Math 120, Sample Final — Fall 2015

Disclaimer: This sample final is intended to help students prepare for the final exam. The final exam will be similar in structure and type of problems, however the actual final will be much shorter than the sample final. There may be topics seen here that do not appear on the final and topics on the final may not appear here. Students should also review all their exams and quizzes.

Fill-in-the-Blank. Simplify all solutions.

1. The equation of the horizontal line passing through $(0,-1)$ is _____.

2. A line perpendicular to $y=\frac{1}{2}x-\frac{4}{5}$ will have a slope of _____.

3. If f^{-1} denotes the inverse of a function *f*, then the graph f^{-1} is a reflection of the graph of f about the _____.

4. The product of the complex numbers $(4-3i)$ and $(8+2i)$ is _____.

5. Simplify. $\sqrt{-80}=$_____.

6. The domain of the function $g(x)=\sqrt{x-7}$ is _____.

7. To graph $y=(x+2)^2-3$, start with the quadratic function $y=x^2$ and shift _____ by ____ unit(s) and _____ by ____ unit(s).

8. For $f(x)=-x^2+1$ and $g(x)=2x-1$, $(g\circ f)(x)=$_____.

9. For $f(x)=-x^2+1$ and $g(x)=2x-1$, $(f-g)(-3)=$_____.

10. If $(3+5i)$ is a complex zero of a polynomial function with real coefficients, then so is _____.

11. Using Descartes Rule of Signs, the function $g(x)=6x^4+x^3-21x^2-x+15$ can have _____ possible positive real zeros.

12. The function $g(x)=6x^4+x^3-21x^2-x+15$ can have at most _____ turns.

True or False. Circle the correct response.

1. T or F : The lines $y=\frac{3}{2}x+5$ and $2x+5y=-25$ have the same *y*-intercept.
2. T or F : To graph $g(x)=-\sqrt{x}$, reflect the function $f(x)=\sqrt{x}$ over the *x*-axis.
3. T or F : The absolute value function, $y=|x|$, is a one-to-one function.
4. T or F : The order of the matrix $A=\begin{bmatrix}8 & -2 & 5\\ 1 & -9 & 3\end{bmatrix}$ is 3×2.
5. T or F : The rational function $f(x)=\frac{3x^2}{2x^3+1}$ has the horizontal asymptote $y=\frac{3}{2}$.
6. T or F : The absolute value equation $|x-3|=-4$ has two solutions.

7. T or F : The circle $(x+3)^2+(y-4)^2=7$ has a radius of 7.

8. T or F : For $f(x)=\begin{cases}6x-1 & if \quad x<0\\ 7x+3 & if \quad x\geq 0\end{cases}$, $f(0)=-1$ and 3.

9. T or F : $\|-5.2\|=-6$.

10. T or F : The solution(s) of a system of two equations is the point(s) of intersection of their graphs.

11. T or F : To complete the square for x^2-6x, you must add 12.

12. T or F : A matrix consisting entirely of zeros is called an identity matrix.

Multiple Choice. Select the best solution.

1. _____ The polynomial function $g(x)=6x^4+x^3-21x^2-x+15$ has end behavior that:
   (a) falls to the left (QIII) and rises to the right (QI).
   (b) falls to the left (QIII) and falls to the right (QIV).
   (c) rises to the left (QII) and rises to the right (QI).
   (d) rises to the left (QII) and falls to the right (QIV).

2. _____ Select the statement that is <u>not</u> true for the square root function, $f(x)=\sqrt{x}$.
   (a) The domain of this function is $x\geq 0$.
   (b) The range of this function is $y\geq 0$.
   (c) This function is a one-to-one function.
   (d) This function is an even function.

3. _____ Select the statement that is true for the absolute value function, $f(x)=|x|$.
   (a) The domain of this function is $x\geq 0$.
   (b) The range of this function is $y\geq 0$.
   (c) This function is an increasing function on the interval $(-\infty,\infty)$.
   (d) This function is an odd function.

4. _____ The equation, $2(x-8)=15-(1-2x)$, is
   (a) a conditional equation.
   (b) an identity.
   (c) a contradiction.
   (d) None of the above

5. _____ The line, $2x-5y+20=0$,
   (a) has an *x*-intercept at $(-10,0)$.
   (b) has an *y*-intercept at $(0,-4)$.
   (c) has a slope $m=2$.
   (d) None of the above

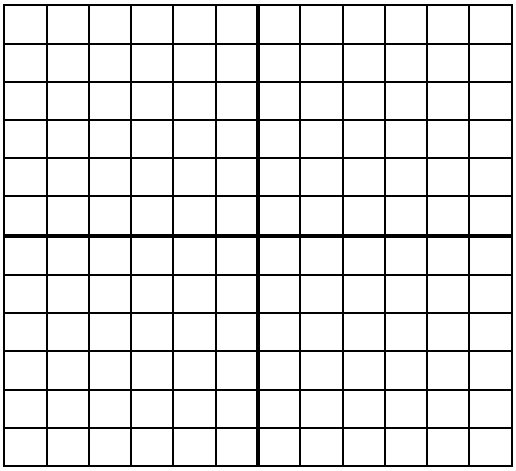
6. _____ The graph of the equation $x^2-y^3=64$,
   (a) has *x*-axis symmetry.
   (b) has *y*-axis symmetry.
   (c) has origin symmetry.
   (d) None of the above

7. _____ When the discriminant of a quadratic equation is negative, this means it has:
   (a) two distinct real solutions.
   (b) one real solution.
   (c) two complex (conjugate) solutions.
   (d) None of the above

Exercises. For the following problems show all steps for full credit. Simplify all solutions.

1. Determine the center and radius of the circle. Sketch the circle and **label** four points on its graph.

$(x+1)^2+(y-2)^2=9$



Center: (h,k) = (_____,_____)

Radius: r = _____

2. Solve the rational equation. $\dfrac{7}{2x+1}-\dfrac{8x}{2x-1}=-4$

3. The price of a digital camera is discounted by 25%. If the sale price is $225, what was the original price of the camera? (You must show an equation that models the problem.)

4. The sum of two consecutive numbers is 525. Find the numbers. (You must show an equation that models the problem.)

5. Solve the quadratic equation by the method of your choice. $2x^2-4x+1=0$

6. Divide the complex numbers. Write the quotient in standard form. $\dfrac{6-7i}{1+2i}$

7. Solve the polynomial equation by factoring. $x^3+2x^2+3x=-6$

8. Solve the radical equation. $\sqrt{2x+7}-x=2$

9. Solve the absolute value equation. $|3x+2|=5$

10. Solve the linear inequality. Express the solution as an inequality. $-2<\dfrac{8-3x}{5}\le 4$

11. Solve the absolute value inequality. Express the solution in interval notation. $|4-9x|>1$

12. Solve the absolute value inequality. Express the solution in interval notation. $|6x-2| \le 3$

13. Solve the polynomial inequality. Graph the solution on a number line. $2x^2 + x \ge 21$

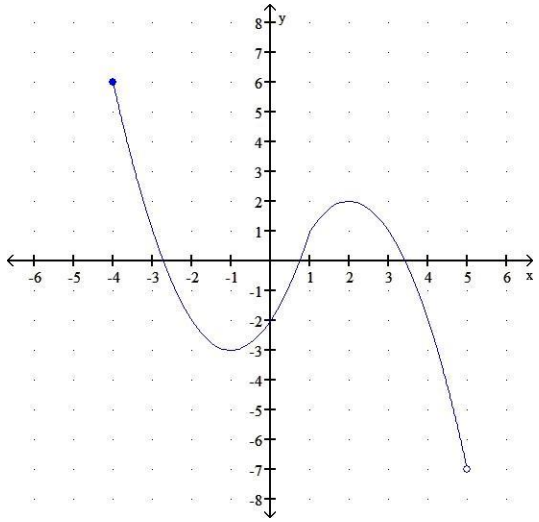
14. Solve the rational inequality. Express the solution in interval notation. $\dfrac{x}{x^2-2x-8} < 0$

15. Write an equation for the line that passes through $(-1,-2)$ and (3, 5). Express your answer using (a) point-slope form, and (b) slope-intercept form.

16. Graph the line: $3x-5y+30=0$. Label two points on the line.

17. Determine whether the equation represents *y* as a function of *x*. Explain briefly. $x^2 + y = 4$

For Problems 18 – 25, use the graph of $f(x)$ below.



18. Determine the domain of *f*.

19. Determine the range of *f*.

20. Find $f(-1)$.

21. Find values of *x* where $f(x)=1$.

22. Determine the interval(s) where *f* is decreasing.

23. Determine the interval(s) where is *f* is increasing.

24. Is *f* even, odd, or neither?

25. Compute the average rate of change of $f(x)=x^3-2$ from $x_1=1$ to $x_2=3$.

26. Find $f(g(x))$ and $g(f(x))$ and determine whether the pair of functions are inverses of each other.

$f(x)=x^3-2$; $g(x)=\sqrt[3]{x+2}$

27. Find the inverse function, $f^{-1}(x)$, for the given function. $f(x)=\dfrac{5}{x-2}$

28. Consider the quadratic function: $g(x)=2(x-1)^2-4$. Determine: (a) whether this parabola opens up or down, (b) the vertex of the parabola, (c) the equation for its axis of symmetry, (d) the *y*-intercept, and (e) the number of its *x*-intercept(s). Sketch its graph.

29. Consider the polynomial function: $h(x)=(x+5)^2(2x-3)^3$.

(a) Find the zeros of the function.
(b) Determine the multiplicity of each zero.
(c) State whether the graph of *h* crosses the *x*-axis, or just touches the *x*-axis at each zero.

(d) Determine the maximum possible number of turning points of its graph.
(e) Apply the Leading Coefficient Test to describe the right-hand and left-hand behavior of its graph.

30. Divide using (a) polynomial long division and (b) synthetic division. State the quotient and the remainder. $\dfrac{4x^3+5x^2+3x-2}{x+2}$

31. Use the Factor Theorem and synthetic division to show that $x=-4$ is a solution of the polynomial equation $x^3-28x-48=0$. Use the result to factor the polynomial equation completely.

32. Use the Rational Zero Test to list all possible rational zeros of: $f(x)=2x^4-5x^3-24x^2+10x-9$. Do not attempt to find the zeros.

33. Use Descartes's Rule of Signs to determine the possible numbers of positive and negative real zeros of: $f(x)=2x^4-5x^3-24x^2+10x-9$. Do not attempt to find the zeros.

34. Use the Intermediate Value Theorem to show that $f(x)=3x^3-x^2+3$ has a real zero between $x=-1$ and $x=0$.

35. Consider the rational function $f(x)=\dfrac{-3x}{x+2}$, and determine the following.

(a) Equation for the vertical asymptote (if any): _____.
(b) Equation for the horizontal asymptote (if any): _____.
(c) Equation for the slant asymptote (if any): _____.
(d) *x*-intercept: _____ *y*-intercept: _____.
(e) Graph the rational function. Include the asymptotes in your sketch.

36. Solve the system by the method of your choice: substitution, addition, or Cramer's Rule.

$$\begin{cases} 3x + 2y = -14 \\ 5x - 4y = 6 \end{cases}$$

37. Let $A=\begin{bmatrix} -5 & 3 \\ -4 & 2 \end{bmatrix}$ and $B=\begin{bmatrix} -5 & -9 \\ 8 & 6 \end{bmatrix}$. Determine the following:

(a) Determinant of A (b) $2A-B$ (c) AB