Appendix 2: Trigonometry

Using figure 1, we define

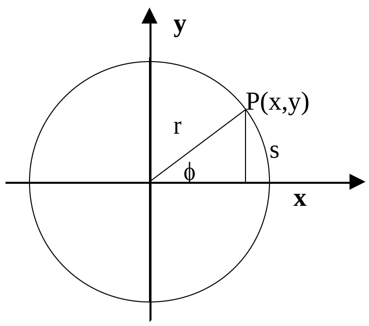
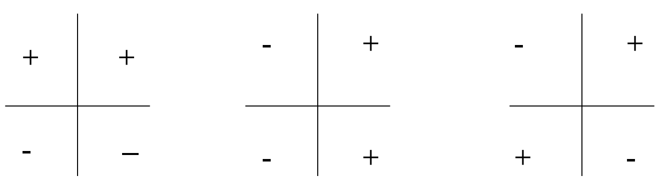


Figure 1

$$\sin\phi = \frac{y}{r}; \cos\phi = \frac{x}{r}; \tan\phi = \frac{\sin\phi}{\cos\phi} = \frac{y}{x}. \qquad \text{(A2.1)}$$

We should note that ϕ is measured *counterclockwise* starting from the positive *x*-axis. Although the distance *r* is always positive, the signs of *x* and *y* depend on the quadrant, and consequently the basic trigonometric functions have the following signs:



Sin ϕ   Cos ϕ   Tan ϕ

Figure 2

Trigonometry can be very useful when working with right triangles, since in a right triangle the following relations obviously hold:

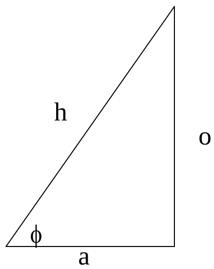


Figure 3

$$\sin\phi = \frac{o}{h};\ \cos\phi = \frac{a}{h};\ \tan\phi = \frac{o}{a}. \tag{A2.2}$$

EXAMPLE A1.1

When the sun is 60° above the horizon, how long is the shadow cast by a 150*m* high skyscraper? Using figure 3, let us set $o = 150m$ and $\phi = 60^\circ$. What we look for is *a*. But $\tan\phi = \frac{o}{a}$, so that $a = \frac{o}{\tan\phi}$. Hence, $a = \frac{150}{\tan 60^\circ}$. By using a scientific calculator, we obtain $a = \frac{150}{1.73} = 86.7m$.

In the examples, we have measured angles in terms of degrees. However, for reasons that need not concern us, in calculus it is better to use *radians*. Consider figure 1; if $s = r$, that is, if the angle ϕ cuts off an arc length *s* equal to the radius *r*, then ϕ measures one radian. If ϕ is two radians then s is twice the radius *r*. In general,

$$s = r\phi\,, \tag{A2.3}$$

where ϕ is measured in radians. When *s* is the whole circumference, then $s = 2\pi r = r\phi$, that is, $2\pi = \phi$. So, there are 2π radians in 360°, which allows us to convert degrees in radians and vice versa. For example, 90° are equivalent to $\pi/2$ radians.[1]

Using (A2.1), if we set $r = 1$ and make ϕ the independent variable (in which case it is customary to denote it with *x*), we obtain the following two plots for $\sin x$ and $\cos x$:

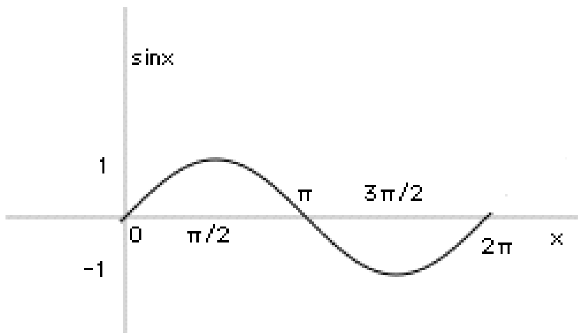


Figure 4

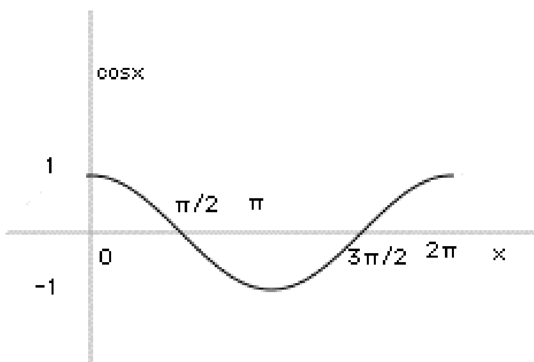


Figure 5

Note that the values of $\sin x$ and $\cos x$ oscillate between -1 and $+1$.

[1] When using a calculator to measure trigonometric functions, make sure you know whether you are using radians or degrees!

Exercises
Exercise A2.1

1. A man scrambles 300m along a road inclined 60° from the horizontal. How high has he gotten? [Hint. Use figure 3 with $h = 300m$ and $\phi = 60^\circ$].
2. A broken pole stuck into the ground makes a right angle with it. If the broken part makes a 60° angle with the ground, and the top of the pole is now 10m high, how tall was the pole before it broke? [Hint. Use figure 3 with $o = 10m$ and $\phi = 60^\circ$].

Exercise A2.2

Determine the radian equivalent of a: 30°; b: 45°; c: 180°; d: 270°.

Answers to the exercises

Exercise A2. 1

1. By using a scientific calculator, we obtain $o = 300m \cdot \sin 60^{\circ} = 259.8m$.

2. The total height is $(10 + h)m$. But $h = \frac{10m}{\sin 60^{\circ}} = 11.62m$.

Exercise A2.2

a: $\pi/6$; b: $\pi/4$; c: π; d: 1.5π.