Math 150 **SAMPLE FINAL** **Spring 2066**

Your Name

Your Signature

Student ID

- Give your answers in exact form. Do not give decimal approximations.
- Calculators are not allowed.
- In order to receive credit, you must show your work. Do not do computations in your head. Instead, write them out on the exam paper.
- **Place a box around YOUR FINAL ANSWER to each question.**
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.

Problem	Total Points	Score
1	5	
2	5	
3	12	
4	8	
5	6	
6	9	
7	12	
8	18	
9	12	
10	8	
11	5	
Total	100	

1. [**5 points total**] Mark each statement below as true or false by circling **T** or **F**.

1. **T** **F** If a function f is continuous at the point a then it is differentiable there.

2. **T** **F** Since the function $f(x) = \frac{(x+3)(x+5)}{(x-2)(x-6)}$ is equal to $\frac{15}{12}$ when $x = 0$, and is equal to -16 when $x = 3$, the Intermediate Value Theorem can be used to conclude that $f(a) = 0$ for some a between 0 and 3.

3. **T** **F** If c is a critical number of a function f and also $f''(c) = 0$, then by the Second Derivative Test, it follows that f achieves neither a local maximum nor a local minimum at $x = c$.

4. **T** **F** If $f(x)$ and $g(x)$ are continuous functions which are defined for all real numbers, then

$$\int_{-2015}^{2015} (x^4 + x^6 \sin x + 12)dx = \int_{-2015}^{2015} (x^4 + 12)dx$$

5. **T** **F** The absolute minimum value of a continuous function $f(x)$ defined on a closed interval $[a, b]$ can only be realized at an endpoint ($x = a$ or $x = b$) or at a point where the graph of f has a horizontal tangent.

2. [**5 points total**] Circle the correct answer.

1. If $f''(x) = 3^{-x}(x-5)(x-14)^{2014}$, then $f(x)$ has inflection point(s) at

A. $x = 5$ and $x = 14$ B. $x = 5$ only. C. $x = 14$ only. D. $x = 0$ only.

2. The absolute minimum value of $f(x) = 1 - x^2$ on $[-1, 2]$ is

A. -3. B. 0. C. 2. D. 3/2.

3. The minimum value of the slope of the tangent line to $h(x) = 2x^3 - 3x^2 - 12x + 5$ occurs at

A. $x = 2$. B. $x = -1$. C. $x = 1/2$. D. There is no such value for the slope

4. Consider the function $h(x) = \ln(g(x))^3$ and assume that $g(2) = 5$ and $g'(2) = -3$. The $h'(2)$ equals

A. $\frac{3}{5}$.

B. $-\frac{3}{5}$.

C. $-\frac{9}{5}$.

D. $\frac{5}{9}$.

5. Suppose f is a function such that $f'(3) = 0$, and $f''(3) < 0$. What can be said about the function?

A. The function has local maximum value at $x = 3$.

B. The function has local minimum value at $x = 3$.

C. The function has neither a local maximum nor local minimum value at $x = 3$.

D. You need more information to determine whether f has a local maximum or minimum at $x = 3$.

3. [**12 points total**] Consider the function $f(x) = xe^{-x}$.

(a) (4 pts) Find the intervals on which f increases and the intervals on which f decreases.

(b) (4 pts) Find the x-coordinates of any local maxima or minima.

(c) (4 pts) Find the intervals on which f is concave up and the intervals on which f is concave down.

4. [**10 points total**] Suppose $f(x) = \frac{1}{3x}$. Using the definition of the derivative, find $f'(2)$. (You will receive NO credit for finding the derivative using a different method.)

5. [**6 points total**] Sketch a well-labeled graph of a continuous function, g, which satisfies all of the following properties.

- $g(0) = -1$.
- $g(1) = 0$.
- $g'(x) = 2$ for $1 < x < 2$.
- $g'(x) = -2$ for $2 < x < 3$.
- g is decreasing for $x > 3$.
- $g''(x) < 0$ for $x > 3$
- g is concave down for $x < 1$.

6. [**9 points total**] Evaluate the following limits. Show work!

(a) $\displaystyle\lim_{x\to 1^-} \frac{x^2-1}{|x-1|}$

(b) $\displaystyle\lim_{x\to\infty} x^2 e^{-x}$

(c) $\displaystyle\lim_{x\to 0} \frac{(1-\cos x)^2}{x\sin x}$

7. [**12 points total**] Find $f'(x)$ (you should simplify and write your final answers without negative exponents) if

(a) $f(x) = x^3 e^{-3x}$

(b) $f(x) = \dfrac{e^{sec^2 x}}{\ln x}$

(c) $f(x) = \int_1^{\sqrt{\cos x}} (t^{2016} + 2015)^{2016} dx$

(d) $f(x) = (\cos x)^x$

8. [**18 points total**] Evaluate the following integrals

(a) $\displaystyle\int x^3\sqrt{x^2-1}\,dx$

(b) $\displaystyle\int \frac{\sin x}{\cos^2 x}\,dx$

(c) $\displaystyle\int_2^3 \frac{3x^2+2x+1}{x}\,dx$

(d) $\displaystyle\int \frac{\sin x}{1+\cos^2 x}\,dx$

(e) $\displaystyle\int_{-1}^{1} |x|\,dx$

(f) $\displaystyle\int \frac{(\cos(\tan x))}{\cos^2 x}\,dx$

9. [**12 points total**] A landscape architect plans to enclose a 4000-square-foot rectangular region in a botanical garden. She will use shrubs costing \$20 per foot along three sides and fencing costing \$5 per foot along the fourth side. What is the minimum total cost?

10. [**8 points total**] The equation $x^2 - xy + y^2 = 1$ describes an ellipse. Find the coordinates (x, y) of all points on the curve where the tangent to the curve is horizontal.

11. [**5 points total**] The acceleration, in m/sec^2, of a particle moving along a line is given as a function of time t (in sec) by the formula

$$a(t) = 2e^t + 3\sin t - t.$$

The initial velocity (at time $t = 0$) is 2 m/sec. What is the particle's velocity at time $t = 4$?