soon as the distances surpass several angstroms, SP can be safely ignored. One could say that crowding is a necessary condition for the indistinguishability of identical particles.

14.2 Statistical Quantum Mechanics

Consider an isolated system constituted by a large number N of particles, for example a certain amount of gas, and suppose that we know its *macrostate*, for example, its energy E and volume V (both of which remain unchanged because the system is isolated). The fundamental goal of statistical physics is to obtain other macroscopic properties of the system by appealing to an assumed microscopic model of the system. Of course, since $N \cong 10^{24}$, it is impossible for us to know the precise state of every particle (the *microstate* of the system), and therefore we have to use statistical methods. Fortunately, it turns out that the macroscopic features of a system do not depend on the state of each particle, but on their average behavior. For example, the temperature of a gas is given by the average kinetic energy of its molecules.

Typically, for every macrostate there will be a very large number of microstates. While a macrostate gives a description of the system bereft of all details, and a microstate provides a detailed description, a *distribution* gives a description with only some detail. For example, imagine a system made up of men, an army, say. Then, providing the average height of the men is analogous to providing the macrostate (no detail at all); providing the height of each man is analogous to providing the microstate (as much detail as possible); saying how many men are 1.6*m*, how many 1.7*m*, and so on, without saying how tall each man is, is analogous to providing a distribution. Typically, for every distribution there will be a number of microstates.

The basic assumption of statistical physics is that all the accessible microstates with the same energy are equally probable.[5] This assumption is crucial in determining the probability of each energy distribution among the N particles, for that probability will be the number of microstates associated with that distribution divided by the total number of microstates.[6] Consequently, how microstates are counted is of crucial importance. It turns out that the counting is very different depending on whether the particles are

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[5] Obviously, a microstate that cannot come about by any physical process (is not accessible) has no probability of obtaining even if it has the same energy as the others. For example, if the system is in a symmetrical state, it cannot transition to a non-symmetrical one.

[6] Since N is so large, the most probable distribution turns out to be overwhelmingly probable, so that we can be practically certain that it is the one describing the system.

distinguishable, bosons, or fermions. The details of statistical physics can be complex; fortunately, we need not tackle them. Instead, we are going to consider a completely unrealistic example that will however clarify the basic ideas.

EXAMPLE 14.2.1

Consider a system with energy $E = 3$ (this is the macrostate) made up of three distinguishable particles, labeled the first, the second, and the third, each of which can have $E = 0,1,2,3$. Let us designate each microstate by three numbers, the leftmost of which gives the energy of the first particle, the middle that of the second particle, and the rightmost that of the third particle. For example, (2,1,0) says that the first particle has energy 2; the second energy 1, and the third energy 0. Obviously, to the macrostate $E = 3$ there correspond 10 equiprobable microstates, to wit, (3,0,0); (2,1,0); (2,0,1); (1,2,0); (1,0,2); (1,1,1); (0,3,0); (0,0,3); (0,2,1); (0,1,2). By simple inspection, we notice that there are only three possible distributions.[7] In the first, one particle has energy 3 and two energy 0; three microstates, (3,0,0), (0,3,0), and (0,0,3), correspond to this distribution. In the second, one particle has energy 0, one has energy 1, and one energy 2; six microstates, (2,1,0), (2,0,1), (1,2,0), (1,0,2), (0,2,1), and (0,1,2), correspond to it. In the third, all three particles have energy 1; only one microstate, (1,1,1), corresponds to it. Since by hypothesis all the microstates are equiprobable, the probability that the first

[7] Actually, we are going in reverse, as it were. In real cases, it is impossible to provide the microstates, whose number is about N^N. Hence, first, one provides the distributions, and then one uses mathematical procedures to obtain the number of microstates corresponding to each distribution. However, the main point is unaffected by our procedure.

distribution occurs is 3/10; the probability that the second occurs is 3/5, and the probability that the third occurs is 1/10.

However, if the particles are indistinguishable, things are different because *the very counting* of the microstates changes in that all microstates that are transpositions of one another are counted as one. So, if the particles are indistinguishable bosons, the first, second, and third distributions are each associated with only one microstate, and therefore each has probability 1/3 of occurring. If the particles are indistinguishable fermions, in addition, the first and the third distributions are impossible because they break the Exclusion Principle, and therefore the only possible distribution is the second one, which is certain to occur. So, the statistics involving distinguishable particles, bosons, and fermions are different. That associated with distinguishable particles is the *Maxwell-Boltzmann* statistics; that associated with bosons is the *Bose-Einstein* statistics; that associated with fermions is the *Fermi-Dirac* statistics.

14.3 Identity

The treatment of identical particles in quantum mechanics raises interesting issues concerning their individual identity. However, before we discuss identity, we need to settle some terminological matters because physicists and philosophers do not use the term "identical" in the same way. We have followed physicists in saying that two particles are identical just in case they belong to the same species. For example, any two electrons are identical in this sense, while an electron and a proton are not. However, we shall follow philosophers in saying that two things are *numerically identical* when they are in fact the same thing. For example, George Washington and the first president of the US are numerically identical (they are the very same person). Similarly, the Morning

Star and the Evening Star are the very same celestial body, namely, Venus. Two things are *qualitatively identical* just in case they have exactly the same properties. Of course, what qualitative identity amounts to depends on what view one has of qualities. However, as a possible illustration, imagine two balls that are exactly alike in all their physical characteristics, or two classical material particles with the same mass and velocity. The relation between numerical and qualitative identity is controversial, for while the former entails the latter (at any given time, a thing is certainly qualitatively identical with itself), whether the converse holds is unclear at best, since the idea of two qualitatively identical things seems self-consistent.

As we saw, the statistics of indistinguishable and distinguishable particles are different because combinations resulting from the mere permutation of indistinguishable particles are not numerically identical: two of them, if one may say so, are counted as one. We are then faced with a disconcerting situation. On the one hand, indistinguishable quantum particles are numerically distinct. On the other hand, switching indistinguishable particles around makes no difference not only physically, but also in terms of the number of combinations (arrangements) in the sense that mere permutation creates no *new* particle arrangement. It is this very last feature that is a real affront to common sense. That indistinguishable and presumably, at least in the standard interpretation, qualitatively identical particles can be (ideally) switched around (made to change roles, as it were) without any observational consequence is not surprising and compatible with classical intuitions. But that two arrangements differing only in the permutation of indistinguishable particles should count not as two but as one, is very hard to swallow. It is as if with two quantum coins, so to speak, there were only one way of getting a tails-heads outcome instead of two.

The conclusion drawn by the founders of quantum mechanics was that quantum particles are not individuals.[8] Radical as this position is, one can see the reason for it. Counting is more basic than physics, and if two particles are individuals, permuting them must give rise to two arrangements, not one. Since assuming two combinations disagrees with experimental evidence, the two particles must not be individuals. The idea has recently been developed formally by constructing logical theories in which identity, while defined for macroscopic objects, is not defined for quantum particles. However, what concerns us here is not a study of such systems, but an understanding of what is involved in saying that a quantum particle is not an individual. Of course, this depends on what one takes to constitute individuality, and for this we need some philosophy.[9]

14.4 Individuals

We are all familiar with individuals in our everyday lives: Peter, the chair one sits on, one's computer, the flower in the pot, are all individuals. Among the many questions that may be asked about them, two stand out. First, what makes an individual an individual? Second, what are the criteria that allow us to determine that something is an individual? Clearly, the two questions are different: the first is ontological, the second

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[8] For a clear statement, see Schrödinger, E., (1984).

[9] For a comparison of two different logical systems dealing with quantum objects, see Dalla Chiara, M. L., Giuntini, R., Krause, D., (1998). For an exhaustive survey of various views on identity, see Gracia, J. E., (1988). Unfortunately, Gracia's excellent book does not cover quantum mechanical issues.

epistemological. Consequently, the answers to them may well be different, since the criteria for something need not be constitutive of it. However, minimally, successful criteria must be properly related to what they are criteria of. For example, wine labels are not constitutive of their wines. Labels saying "Barolo" do not make the wine they label the real thing, otherwise there would be no room for fraud. However, they are properly related (if they are good and honest labels) to the content of their bottles. That is, they label only those bottle whose contents come from such and such vineyard, were aged in such and such barrels, and so on. In addition, at times criteria are constitutive of the thing; after all, that a wine comes from such and such a vineyard, and so on, is what makes a certain wine Barolo, and is also a criterion for it. Consequently, a reasonable way to begin to answer the ontological question is to consider how we address the epistemological one.

Typically, we determine that something is an individual by noticing that it is somehow different from other things. For example, we may claim that Peter is an individual because he has some feature that distinguishes him from Paul, Mary, and whatever else. Perhaps, Peter has a snub nose and Paul an aquiline nose, and so on. If we are optimists, we may hope that in this case the criterion is constitutive, and make the general ontological claim that an individual is something that is different, in ways to be specified, from anything else. In other words, being appropriately different is a necessary and sufficient condition for being an individual. Since appropriate difference entails numerical difference (at any given time, different things are not one), an individual is numerically distinct from anything else: if Peter and Paul are individuals, they are different from each other, and therefore they are two and not one. Note that such a claim does not entail the converse, namely that if two things are numerically distinct then they are two individuals. In other words, the view that being appropriately different is being an individual entails that individuals are numerically distinct but is compatible with the view that some numerically distinct items are not individuals.

An obvious candidate for differentiating individuals is properties, as the previous example shows. If we denote qualitative difference with "QD", numerical difference with "ND", and individuality with "I", the following relations obtain:

$$QD \leftrightarrow I, \tag{14.4.1}$$

$$QD \rightarrow ND, \tag{14.4.2}$$

and therefore

$$I \rightarrow ND.^{10} \tag{14.4.3}$$

If we want to parlay this into the orthodox interpretation, we must keep in mind EE. At any given time, any particle in a unique eigenstate is qualitatively different from other particles, and therefore an individual. Hence SP, through the Pauli Exclusion Principle, forces individuality on (all) fermions, although not on bosons. For example, the two (numerically distinct) electrons orbiting the helium nucleus must have opposite spin states, and therefore they are individuals.

As we noted before, it would seem that (14.4.1)-(14.4.3) are consistent with

$$\sim (ND \rightarrow QD), \tag{14.4.4}$$

and therefore with

$$\sim (ND \rightarrow I). \tag{14.4.5}$$

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[10] Here we use $A \rightarrow B$ for 'if *A*, then *B*'; $A \leftrightarrow B$ for '*A* if and only if *B*'; $\sim A$ for 'it is not the case that *A*'.

In particular, the founders of quantum mechanics seem to have considered (14.4.5) established by the need to engage in non-Maxwellian statistics. At this point, one might argue that the orthodox interpretation cannot really accept (14.4.4)-(14.4.5). For, if spatio-temporal position (being at a certain place at a certain time) is a genuine property and quantum particles are impenetrable, then no two numerically different particles can occupy the same spatio-temporal location, with the result that if two particles are numerically distinct, then they are different, and therefore individuals, which amounts to negating (14.4.4) and (14.4.5). There is little doubt that quantum particles are impenetrable, as phenomena like the Compton effect are explained in terms of elastic collisions, and much of our experimental evidence about quantum particles is obtained by bombarding them head on with other particles in order to beak them into their constituents. However, the objection is misguided. If the two particles are in different position eigenstates, then EE guarantees that they have different positions, and therefore that they are qualitatively different, and hence individuals, per (14.4.1). By contrast, if they are not in eigenstates of position, then EE guarantees that they have no position at all, and therefore the property of impenetrability becomes irrelevant.

Up to now, we have considered the view that things are distinguished by their properties. However, there seems to be no a priori reason why properties should not be shareable. To be sure, we have noted that particles are impenetrable, but this property comes into play only if we consider position a genuinely intrinsic quality. But position seems to be a relational quality, as a particle has a position in relation to other particles. The problem here is that one might reject relational properties as genuine properties. So, if one believes that all real properties are shareable, that something is an individual because of its intrinsic features that make it different, then its individuality depends on whether another qualitatively identical thing happens to come about: it is as if one's individuality could die of competition. This is rather bizarre because it grates against one's intuition that a thing's individuality should be an intrinsic feature not dependent on anything else. If one is moved by this sort of considerations, and rejects the idea that qualitative identity entails numerical identity, then one must find the source of differentiation in something else than properties.[11]

A traditional candidate is some sort of primitive 'thisness' (*haecceitas*, as Duns Scotus called it) that is taken to constitute a thing's individuality. Peter and Paul could be qualitatively identical and yet they are individuals because each has a 'thisness' that makes each of them a particular. Since the notion of 'thisness' is primitive, it cannot be analyzed into simpler ones, and therefore it is perhaps bound to remain obscure, although we can certainly say that a 'thisness' is not a property and therefore it is not shareable. This last characteristic entails that numerically distinct items cannot have the same 'thisness', and consequently numerical distinction entails individual distinction. According to this view, then, an orthodox theorist could claim that indistinguishable particles are individuals simply because they are numerically distinct. [12]

Some have proposed other views about individuality. Individuals such as Peter and Paul are sometimes contrasted with universals such as humanity or animality. The fundamental difference between the two is that while universals can be instantiated,

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[11] That qualitative identity entails numerical identity is embedded in the Principle of the Identity of Indiscernibles, a view most famously espoused by Leibniz.

[12] For a different interpretation involving 'thisness', see Teller, P., (1998).

individuals cannot: there are many instances of humanity but no instance of Paul. To put it differently, humanity is a type of which Paul is a token. In this view, numerical difference is not a sufficient condition for individuality, as two universals such as humanity and animality are numerically distinct but not individuals. Hence, the numerical difference of indistinguishable particles does not entail their individuality. The real test is whether they can be instantiated. There seems to be little doubt that they cannot. The most one could say is that a quantum particle could be duplicated, but duplication is not instantiation.[13] The pen on the desk might be duplicated; perhaps it might be possible to create another pen that is indistinguishable from it, but even so, no instantiation of it has occurred in that all the items involved are individuals. Analogously, duplicating humanity, if that made any sense, would amount to creating another universal; instantiating humanity, that is creating at least one human being, is a completely different affair. In short, in this view, an orthodox physicist could hold that indistinguishable particles are individuals.

14.6 Restricting Accessible States

Suppose that one becomes convinced that numerical diversity does entail individuality, and that consequently quantum particles are individuals. How can one then account for the strange statistics? Obviously, one way is to change the probabilities of the various combinations. Think of two quantum coins; all one needs to do to get the

[13] Actually, there is good evidence that one may not even say that. The cloning of a quantum particle, that is, the production of a second quantum particle in the very same state as the first without the destruction of the first, seems incompatible with Special Relativity.

Bose-Einstein distribution is to assume that each of the two combinations with one heads and one tails has probability 1/6 instead of 1/3. This is tantamount to making them probabilistically dependent. When one gets heads (tails) on one quantum coin, the probability of getting tails (heads) on the other goes down; that is, getting heads (tails) on one coin increases the probability of getting the same on the other.[14]

Some have argued that the lack of accessibility is not due to exchange forces but to the fact that symmetric states must remain symmetric and anti-symmetric states must remain anti-symmetric. Consider now two indistinguishable particles and two state functions χ and φ. The indistinguishability of the particles demands that the arrangement in which one particle is in one state and one in the other is expressed as

$$\psi_+(\mathbf{r}_1,\mathbf{r}_2) = \chi(\mathbf{r}_1)\varphi(\mathbf{r}_2) + \varphi(\mathbf{r}_1)\chi(\mathbf{r}_2) \qquad (14.6.1)$$

if the particles are bosons and as

$$\psi_-(\mathbf{r}_1,\mathbf{r}_2) = \chi(\mathbf{r}_1)\varphi(\mathbf{r}_2) - \varphi(\mathbf{r}_1)\chi(\mathbf{r}_2) \qquad (14.6.2)$$

if they are fermions, with no physical possibility for a boson to end up in state (14.6.2) or for a fermion to end up in state (14.6.1) because the exchange operators is a constant of the motion. Consequently, half of the states corresponding to the arrangement in

[14] The point is cogently made in Reichenbach, H., (1956): 224-35. More generally, the idea that quantum mechanics leaves the metaphysics of individuality undetermined has been forcefully argued in French, S., (1989) and Huggett, N., (1997).

question are not available to bosons, and the same holds for fermions. Not surprisingly the probabilities associated with (14.6.1) and (14.6.2) are half their classical value.[15]

14.7 Bohmian Particles

For BM, each particle has an individual and determined trajectory which, together with the fact that even identical particles are numerically distinct, can be reasonably taken to entail that they are individuals. However, at times different trajectories are compatible with the same experimental evidence, in which case the particles are (for us) indistinguishable. For example, consider the following (simplified) scenario. We shoot two electrons, 1 and 2, at each other, making 1 travel to the right and 2 to the left. During the collision, the two wave packets overlap, and the conservation of total momentum is compatible with both the trajectories of figure 1.

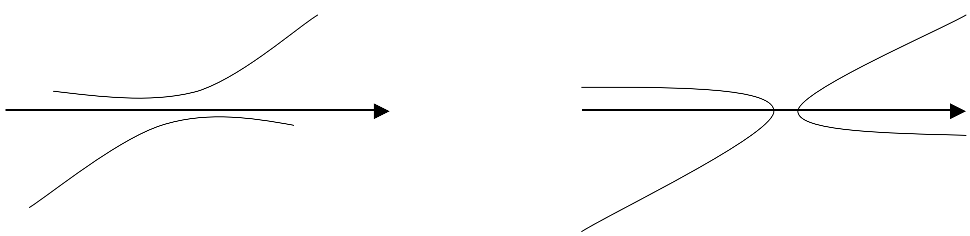

Figure 1

By no measurement can we determine which path 1 or 2 followed, although for BM the two scenarios cannot both be true. Indistinguishability is merely a matter of epistemology with no ontological implications.

The statistics proper to bosons and fermions arise from the fact that once the wave function has been rendered symmetric or anti-symmetric, the quantum potential will

[15] This point is made by French in French, S., (1989) and reiterated by him in several subsequent articles.

bring about the necessary forces.[16] Appealing to Bohmian exchange forces effectively restricts the number of physically accessible microstates without changing their purely combinatorial number. In other words, as far as counting microstates is concerned, it makes no difference whether distinguishable or indistinguishable particles are involved. However, with indistinguishable particles, some states are not accessible, and therefore should not be considered when probabilities are determined.

[16] See, for example, Bohm, D., and Hiley, B. J., (1993): 153-56.

Exercises

Exercise 14.1

Show how the symmetrization postulate affects the quantum potential. [Hint. Consider a two particle system satisfying the symmetrization postulate, so that $\Psi_{\pm}(\mathbf{r}_1,\mathbf{r}_2) = N_{\pm}\left[\varphi(\mathbf{r}_1,\mathbf{r}_2) \pm \varphi(\mathbf{r}_2,\mathbf{r}_1)\right]$, and show how it affects the quantum potential via R.]

Exercise 14.2

1. Show that $\hat{P}$, the exchange operator, is Hermitian.
2. Show that $\psi_{-}(\mathbf{r}_1,\mathbf{r}_2) = \psi(\mathbf{r}_1)\varphi(\mathbf{r}_2) - \varphi(\mathbf{r}_1)\psi(\mathbf{r}_2)$ satisfies the anti-symmetry requirement.
3. Show that $\psi_{+}(\mathbf{r}_1,\mathbf{r}_2) = \psi(\mathbf{r}_1)\varphi(\mathbf{r}_2) + \varphi(\mathbf{r}_1)\psi(\mathbf{r}_2)$ satisfies the symmetry requirement.
4. Right or wrong: (13.7.12) shows that switching particles changes the state function. Hence, it should be possible experimentally to decide whether switching has occurred.
5. Right or wrong: Given two indistinguishable particles and an observable O, $\left\langle \psi(r_1,r_2) \middle| O \middle| \psi(r_1,r_2) \right\rangle = \left\langle P\psi(r_1,r_2) \middle| O \middle| P\psi(r_1,r_2) \right\rangle$.

Answers to the exercises

Exercise 14.1

Since $\Psi_{\pm}(\mathbf{r}_1,\mathbf{r}_2) = N_{\pm}\left[\varphi(\mathbf{r}_1,\mathbf{r}_2) \pm \varphi(\mathbf{r}_2,\mathbf{r}_1)\right]$,

$R^2 = N^2{}_{\pm}\left[\left|\varphi(\mathbf{r}_1,\mathbf{r}_2)\right|^2 + \left|\varphi(\mathbf{r}_2,\mathbf{r}_1)\right|^2 \pm \varphi^*(\mathbf{r}_1,\mathbf{r}_2)\varphi(\mathbf{r}_2,\mathbf{r}_1) \pm \varphi^*(\mathbf{r}_2,\mathbf{r}_1)\varphi(\mathbf{r}_1,\mathbf{r}_2)\right]$. As the quantum potential is $Q = -\frac{h^2}{2m}\frac{\partial^2 R}{R\partial x^2}$, the interference terms represent the exchange forces.

Exercise 14.2

1. P's matrix is symmetrical with respect to the main diagonal, which consists of real numbers, $+1$ or -1, in fact. Hence, the matrix is Hermitian and so is the operator it represents.

2. $\psi(\mathbf{r}_1)\varphi(\mathbf{r}_2) - \varphi(\mathbf{r}_1)\psi(\mathbf{r}_2) = \psi_-(\mathbf{r}_1,\mathbf{r}_2) = \varphi(\mathbf{r}_2)\psi(\mathbf{r}_1) - \psi(\mathbf{r}_2)\varphi(\mathbf{r}_1)$.

3. $\psi(\mathbf{r}_1)\varphi(\mathbf{r}_2) + \varphi(\mathbf{r}_1)\psi(\mathbf{r}_2) = \psi_+(\mathbf{r}_1,\mathbf{r}_2) = \psi(\mathbf{r}_2)\varphi(\mathbf{r}_1) + \varphi(\mathbf{r}_2)\psi(\mathbf{r}_1)$.

4. Wrong, because Ψ and $-\Psi$ describe exactly the same physical state.

5. Right, because $\hat{P}$ is Hermitian, and therefore $\left\langle P\Psi\middle|O\middle|P\Psi\right\rangle = \left\langle P^2\Psi\middle|O\middle|\Psi\right\rangle = \left\langle\Psi\middle|O\middle|\Psi\right\rangle$.