Instructor Notes

Lesson: Math using common manipulatives in the ABE & GED classroom.

Why use manipulatives to teach math? There is a link between *seeing* math and *understanding* math.

Building with manipulatives incorporates the visual, kinesthetic, figurative, and tactile; **Saying** taps into the auditory and semantic; **Writing** teaches the symbolism of math and bridges the concrete and the abstract. The result? When a student sees the numbers on paper (abstract) he or she is able to visualize the concept.

Objectives: In this lesson, the participants will:

- Use common items (many are provided in this kit) in an effort to give students a concrete example of math principles.
- Utilize various critical thinking skills related to learning about math.
- Understand the importance of good math skills in everyday life as well as in workplace settings.
- Reduce stress often associated with learning math concepts by making the lessons interactive and fun.

Subjects: math, workplace skills, reading, writing, social studies, and critical thinking

Procedure:

1. Humans – the always accessible manipulative! Use the students in class to make bar graphs, walk number lines, or demonstrate commutative and associative properties. Students will remember these concepts when they have fun and move around.
2. Fractions can be difficult to comprehend so some ideas are included in this kit. Use the egg cartons and measuring cups to start the fractions lesson. If you really need a reinforcement activity, splurge on some candy bars! Four sets of laminated make-your-own fraction circles are also included for students to use.
3. Popsicle sticks are a great, inexpensive way to teach place value, carrying, and borrowing. A game with Popsicle sticks is included as well.

4. Building a brick wall uses critical problem solving skills as well as math. Enjoy allowing your students to use dominoes to create a wall and to think strategically.
5. “Make Fifteen” and “24” are two games using number cubes to reinforce basic number facts and order of operations. This game is a real winner because it can be altered slightly to make it easier or more difficult depending on the needs of your students.
6. Convincing students to focus on math facts can be difficult. Use the *"Fact Family Houses"* to reinforce addition or multiplication facts. This activity works especially well when an independent learning activity is needed.
7. As a hands-on activity to help students visualize what is happening in the *"Fact Family Houses",* try the *"Math on a Plate"* activity.
8. Even more reinforcement and ideas for integer rules can be found on the *"Developing Integer Rules"* worksheet. Some students will enjoy learning a short history on the subject. Use the chips to let students have a hands-on approach to this lesson.
9. Positive and negative numbers were introduced in the first activity with the number line. If your students are ready to move on and practice their knowledge in this area, use the deck of cards and instructions included in activity # 9 to have fun with this concept that is sometimes confusing.
10. Pentominoes are shapes that use five square blocks joined edge to edge to form various combinations. Pentominoes allow the introduction of important mathematical concepts and skills in a fun way. Some of those concepts include: measurement, geometry, and multiples.
11. Probability can be a difficult, abstract notion. Use the probability lesson and let students actually try it out to let the idea make some sense. It will also give practice with graphing.
12. The *"Math Readiness Activities"* sheet includes ideas for those working with ABE students. All of the ideas are concrete examples to illustrate basic math concepts.
13. A list of more advanced math ideas is included for those teaching GED students. These are all practical life or work skills activities that utilize these specific math applications.

Tactile Math – Using Humans – the Always Accessible Manipulative

- Use humans for bar graphing. Have the students physically move and make rows for: gender, hair color, favorite subject, number of children, or any other topic.

- Ask students to actually walk through a number line of positive and negative integers. Place the numbers on the floor on pieces of paper writing positive in black and negative in red (accounting). Let students walk up and down the number line to act out addition or subtraction problems using positive and negative numbers.

- Demonstrate commutative and associative properties in math using humans. Have several students go to the front of the room and then switch places or switch groupings to illustrate these properties.

  For example: addition is **associative** if you can *group* numbers in any way without changing the answer. It doesn't matter how you combine them, the answer will always be the same. Addition and multiplication are both associative.

  Use your human manipulatives to show the following:

  1+(2+3)=(1+2)+3

  1+(5)=(3)+3

  6=6

  An operative is **commutative** if you can *change the order* of the numbers involved without changing the result. Addition and multiplication are both commutative. Subtraction is not commutative: 2-1 is not equal to 1-2.

Fractions

1. Use different color egg cartons to demonstrate fractions. Use 4 egg cartons to find one-half, one-third, and one-fourth of twelve. Show 2 cartons, one white and one blue. Have the student cut the blue carton in 1/2. Place the 2 equal parts (halves) in a white carton. With student watching, remove one part and ask, "How much of a whole carton is this?" (1/2) Place the 2 halves back into the carton and ask, "How many eggs will a carton hold?" How many eggs will ½ of a carton hold? How much is ½ of 12? Write the following and ask the student to complete:

   ½ of 12 = ____

   Display a pink carton and cut it in thirds. Do the same as above and then offer this equation:

   1/3 of 12 = ____

   Do the same with a yellow egg carton to show fourths.

   ¼ of 12 = _____

2. Fill a ½-cup measuring cup and empty it into a 1-cup measuring cup. Talk with the student about how much is filled. Repeat and talk about how many ½-cup measuring cups it takes to fill the 1-cup.

3. Use Kit-Kat or Hershey bars for this activity. Discuss how many parts make up a whole candy bar. Break off pieces and discuss what fraction of the candy is broken off and what fraction is remaining.

Popsicle Stick Activities

1. Use Popsicle sticks bundled in sets of ten and some individual sticks. Write a number from 10-99 and ask the students to model the number using the sticks.

2. Show nine Popsicle stick bundles of ten sticks and nine ones. Ask the student to count the number of 10s and ones and record the number. Place one more stick with the 9 ones and ask how many sticks there are. Observe to see if the student bundles the 10 ones to make 1 ten. If not, ask: "What do we do when we have 10 ones?" (make a ten). Then ask how many 10s do we have now? (10) We have 10 tens. What do we do when we have 10? (We bundle them) So we will bundle the 10 tens and we have? (100) One bundle of tens is 100. Make certain to have rubber bands on the table for this activity.

3. Follow up by having the student do problems to practice carrying/borrowing using the Popsicle sticks.

NIM

Place 20 Popsicle sticks in a row. Each of 2 players will alternately pick up 1, 2, or 3 Popsicle sticks until they are all picked up. The player who picks up the last Popsicle stick wins the game. Develop a strategy!!

Make Fifteen

Game for 2 players that provides practice for building and solving equations.

In turn, each player throws 3 dice and uses the number showing on top to form equations naming numbers 1 to 15 in that order. The number on each die must be used once, and only once, in an equation. When a player is unable to form an equation that names the next number, the play passes to the next player.

Example: First player throws 2, 4, and 6 and forms the equations below.

$(2 + 4) \div 6 = 1$
$(2 \times 4) - 6 = 2$
$(2 \times 6) \div 4 = 3$
$(6 + 2) - 4 = 4$
$(6 + 4) \div 2 = 5$

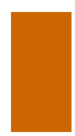
The player is unable to name 6, so the dice are passed to the next player. Player number one will begin naming numbers at 6 on the next round of play. The first player to name all numbers to 15 is the winner; however, if both players reach 15 in the same round, extend the goal to 21.

Building a Brick Wall

As you consider the different ways to make a wall with a certain number of bricks, look for a pattern or a system to predict what will happen with any number of bricks.

Brick wall made with one brick

Since the wall must be **2 units high**, there is only one possibility.



Brick wall made with two bricks

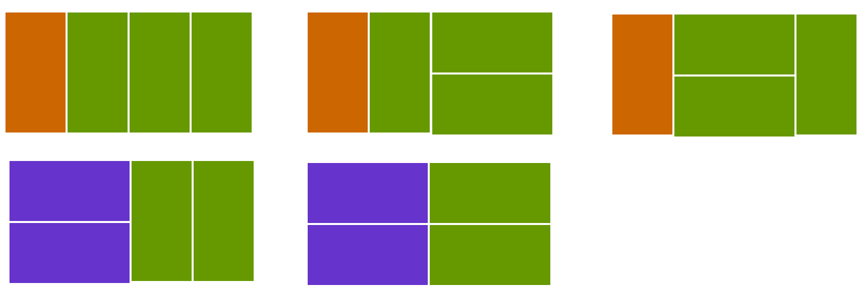
There are **two** possibilities if the wall can be made with **2** bricks. A height of **2 units** can be made either using two horizontal bricks or two vertical bricks.

Brick wall made with three bricks

There are **three** possibilities if the wall can be made with **3** bricks. Now the pattern begins. If you take the two possibilities that could be made using 2 bricks and **add a vertical** brick in front of both of them you will have the first two figures to the left below. Then look at the brick wall that could be made with 1 brick (2 before the one you are currently working on). Put two horizontal bricks in front of it and that will give you the third possibility.

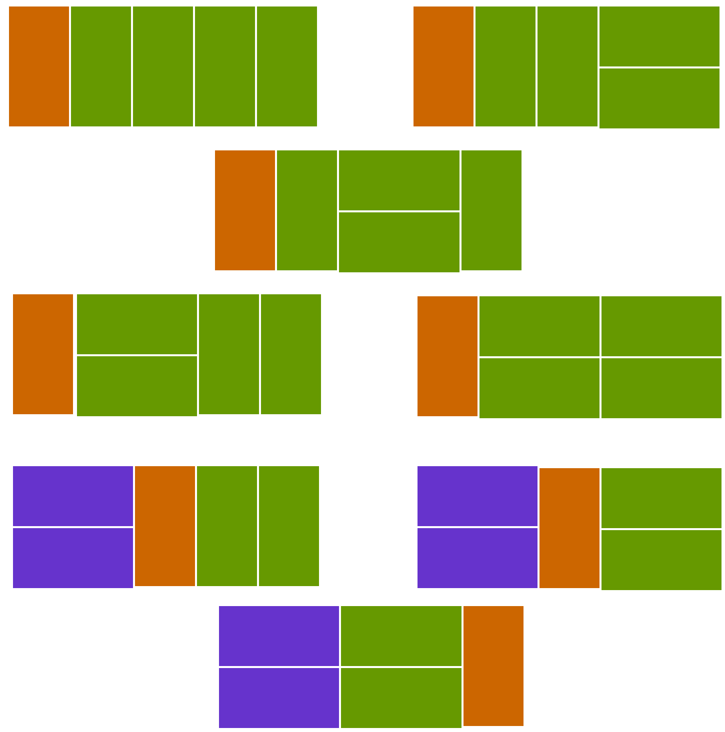
Brick wall made with four bricks

There are **five** possibilities if the wall can be made with **4** bricks. If you take the three possibilities that could be made using 3 bricks and **add a vertical** brick in front of each of them you will have the first three figures. Then look at the brick walls that could be made with 2 bricks (2 before the one you are currently working on). Put two horizontal bricks in front of them and that will give you the fourth and fifth possibilities.



Brick wall made with five bricks

There are **eight** possibilities if the wall can be made with **5** bricks. If you take the five possibilities that could be made using 4 bricks and **add a vertical** brick in front of each of them you will have the first five figures. Then look at the brick walls that could be made with 3 bricks (2 before the one you are currently working on). Put two horizontal bricks in front of them and that will give you the sixth, seventh and eighth possibilities.



Brick wall made with n number of bricks

If you look at the pattern that emerges if you are finding the number of configurations for any number, n, bricks:

you add 1 vertical brick to all of the possibilities listed under (n-1)
you add 2 horizontal bricks to all of the possibilities listed under (n-2).

This can be stated as a function: $F(n) = F(n-1) + F(n-2)$. This is the same function that generates the Fibonacci numbers.

Dominoes Activity

Materials:

- 2 grid sheets (one for each pair of students)
- 30 dominoes (15 for each pair of students) or cut out the paper dominoes
- Scissors (if necessary)

Procedure:

1. Work in pairs within your group to **show if it is possible to cover the 6X5 grid with your dominoes.** Take the time given to thoroughly complete the task.
2. Compare answers with the other people in your group.
    a. Did everyone have the same answer?
    b. If yes, can you find more than one answer?
    c. If no, how many possible answers there are?
    d. Describe how the dominoes cover the grid. Use the terms **vertical** and **horizontal** in your description.

Materials:

- 4 sheets of graph paper
- 30 dominoes cut out paper dominoes
- Scissors (if necessary)

Procedure:

Your Brick-Wall company wants to produce a **catalog** of designs to show to customers. If they miss out on a design then a competitor may offer it. So your company had better include all the designs you can. The brick walls are to be **two units tall**. The bricks are all the same size, **2 units by 1 unit**.

How many different ways can you make a wall using:

1. one brick?
   Hint: There is only 1 possible pattern.
2. two bricks?
   Hint: There are 2 possible patterns.
3. three bricks?
   Hint: There are 3 possible patterns.
4. four bricks?

5. five bricks?

6. six bricks?

7. any number of bricks?

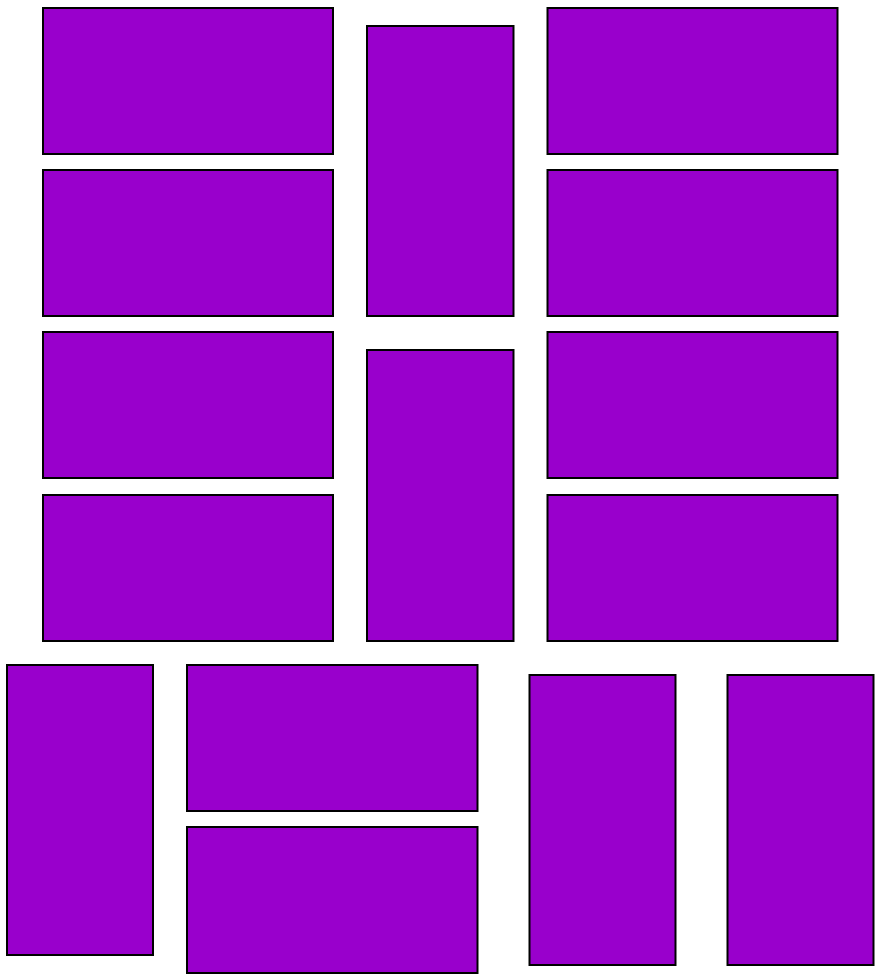
As your group investigates this problem each student records the diagram, process, and solution. There is a pattern that emerges as you investigate this problem.

Use your findings to create a Brick-Wall catalog including:

- A title for the company
- Sketches of the possible wall designs
- An explanation of how to figure out how many possible designs there are for each certain number of bricks to be used before repeating the design.

Domino Grid Sheet

Paper Dominoes



"24" Game

By: Elinor Crecelius, Wyoming Girls School;
Sheridan, Wyoming

Here is a game designed to review basic facts and the order of operations in a fun, problem-solving situation. I use it at the beginning of the period to warm-up and focus on math.

OBJECTIVE: Students will use four digits obtained from rolling four dice to create a mathematical expression equal to 24.

PURPOSE: Basic facts are important to learn, but boring to teach. This game has appealed to all of my students and has stimulated them to use what they do know and master what they don't.

MATERIALS:

- Four dice
- Paper
- Pencils
- calculators if desired.

PROCEDURE:

1. The first four students to class each get a die to roll and call out the number. I record the four numbers on the board. The students are then on their own to come up with a way to reach 24 using any or all of the four operations. For example, if 6,5,6, and 4, were rolled, one solution would be:
   4 x 6 / (6 - 5) = 24.

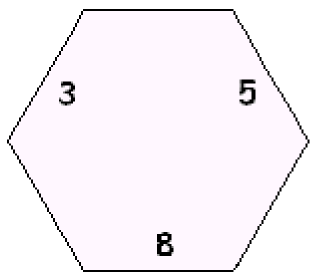
2. When a student has found a way, she writes it on the board as a mathematical expression using the most concise way to communicate her answer. When one way is found and agreed to be correct, many are ready to quit looking. I challenge them to find as many ways as possible to obtain 24 causing the given numbers. No answer counts unless it is written with the appropriate symbology. Although which operation used is not restricted, each number must be used only once.

TYING IT ALL TOGETHER:

This activity appeals to all levels. If appropriate, the use of exponents can be added. For instance, if the numbers 6, 4, 2, and 1 are rolled, one solution would be, 1 to a power of 3(6x4). As well as forcing a recall of basic facts, order of operations becomes a point of interest, and the idea of math symbols being used to communicate one student's thoughts to the rest of the class is stimulating.

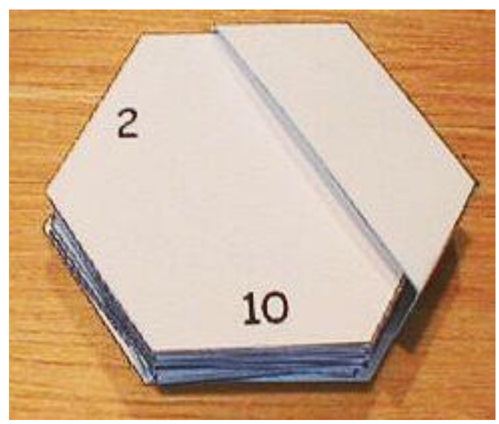
How to Use Fact Family Cards

Each card has three numbers on it. The number at the bottom is the product of the two numbers above it. These three numbers are a fact family. This card shows four related facts:



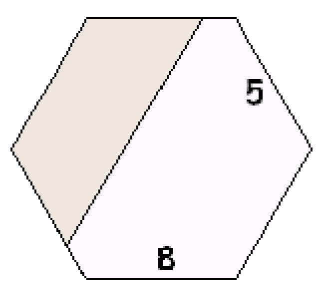
3 + 5 = 8
5 + 3 = 8
8 - 3 = 5
8 - 5 = 3

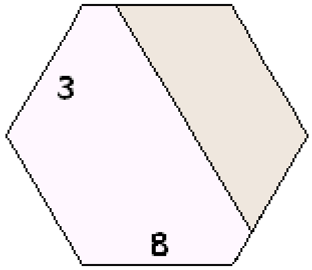
You can cut and glue a "fact family house" to hide one number at a time.
Place a stack of fact family cards in the house, say the missing number on the card (see the examples below), then pull the card off the stack and go on to the next card. At any time, you can move the position of the house so that it is hiding a different side of the cards.



You can say each missing number as part of an addition problem or a subtraction problem. Look at the examples below.

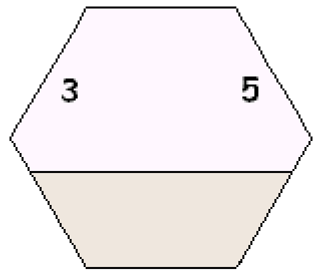
3 + 5 = 8
What plus 5 is 8? 3
8 - 5 = 3
What is 8 - 5? 3



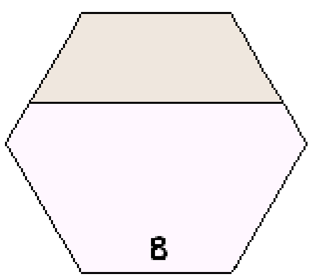


3 + 5 = 8
3 plus what is 8? 5
8 - 3 = 5
What is 8 - 3? 5

3 + 5 = 8
3 plus 5 is what? 8
8 - 3 = 5
What - 3 = 5? 8
8 - 5 = 3
What - 5 = 3? 8



You can even place the house so that its roof hides the top two numbers, then say all the pairs of numbers which add up to the number you can see. You won't know which pair is hidden under the roof, but you can turn this into a guessing game.

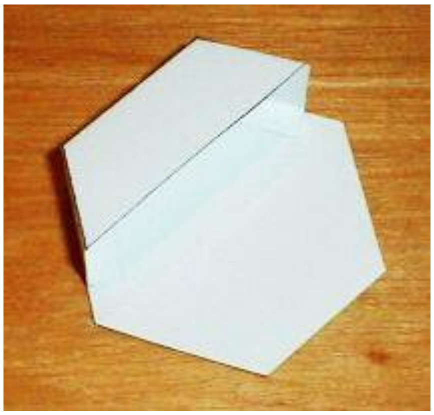


3 + 5 = 8
2 + 6 = 8
1 + 7 = 8

5 + 3 = 8
6 + 2 = 8
7 + 1 = 8

4 + 4 = 8

How to Make the Fact Family House



1) Print and cut out the model. It is best to use thick paper such as card stock (used for report covers).

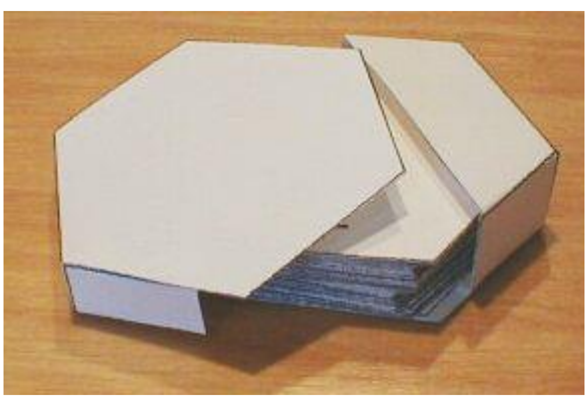
2) Fold and crease all of the lines.

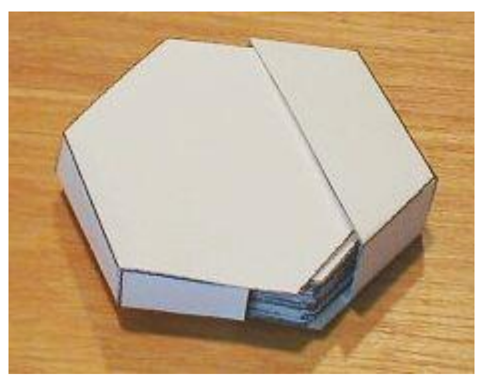
3) Decorate your house before gluing. You could color it or write your name on it.

4) Glue the two tabs marked with arrows to the rectangle, and glue the two tabs marked with "T" to the hexagon.

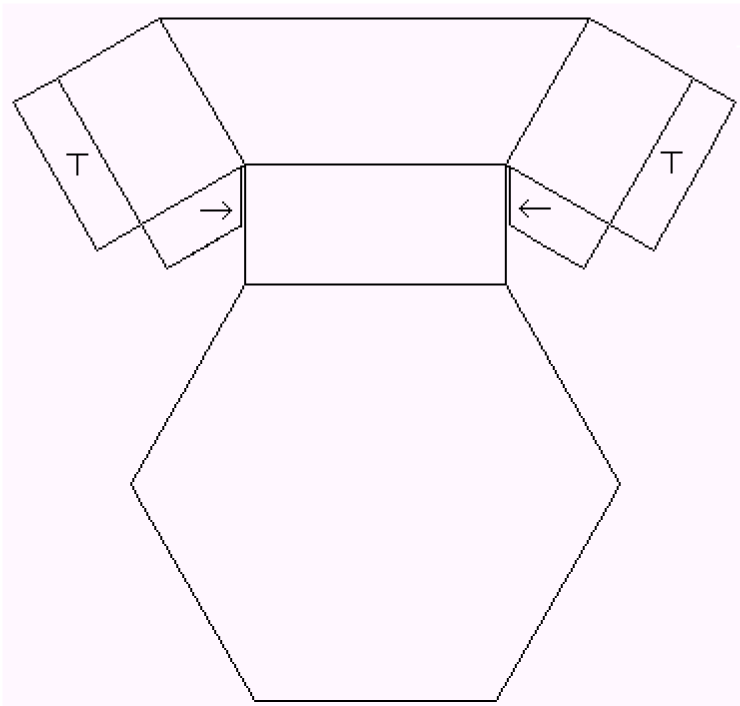
5) Place your fact family cards in the "house." See the links at the bottom of the page for ideas on how to use the cards.

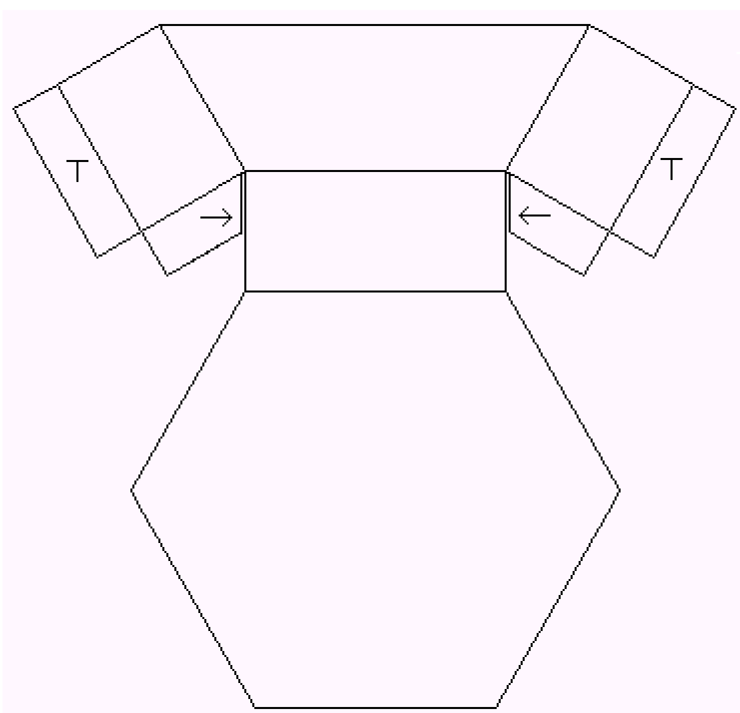
6) If you make a second fact family house, you can fit them together to store your cards. You can use a rubber band to keep them from spilling.



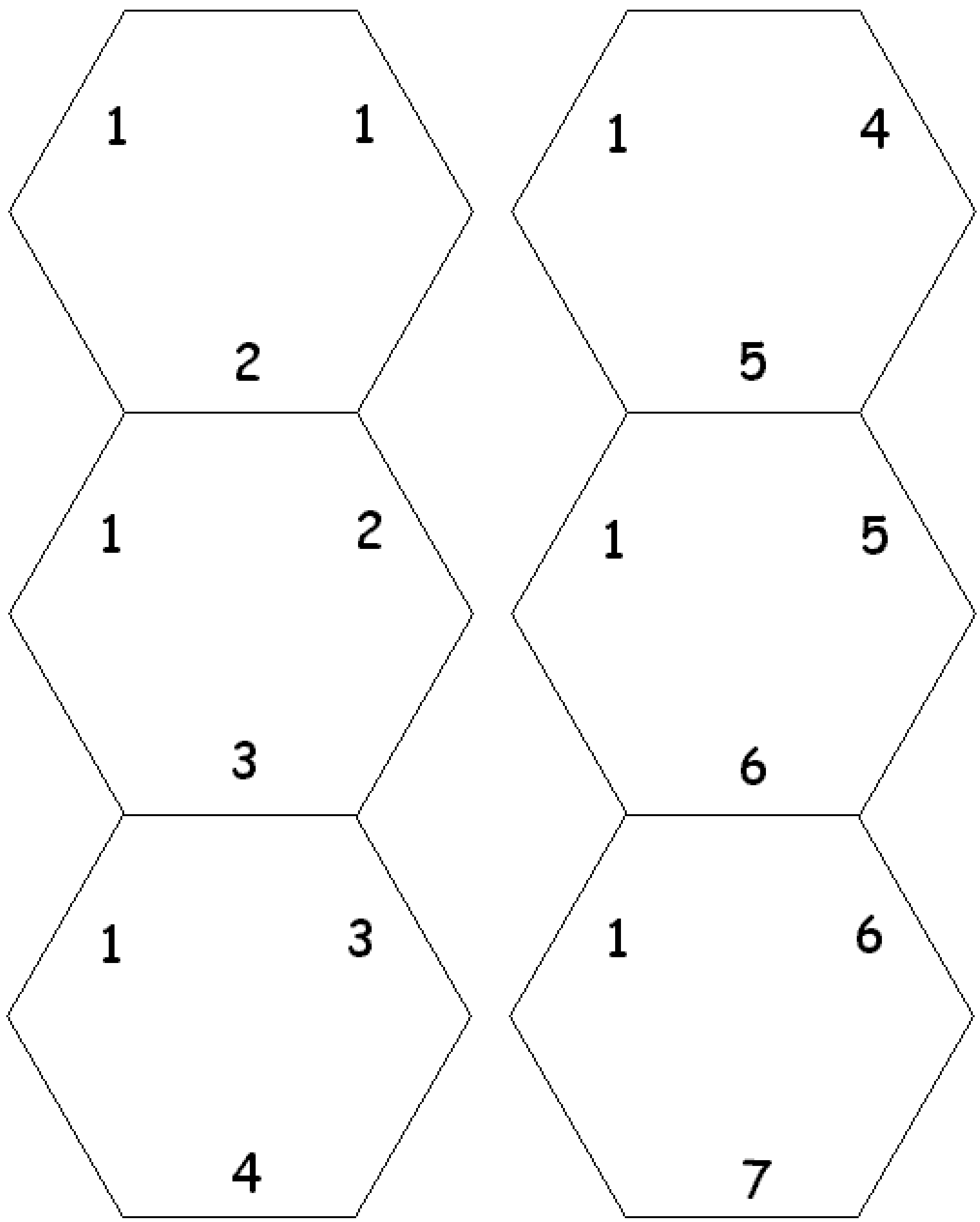


Activity # 6 (Page 4 of 13)

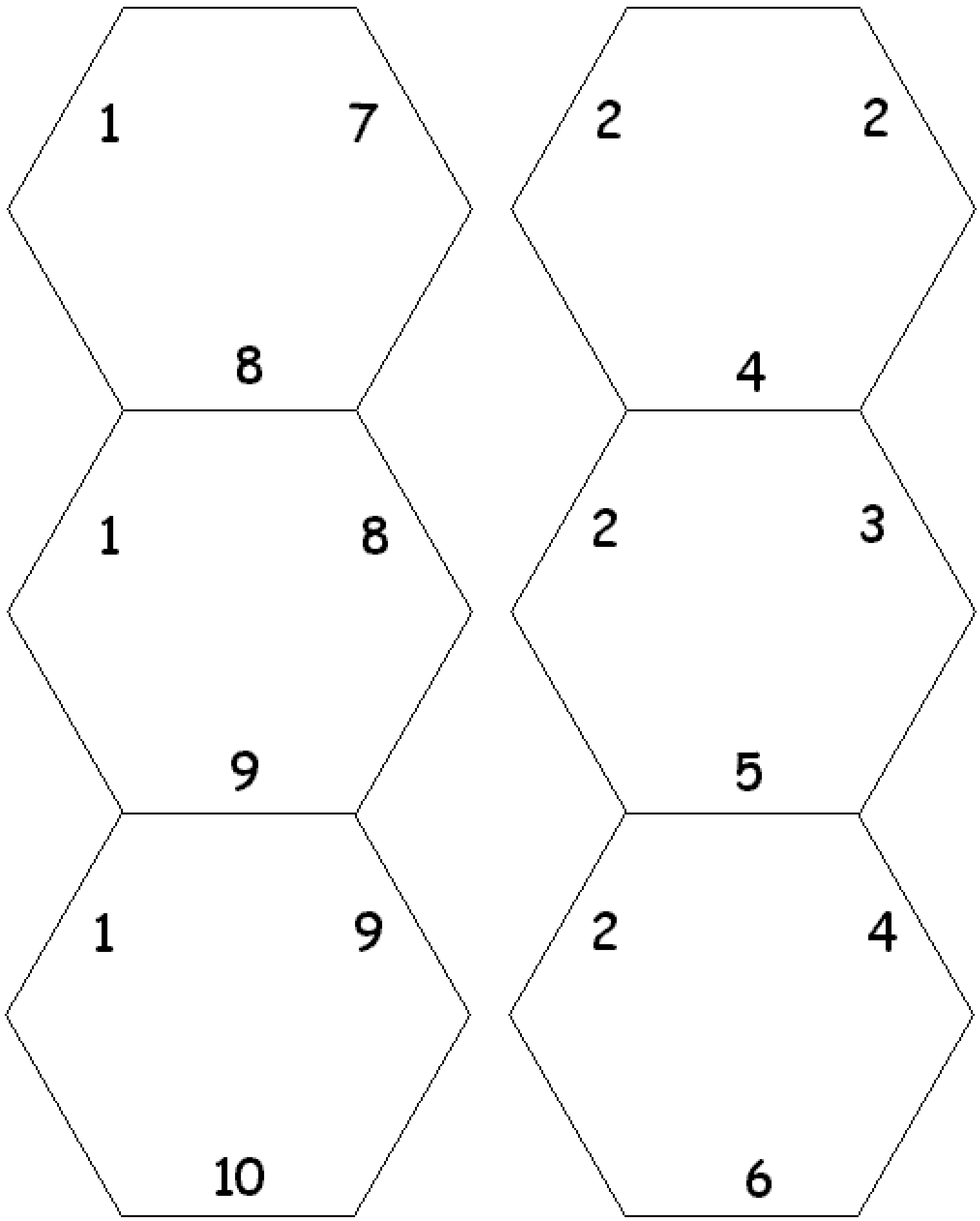




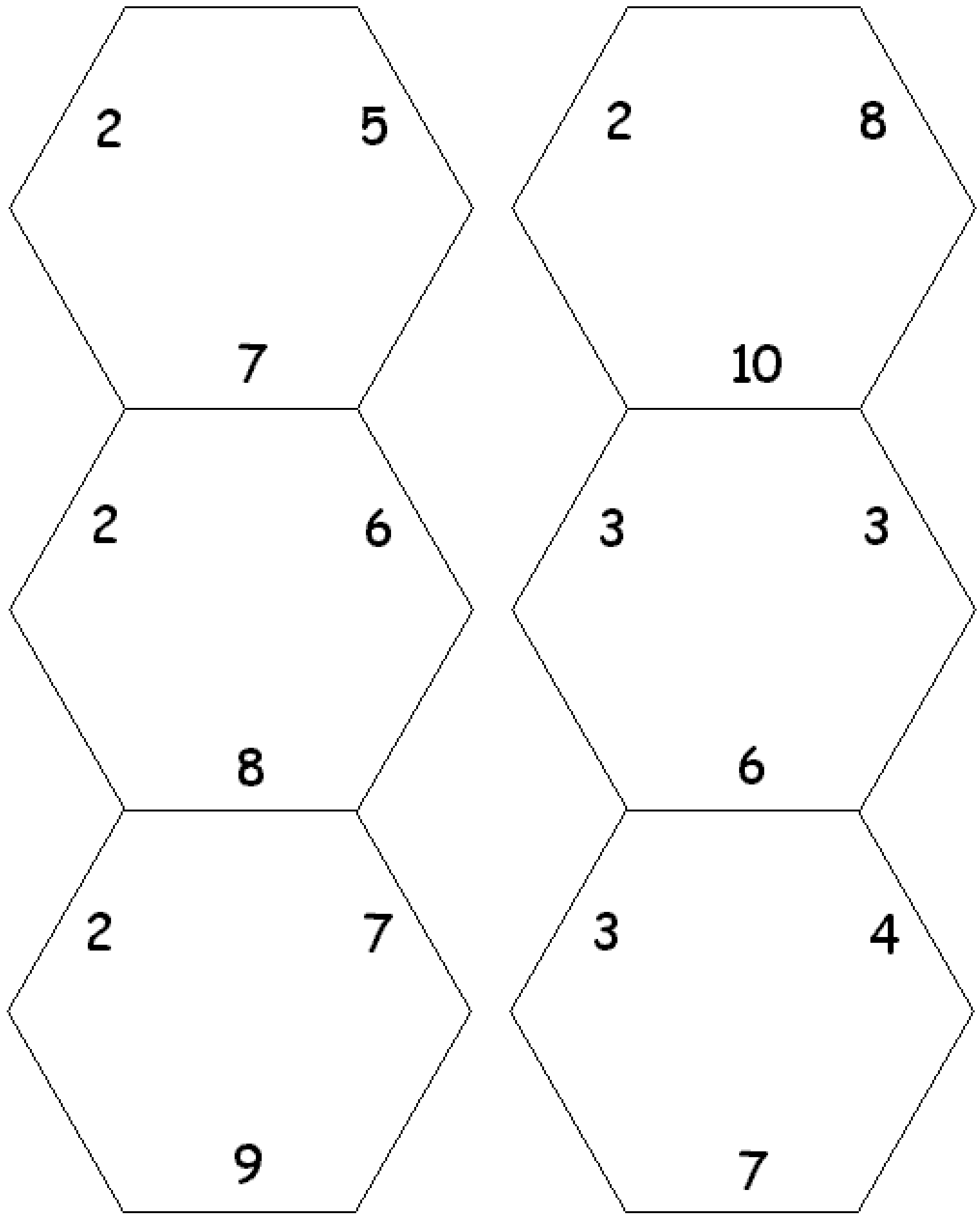
Addition 1



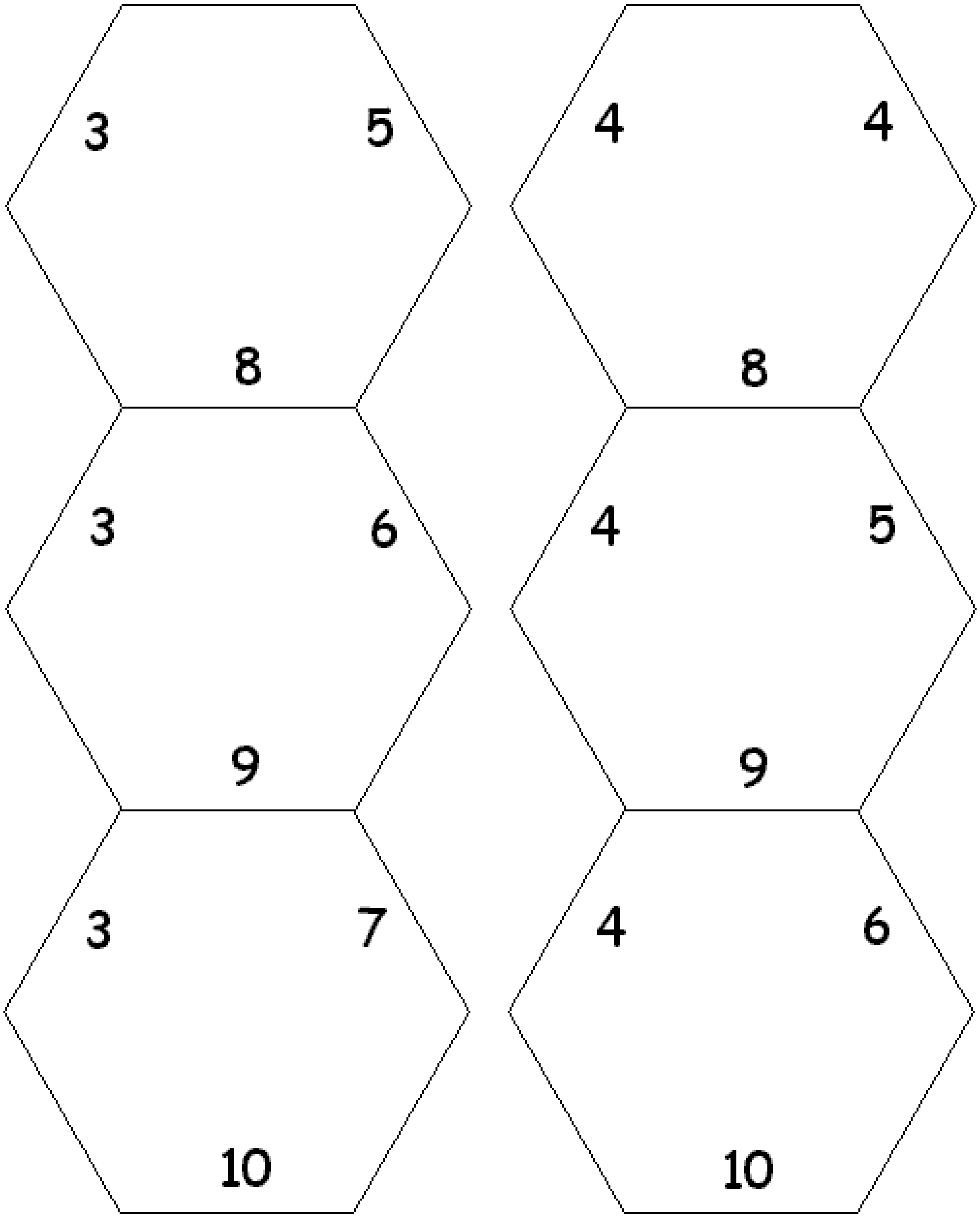
Addition—2



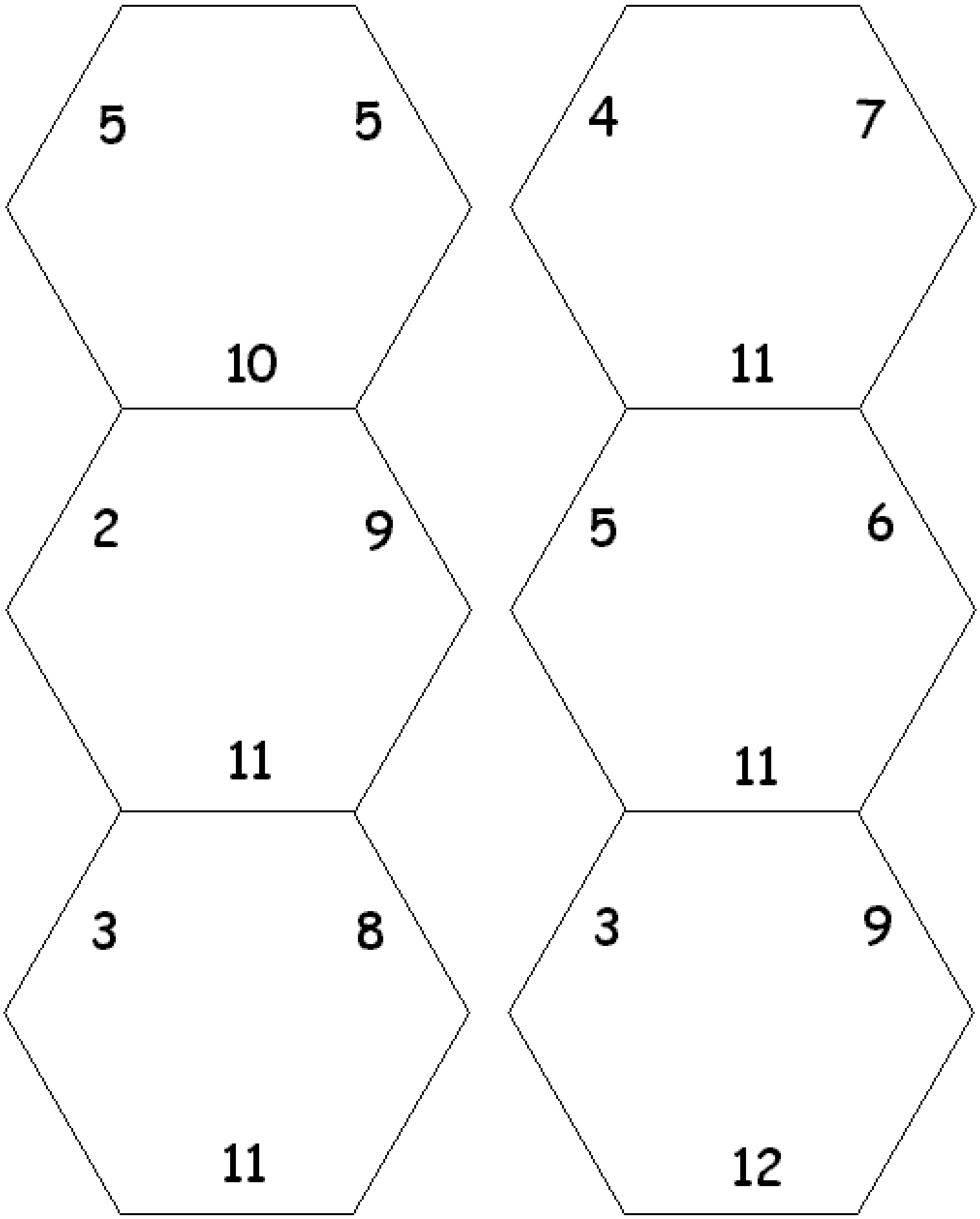
Addition—3



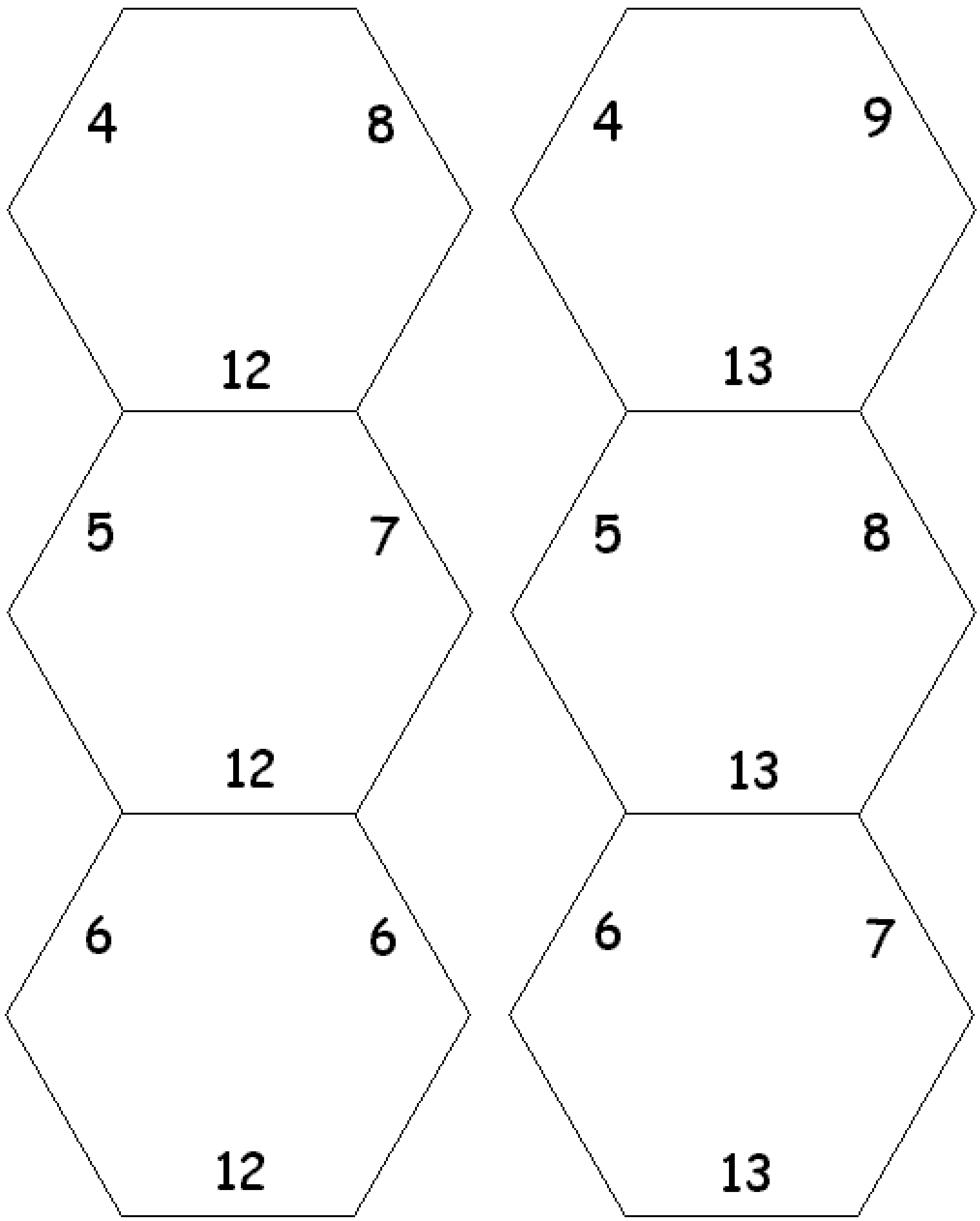
Addition—4



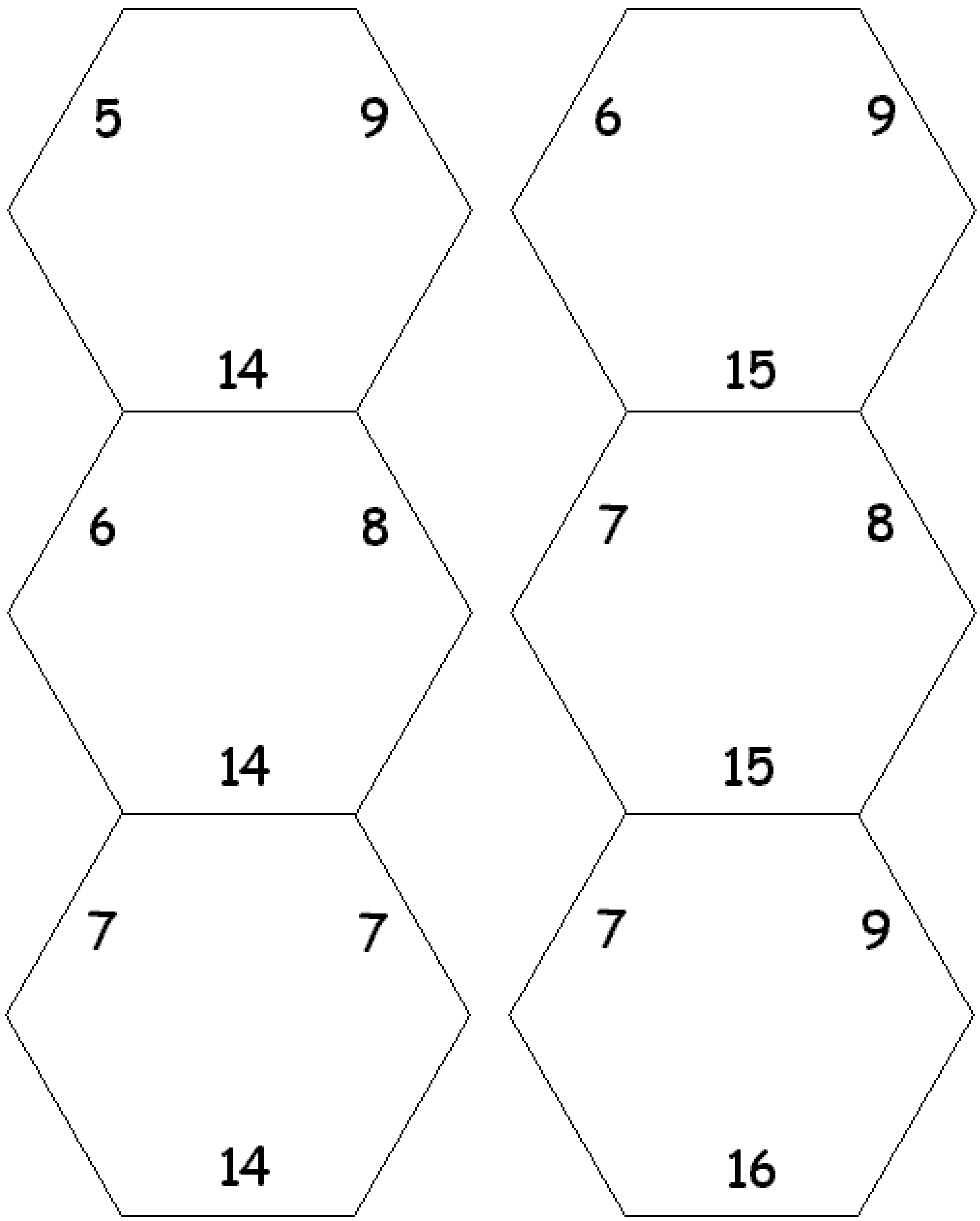
Addition—5



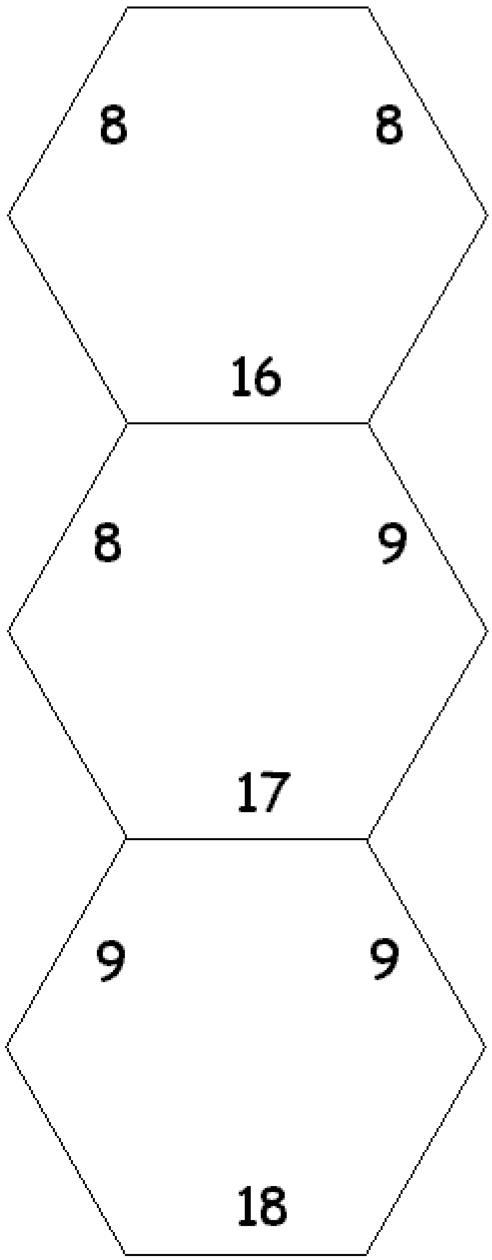
Addition—6



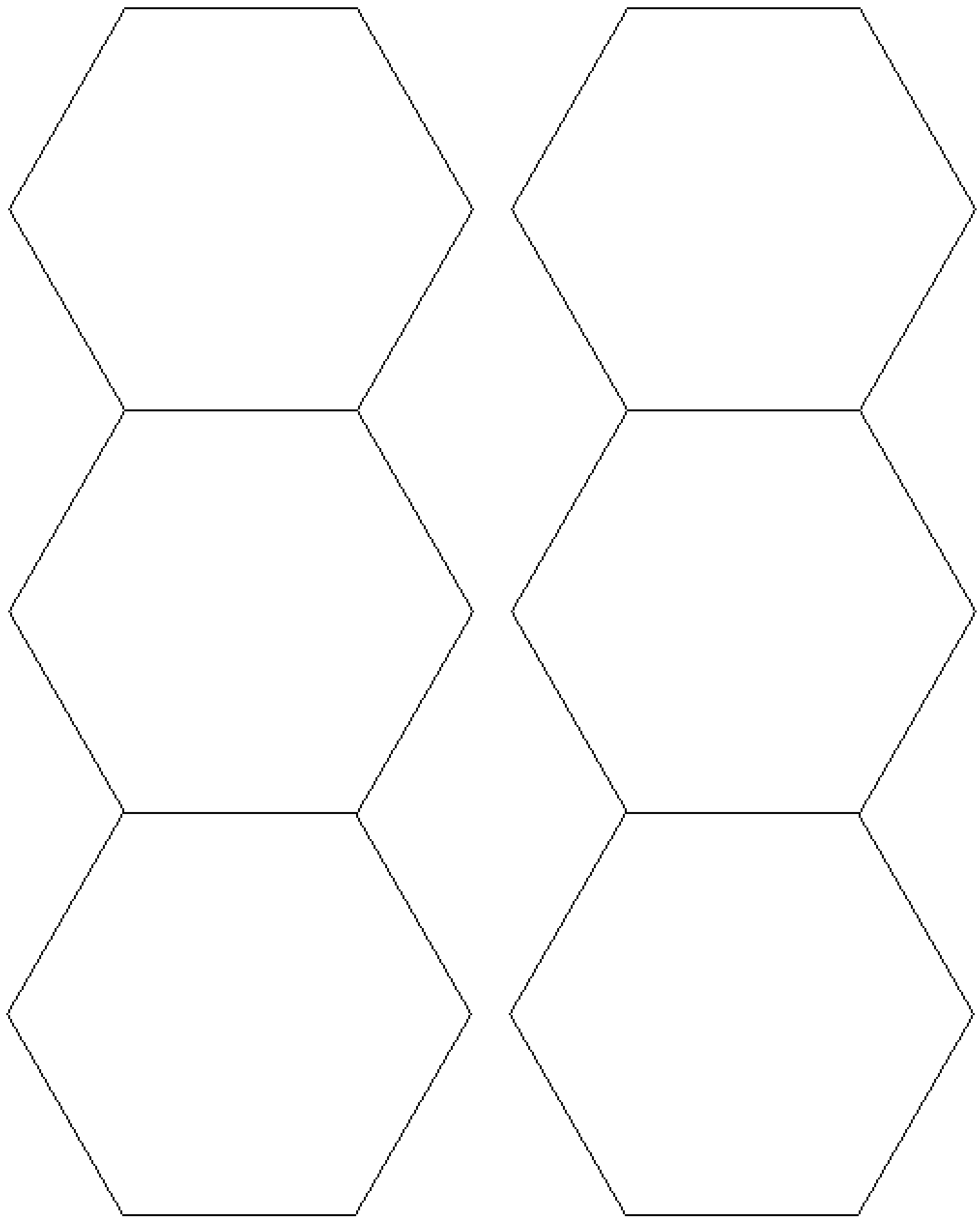
Addition—7



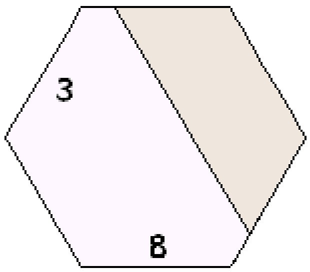
Addition—8



Blank Fact Family Cards

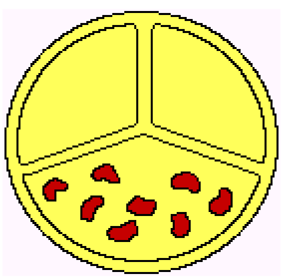


Math on a Plate

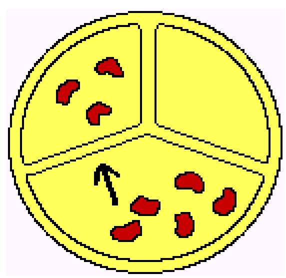


Place the fact family "house" to cover one of the smaller numbers on the cards. You will need:

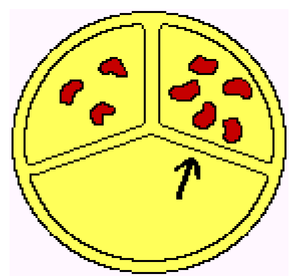
* a plastic or paper plate divided into 3 sections
* objects to count, such as:
  - dried beans
  - uncooked pasta
  - dry cereal
  - pebbles



1) Using the card above as an example, fill the large bottom section of the plate with 8 objects (to match the bottom number on the card).



2) Move 3 objects into one of the smaller sections.



3) Move the rest of the objects into the other small section. How many are there? That's the missing number hidden by the fact family "house."

Positive and Negative Integers: A Card Game

This card game provides practice adding and subtracting positive and negative integers.

Materials:

- standard deck(s) of cards

Procedure:

1. Arrange students into groups of two or more. Have students deal out as many cards as possible from a deck of cards, so that each student has an equal number of cards. Put aside any extra cards.

2. Explain to students that every black card in their pile represents a positive number. Every red card represents a negative number. In other words a black seven is worth +7 (seven), a red three is worth –3 (negative 3). *Note: If this game is new to students, you might want to discard the face cards prior to dealing.* If students are familiar with the game, or if you want to provide an extra challenge, leave the aces and face cards in the deck. In that case, explain to students that aces have a value of 1, jacks have a value of 11, queens have a value of 12, and kings have a value of 13.

3. At the start of the game, have each player place his or her cards in a stack, face down. Then ask the player to the right of the dealer to turn up one card and say the number on the card. For example, if the player turns up a black eight, he or she says "8".

4. Continue from one player to the next in a clockwise direction. The second player turns up a card, adds it to the first card, and says the sum of the two cards aloud. For example, if the card is a red 9, which has a value of -9, the player says "8 + (-9) = (-1)"

5. The next player takes the top card from his or her pile, adds it to the first two cards, and says the sum. For example, if the card is a black 2, which has a value of +2, the player says "(-1) + 2 = 1."

6. The game continues until someone shows a card that, when added to the stack, results in a sum of exactly 25.

Extra Challenging Version

To add another dimension to the game, you might have students always use subtraction. Doing this will reinforce the skill of subtracting negative integers.

For example, if player #1 plays a red 5 (-5) and player #2 plays a black 8 (+8), the sum is -13: (-5) - (+8) = -13

If the next player plays a red 4, the sum is -9: (-13) - (-4) = -9. [Recall: Minus a minus number is equivalent to adding that number.]

Adapting for Special Students
For students who find the game too challenging, you might change the sum you're aiming for to a number less than 25. The game will end more quickly. As students become more comfortable with the game, you can gradually increase the numeric goal.

Writing Extension
After the game ends, have the students write about it in their math journals. For example, you might have them explain the rules in their own words.

Adapted from: A Teacher Submitted Lesson Plan by Pam Harper, Rockville Jr/Sr High School, Rockville, Indiana on the Education World website.

Probability

If a person tosses a coin, only 1 out of 2 sides can show when it lands, so the probability for either side is 1 out of 2 or ½. However, if a die with numbers 1-6 is rolled, the probability of any number showing is 1 out of 6 or 1/6. If there are 3 socks in a drawer—2 red and 1 blue—the probability of blindly pulling out a red sock is twice as great as that for a blue sock. Red sock—2 out of 3 or 2/3 and blue sock—1 out of 3 or 1/3.

Activities:

1. Show a die and ask which number will be on top when you roll it. Have the student roll one die 20-30 times and record the result on a graph.

2. Show the student the following graph and two dice. Have the student roll the dice several times and after each roll, color in a square above the sum of the 2 numbers. After about 30 rolls, ask which sum was rolled the most times? (Probably 7) Which sums were rolled least? (Probably 2 and 12) Discuss why.

Activity # 11 (Page 2 of 2)

2	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**

Math Ideas

1. **Remodeling a home** – get samples and prices from local hardware stores and use newspaper ads for pricing. You will need samples of wallpaper, paint, carpet, etc. This is a great thing to use when teaching area and perimeter. You can use house plans found in magazines to make it more realistic.

2. **Studying proportion and scale** – Using the Illinois state maps provided, study the road construction going on in the state. A spin off of this activity would be to also figure distances from location to location using the scale. Use the Internet to research several types of cars to see which one has the best gas mileage. Figure gas cost for a trip with this vehicle. *(Additional maps can be obtained from the Secretary of State's office.)*

3. **Planning a party –** set up the budget, the cost, food to buy, menu, and recipes. Use newspaper ads to figure cost and the recipes are great for working fractions.

4. **How much TV does the student watch in a week?** This activity is great for working on averages and also for charting. A TV journal worksheet is included so that the students can keep a log. Type of show can be a sitcom, drama, game show, soap opera, news, history, PBS, etc. If you have the students keep track of how they were feeling and what they ate during the times they were watching TV, the information can be used for a nutrition lesson or give insights into mental health.

5. **How much do cigarettes cost in a week? A month? A year?** Use this math activity to also teach a life skills lesson. If the student does not smoke, use the cost of a vending machine soda or snack to calculate the cost for a week, a month, and/or a year? What could the students buy at the end of the year if they weren't purchasing soda, snacks, or cigarettes? Could they get a new TV or a car?

6. **Use the bus schedules.** Figure out the fastest way to get from point A to point B. If you had to use public transportation to get to work, which bus number would get you there in time? How much does it cost? *(Bus schedules are included.)*

7. **Finding Volume.** You can use Legos to find volume.
   Have students use Legos to build boxes (rectangular prisms) of given dimensions. Determine the volume of each box. Have students record the data and then look for patterns to develop the formula for finding the volume of a box.

Length (# of bumps)	Width (# of bumps)	Height	Volume
2	2	2	
4	2	4	
3	2	5	

TV LOG

Type of show—sitcom (i.e. *Friends*), game show, soap opera, educational (History channel), drama (i.e. *C.S.I.*), or the news.
Food group consumed—vegetables, fruit, fats (chips, ice cream, cookies, etc.), dairy, grains (bread, rice, etc.), and protein (meat, eggs, beans)
Mood or feeling—happy, sad, depressed, worried, tired, excited, or angry.

DATE	# OF MINUTES	TYPE OF SHOW	FOOD GROUP CONSUMED	MOOD OR FEELINGS
	TOTAL			

Developing Integer Rules
(Lessons for Signed Numbers)

This lesson presents an approach to introducing signed numbers in a way that is meaningful and enjoyable for adult learners.

Materials:

- The kit contains white and either red or blue chips. Use the white chips to represent positive numbers and the red or blue chips to represent negative numbers. You will need about 20 chips of each color for every student or pair of students.

Procedure:

1. Read the background information about signed numbers. It provides a meaningful context within which you can teach this concept.
2. Start the session with a discussion about positive and negative quantities and where they come into our lives. This will provide students with a context for learning to use signed numbers in mathematics. (Use the chart, *Categorizing Situations Using Signed Numbers,* as a springboard for your discussion.)
3. Have the students complete the *"Signed Numbers Worksheet".*
4. Discuss the results of the worksheet and any observations students have made about the patterns in their answers. Here are two important conclusions which students should have reached by this point.
    - Adding a negative number has the same result as subtracting a positive number.
    - Subtracting a negative number has the same result as adding a positive number.
      Beware of the old rule "two negatives made a positive" as students can get confused in its use.
      Example: -2 -3. Students are tempted to say "two negatives made a positive" and give an answer of +5.

Developing Integer Rules
(Lessons for Signed Numbers)

Background:
Negative numbers were first used by the Chinese around 200 B.C., and the first algebra textbook was written in AD 825 by Al-Khowarizmi of Baghdad. But the problem of negative numbers he left unsolved. After all, what could a negative number mean? Who ever held in their hand less than nothing?

Little was done about negative numbers until Fibonaccik, an Italian mathematician, had another look in the thirteenth century. At that time, mathematicians spent a lot of time making up and solving puzzles. He was tackling a financial problem and saw that it could not be solved unless the solution was a negative number. We don't know exactly why the problem was, but it was probably of the type:

A man had a certain amount of money. After being given 20 ducats he paid a bill of 10 ducats and then had nothing. How much did he start with?

This problem can't be solved unless it is conceded that the first man had a debt.

Despite Fibonacci's tentative recognition that an equation might have a negative solution, most mathematicians viewed negative numbers with disbelief. The rise of a banking system came into being in the towns of Northern Italy (Florence and Venice) during the fourteenth century and made possible the seemingly absurd subtraction of, say, 7 from 5. The new bankers simply allowed their clients to draw seven gold ducats while their deposit stood at five. All they had to do was write the difference, 2, on another side of the ledger—the debit side.

As our scientific knowledge developed, so did the acceptability of negative numbers, which were increasingly incorporated into the scientific models, formulas and equations we use to describe and understand our world. For instance, negative number concepts are essential to our models of atomic structure, electricity, chemical reactions and the equations describing the laws of motion.

Giving Negative Numbers Meaning
In introducing signed numbers to students for the first time, it helps to discuss how positive and negative numbers can be used to describe particular situations. The chart *(Categorizing Situations Using Signed Numbers*) illustrates how a range

of situations can be categorized into three states (positive, negative, or zero). Use some of these ideas to stimulate discussion in your classroom about positive and negative numbers. For instance, you may suggest a situation and ask your students to describe the corresponding positive, zero and negative states. Also, ask students for examples from their own experiences.

These analogies, with their division of numbers into positives or negatives explain the *position* of a number very well, and even *direction of movement*, but not *operations* with signed numbers. How can we handle this?

Using manipulatives

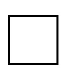
Two colored chips are a physical model which can be used for addition, subtraction and multiplication of signed numbers in a consistent and logical manner. They work for all operations, and can be used successfully for teaching linear algebra, expansion and factorization of quadratics, and even for solving quadratic equations.

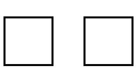
How to introduce the chips

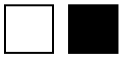
It is important to start by using the chip in an exploratory way, guiding your students to recognize white as positive and red/blue as negative. The basic idea used here is related to positive and negative charges of electricity. From this follows the important idea of positive and negative neutralizing each other. Thus zero, besides being an absence of chips is also, and more importantly, an equal quantity of white and red/blue chips.

It may be of help to your students at first to introduce an analogy from chemistry, with white representing sodium (Na) ions and red/blue representing chlorine (Cl) ions. Pairs of red/blue and white become sodium chloride—NaCl-- (salt) molecules, which are now chemically neutral.

Have the students place the following chips on their desk or draw them on a overhead transparency. You can also use different colored transparency blocks for demonstration. For example:

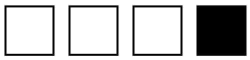
 One red/blue chips represents -1  One white chip represents + 1 or 1

 Two white blocks represent + 2  Three red/blue chips represent -3

 One white and one red/blue chip represent zero or neutral

**note: the positive sign will be omitted when referring to a positive number

Have the students place the following chips on their desks.



What number does it represent? Answer *(+2)*

Have the students find and display other ways of representing +2 using the chips.

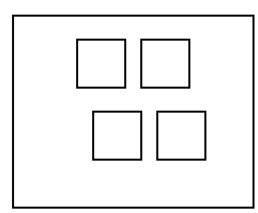
Emphasize that variations in the number of zeros has **no effect on the result**. You can therefore put in or take out zeros without changing anything.

Have students place the following chips on their desk.

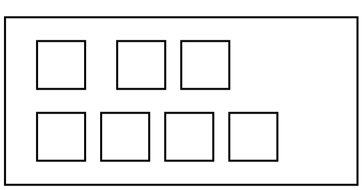
What number does this represent? Answer (-3)
Now introduce the idea of a stage. It can be a sheet of paper, or a circle drawn on the table with chalk or on a piece of paper with crayon. Each problem will be "acted out" on the stage.

Addition:

Demonstrate the following examples. Explain addition as joining together or adding actors to the stage.

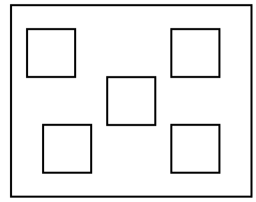


1. Start with an empty stage.
2. Have the students put four white chips on the stage.
3. Now add three white chips on the stage.

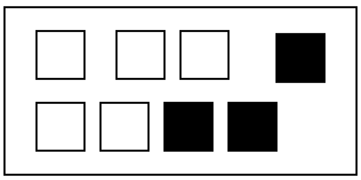


The result is seven white chips so the answer is +7. A possible story problem to go along with this example is: I had $4 and someone gave me $3 more. Now I have $7.
The equation would be 5 + 2 = 7.

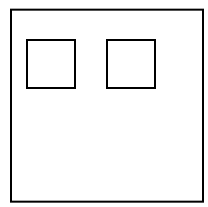
Try another one adding a negative number.



1. Start with an empty stage.
2. Have the students put five white chips on the stage.
3. Now add three red/blue chips on the stage.

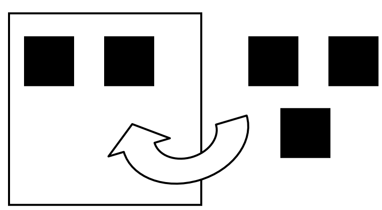


4. Pair up the white and red/blue chips together. Remind them that when one positive and one negative meet they cancel each other out.
5. Remove the three zero pairs.
6. What is remaining? In this case two positive (white) chips remain. That means that 5 + -3 = +2



Try adding two negative numbers.

1. Start with an empty stage.
2. Put 2 red/blue chips on the stage.
3. Put in 3 more red/blue chips.
4. The result is 5 red/blue chips.
   That means -2 + -3 = -5

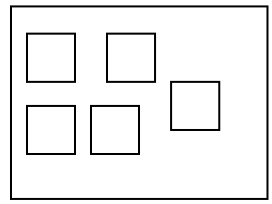


Thus you can now develop addition rules for working with signed numbers.

a. If the signs of the numbers are the same, add the numbers and use the sign that they both have.
b. If the signs of the numbers are different, subtract the smaller number from the larger number and use the sign of the larger number.

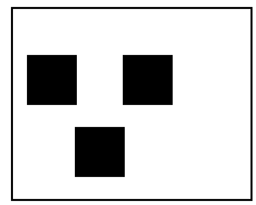
Subtraction

Subtraction means to take away or remove actors from the stage.

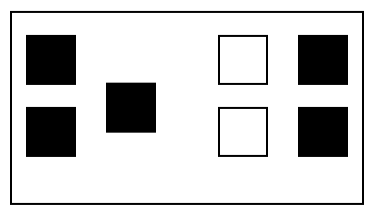


1. Start with an empty stage.
2. Put in 5 white chips.
3. Remove 3 white chips.
4. The result is 2 white chips. The equation would be 5 – 3 = 2

Demonstrate subtraction with two negative numbers.



1. Start with an empty stage.
2. Put in 3 red/blue chips.
3. Ask the students to take away 5 red/blue chips. Their first comment will be that it is impossible. The next question is "What can be added on to a number without changing its value?" You can add zero which can be represented by a positive (white) and negative (red/blue) chip together.
4. Have the students add enough zeros to the stage until there are enough negatives to take away 5 negative chips.
5. Take away the 5 negative (red/blue) chips.
6. What is remaining? In this case two positive chips remain. This means that -3 - -5 = +2.
   Each time a subtraction problem is done write the related addition problem with it.
   For example:
   -2 - -3 = +1 is the same as -2 + +3 = +1



From comparing the subtraction problems with the addition problems, the rule can be developed that each subtraction problem can be changed to an addition by changing the subtraction to addition and changing the sign of the number being subtracted. In this way students do not need to learn new rules for subtraction. They just change each subtraction to addition and then use the addition rules.

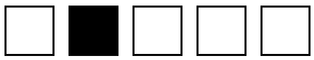
Complete the Signed Numbers Worksheet.

Some information obtained from: *Mathematics A New Beginning* by Beth Marr and Sue Helme

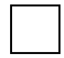
CATEGORIZING SITUATIONS USING SIGNED NUMBERS

Situation	Negative (-)	Neutral (zero)	Positive (+)
Temperature	Below freezing	Freezing point	Above freezing
Quantity of goods	Shortage	Supply equals demand	Surplus
Feelings	Feeling bad	Feeling OK	Feeling good
Height & depth	Below sea-level	At sea level	Above sea level
Checking account	Account in debt	Zero balance	Account in credit
Attitudes	Dislikes	Neutral	Likes
Electrical charges of parts of the atom	Electrons	Neutrons	Protons
Statistics	Less than the average	Average	More than the average
Movement on a staircase	Going downstairs from landing	At landing	Going upstairs from landing
Travel	Going south Going west	At starting point	Going north Going south
Prices	Deflation (going down)	Stable	Inflation (going up)
America's trade performance	Imports exceed exports (deficit)	Imports balance exports	Exports exceed imports (surplus)

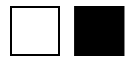
Signed Numbers Worksheet

1. Put the following onto the stage and simplify:
   a. 
   b. 
   c. 

Remember:

 = positive

 = negative

 = zero

2. Use the blocks to put these numbers onto the stage.
   What result do you get?
   a. 3, -4
   b. 5, -2
   c. 3, -2, 4
   d. -3, 3
   e. 4, -1, 2
   f. -2, 5, 6, -1
   g. -5, 0, 2, 7, -3

3. Use the stage to find the result for the following additions:
   a. 2 + -1
   b. -3 + 5
   c. 2 + -6
   d. 6 + -5
   e. -5 + -3
   f. -7 + -2

4. Use the stage to find the result of the following subtractions:
   a. 3 – 2
   b. 3 – 7
   c. -5 - -3
   d. -3 - 5
   e. 8 - 6
   f. -5 - 5
   g. -3 - -4
   h. 5 - -1

Answer Key:

1. (a) 1	(b) 3	(c) 2				
2. (a) -1	(b) 3	(c) 5	(d) 0	(e) 5	(f) 8	(g) 1
3. (a) 1	(b) 2	(c) -4	(d) 1	(e) -8	(f) -9	
4. (a) 1	(b) -4	(c) -2	(d) -8	(e) 2	(f) -10	(g) +1
(h) 6						