Math 125 Review Packet for the Common Final

Remarks:
- The final will not be identical to the questions in this packet.
- The final will be in the same format as this packet, but not as long.
- The use of any calculator, smart phone or other transmitting devices such as a smart watch is prohibited for the final.

1. Circle the correct answer: **T** (True) or **F** (False)

   (a) **T or F:** $\dfrac{x}{x^2+3} + \dfrac{7}{6x-1}$ is simplified to $\dfrac{x+7}{x^2+6x+2}$.

   (b) **T or F:** $\dfrac{x}{x^2+3}$ is simplified to $\dfrac{1}{x+3}$.

   (c) **T or F:** If $x(x+1) > 0$, then x and $x+1$ are either both positive or both negative.

   (d) **T or F:** $f^{-1}(f(x)) = x$ for every x in the domain of f.

   (e) **T or F:** $\pi = 180$

   (f) **T or F:** The angle $\dfrac{2\pi}{3}$ is coterminal the angle $\dfrac{\pi}{3}$.

   (g) **T or F:** $\sin^{-1} x = \csc x$

   (h) **T or F:** $\sin t \cos t > 0$ in the second quadrant.

   (i) **T or F:** $\cos \dfrac{9\pi}{10} > 0$

   (j) **T or F:** $\tan(-\theta) = -\tan\theta$

   (k) **T or F:** $\sin(\theta + 2\pi) = \sin\theta$

   (l) **T or F:** The period of $f(x) = \tan x$ is 2π

   (m) **T or F:** $\sin^{-1}\left(\sin\dfrac{5\pi}{6}\right) = \dfrac{\pi}{6}$

   (n) **T or F:** The amplitude of the graph of $y = -3\sin\left(\dfrac{x}{2}\right)$ is -3.

   (o) **T or F:** $\sec\theta = \csc\left(\dfrac{\pi}{2} - \theta\right)$

   (p) **T or F:** In triangle ABC, $\dfrac{\sin A}{a} = \dfrac{\cos B}{b} = \dfrac{\tan C}{c}$

   (q) **T or F:** In triangle ABC, $a^2 = b^2 + c^2 - 2bc\cos A$

   (r) **T or F:** The domain of the function $f(x) = e^x$ is all real numbers.

   (s) **T or F:** The graph of the function $f(x) = \left(\dfrac{1}{3}\right)^x$ goes through the point $(1, 0)$.

   (t) **T or F:** $\log(A + B) = \log A + \log B$, with $A > 0$ and $B > 0$

2. Fill in the blank.

(a) The graph of f^{-1} is obtained by reflecting the graph of f in the line _____

(b) The radian measure of the angle $-330°$ is _____.

(c) The degree measure of the angle $\dfrac{11\pi}{3}$ is _____.

(d) Find the radius of the circle if an arc of length 6 meters on the circle subtends a central angle of $135°$: _____

(e) For the graph of the function $f(x) = 5\sin(3x + \pi)$, the amplitude is _____, the period is _____ and the phase shift is _____.

(f) For the question (g), use the figure below to state the trigonometric ratios.

$\sin\theta =$ _____ $\cos\theta =$ _____

$\tan\theta =$ _____ $\csc\theta =$ _____

$\sec\theta =$ _____ $\cot\theta =$ _____
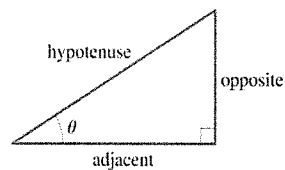
(g) Find the exact value: $\sin\dfrac{2\pi}{3} =$ _____

(h) Find the exact value: $\cos\dfrac{2\pi}{3} =$ _____

(i) Find the exact value: $\tan\dfrac{5\pi}{4} =$ _____

(j) $y = \cos^{-1} x \Leftrightarrow x = \cos y$ for _____ $\le x \le$ _____ and _____ $\le y \le$ _____.

(k) Find the exact value: $\sin^{-1}\left(-\dfrac{\sqrt{2}}{2}\right) =$ _____

(l) Find the exact value: $\cos^{-1}\left(-\dfrac{\sqrt{2}}{2}\right) =$ _____

(m) Find the exact value: $\cos^{-1}\left(\cos\dfrac{5\pi}{6}\right) =$ _____

(n) Find the exact value: $\tan^{-1}\left(\tan\dfrac{5\pi}{6}\right) =$ _____

(o) Find the exact value: $\cos\left(\sin^{-1}\dfrac{3}{5}\right) =$ _____

(p) State the reciprocal identity: $\cot x =$ _____

(q) State the quotient identity: $\cot x =$ _____

(r) State the Pythagorean identity: $\sec^2 x =$ _____

(s) State the addition formula: $\sin(x + y) =$ _____

(t) State the subtraction formula: $\cos(x - y) =$ _____

(u) State the three variations of the double-angle identity for cosine:

$\cos 2x =$ _____

(v) State the half-angle formula: $\sin\dfrac{x}{2} =$ _____

(w) Find all solutions of the equation $\tan x = \sqrt{3}$: $x =$ _____

(x) The graph of the function $f(x) = e^x - 2$ has a horizontal asymptote and it is _____ .

(y) Express the equation $\log_3 8 = x$ in exponential form: _____

(z) The inverse of $f(x) = e^x$ is $f^{-1}(x) =$ _____ .

(aa) $\log_a a^x =$ _____ for $a > 0$, $a \ne 1$ and $x \in \Re$

(bb) $a^{\log_a x} =$ _____ for $a > 0$, $a \ne 1$ and $x > 0$

(cc) Find the exact value: $\ln 1 = $ _____

(dd)   Find the exact value: $\log 10 = $ _____

(ee) Find the exact value: $\log_2 160 - \log_2 5 = $ _____

(ff) Find the exact value: $\ln e^7 = $ _____

(gg)   Expand: $\log\left(\dfrac{x^2 y}{\sqrt{x+1}}\right) = $ _____

(hh)   Condense: $\ln(x+y) + \ln(x-y) - 3\ln x = $ _____

3. Perform the indicated operations and simplify.

(a) $\dfrac{1}{x-1} + \dfrac{x}{(x-1)^2}$ (b) $\dfrac{x^2 + 2x - 3}{x^2 + 8x + 16} \cdot \dfrac{x^2 + 4x}{x^2 - 1}$

4. Solve the equation.

(a) $2x^3 - 3x^2 - 8x + 12 = 0$ (b) $|2x + 3| = 9$ (c) $\sin x - 2\sin^2 x = 0$

(d) $\sin x = \cos 2x$ (e) $2^{3x-5} = 7$ (f) $\log_2(1 - x) = 4$

(g) $3^{2x} - 3^x - 6 = 0$ (h) $\log x + \log(x + 1) = \log 12$
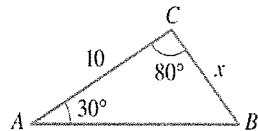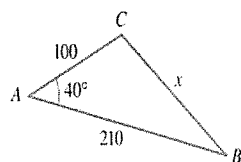
5. Solve the inequality.

(a) $|2x - 5| \le 3$ (b) $|3x - 2| > 5$

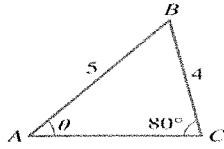6. Find the side labeled x or the angle labeled θ.
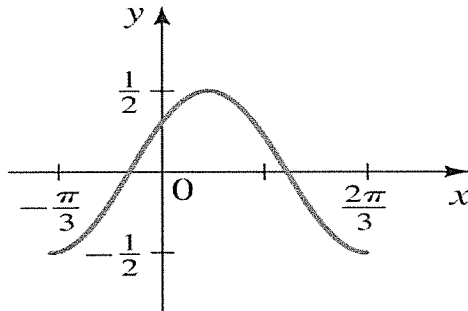
(a) $x = $ _____

(b) $x = $ _____

(c) $\theta =$ _____

7. Find the amplitude, period and phase shift of the function and graph one complete period.

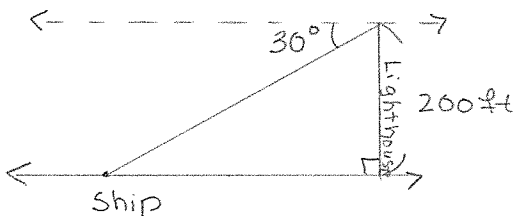$$y = 2\sin\left(x - \frac{\pi}{2}\right)$$

8. The graph of one complete period of a sine or cosine curve is given.

(a) Find the amplitude, period and horizontal shift.

(b) Write an equation that represents the curve in the form $y = a\sin k(x - b)$ or $y = a\cos k(x - b)$.

9. From the top of a 200-ft lighthouse, the angle of depression to a ship in the ocean is $30°$. How far is the ship from the base of the lighthouse?

10. Because of prevailing winds, a tree grew so that it was leaning $5°$ from the vertical. At a point 40 meters from the tree, the angle of elevation to the top of the tree is $35°$. Find the height h of the tree.
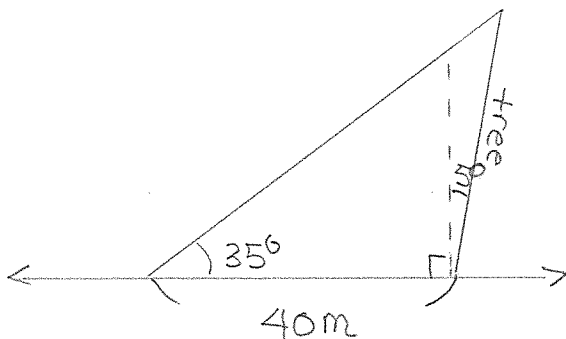
11. Two tugboats that are 120 feet apart pull a barge, as shown. If the length of one cable is 210 feet and the length of the other is 230 feet, find the angle formed by the two cables.

tugboat

Barge

210 ft

120 ft

230 ft

tugboat
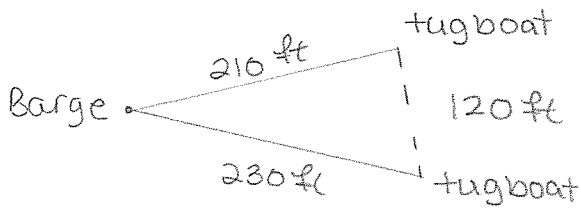
12. Find $\sin 2x$ and $\cos 2x$ from the given information: $\tan x = -\dfrac{1}{3}$, x in Quadrant II

13. Verify the identity.

(a) $\cos^2 x \csc x - \csc x = -\sin x$

(b) $\dfrac{1}{1 - \sin^2 x} = 1 + \tan^2 x$

14. Sketch the graph of (a) $f(x) = 2^x$ and (b) $g(x) = 2^{-x}$.

15. Match the logarithmic function with its graph.

(i) $f(x) = \log_3(x - 1)$ (ii) $f(x) = -\log_3(x + 2)$ (iii) $f(x) = \log_3(1 - x)$

(iv) $f(x) = -\log_3(-x)$