Your Name

Your Signature

Student ID

- Give your answers in exact form. Do not give decimal approximations.
- Calculators are not allowed.
- In order to receive credit, you must show your work. Do not do computations in your head. Instead, write them out on the exam paper.
- **Place a box around YOUR FINAL ANSWER to each question.**
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.

Problem	Total Points	Score
1	5	
2	5	
3	13	
4	5	
5	9	
6	20	
7	20	
8	8	
9	8	
10	7	
Total	100	

1. [**5 points total**] Mark each statement below as true or false by circling **T** or **F**. No partial credit.

1. **T** **F** The function $f(x) = |x - 1|$ is continuous at $x = 1$.

2. **T** **F** $\dfrac{d}{dx}\ln(\pi) = 0$

3. **T** **F** If $f''(a) = 0$, then f has an inflection point at a.

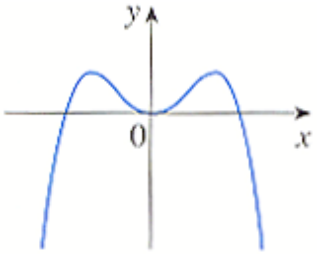
4. **T** **F** If f and g are continuous on $[a, b]$ and c is a real number, then

$$\int_a^b cf(x)g(x)dx = c\int_a^b f(x)dx \cdot \int_a^b g(x)dx$$

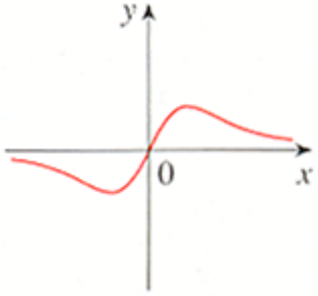
5. **T** **F** If $f(x)$ is continuous on a closed interval, then it is enough to look at the points where $f'(x) = 0$ in order to find its absolute maxima and minima.

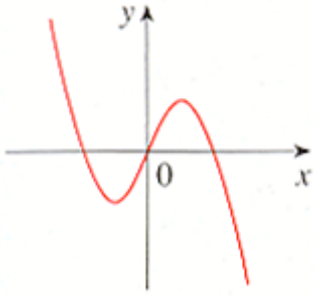
2. [**5 points total**] Circle the correct answer. No partial credit.

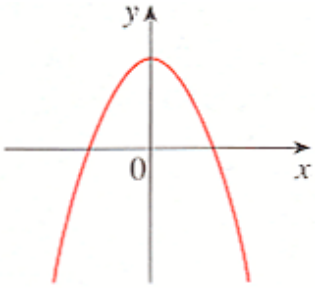
1. Consider the graph of $y = f(x)$:



Which is the graph of $y = f'(x)$?

A. 

B. 

C. 

2. Suppose f has a local minimum at a. What can you say about $f''(a)$?

   A. $f''(a) > 0$.

   B. $f''(a) < 0$.

   C. $f''(a) = 0$.

   D. You cannot say anything about $f''(a)$ without more information.

3. A slow freight train chugs along a straight track. The distance it has traveled after x hours is given by a function $f(x)$. An engineer is walking along the top of the box cars at the rate of 3 miles per hour in the same direction as the train is moving. The speed of the man relative to the ground is

   A. $f(x) + 3$.

   B. $f'(x) + 3$.

   C. $f(x) - 3$.

   D. $f'(x) - 3$.

4. If $f'(a)$ exists, $\lim_{x \to a} f(x)$

   A. Equals $f'(a)$.

   B. Equal $f(a)$.

   C. It must exist, but there is not enough information to determine it exactly.

   D. It may not exist.

5. If f is an antiderivative of g, and g is an antiderivative of h, then

   A. h is an antiderivative of f.

   B. h is the second derivative of f .

   C. h is the derivative of f''.

   D. None of the above.

3. [**13 points total**] Consider the function $f(x) = xe^{-x}$.

(a) (3 pts) Determine if f has any asymptotes (horizontal and vertical).

(b) (3 pts) Find the intervals on which f increases and the intervals on which f decreases.

(c) (2 pts) Provide the (x, y) coordinates of any local extrema if any.

(d) (3 pts) Find the intervals on which f is concave up and the intervals on which f is concave down.

(e) (2 pts) Provide the (x, y) coordinates of any inflection points if any.

4. [**5 points total**] For what values of a and b is

$$f(x) = \begin{cases} x+4, & x \leq 1 \\ ax+b, & 1 < x \leq 3 \\ 3x-8, & x > 3 \end{cases}$$

continuous at every x?

5. [**9 points total**] Evaluate the following limits with justification. If the limit does not exist, explain why. If there is an infinite limit, then explain whether it is $+\infty$ or $-\infty$.

(a) $\lim_{x \to 1^-} \dfrac{x^2 - 1}{|1 - x|}$

(b) $\lim_{x \to \infty} \dfrac{\ln x}{x^3}$

(c) $\lim_{x \to 0} \dfrac{\int_0^x \ln(t+1)dt}{x^2}$

6. [**20 points total**] Compute the indicated derivative of each of the following functions. (Do not simplify)

(a) $f(x) = x^2 \ln\left(\frac{x}{3}\right)$. Find $f'(x)$.

(b) $y = (\cos x)^{\sin x}$. Find $\dfrac{dy}{dx}$

(c) $f(x) = \displaystyle\int_2^{\sqrt{x}} t \ln t \, dt$. Find $f'(e^8)$.

(d) $f(\theta) = \tan^2(\sin\theta) + \arctan\left(\dfrac{1}{\theta}\right)$. Find $f'(\theta)$.

7. [**20 points total**] Evaluate the following integrals

(a) $\int [e^{-\frac{x}{2015}} + \cos(2016x)]\, dx$

(b) $\int \dfrac{\sin x}{1 + \cos^2 x}\, dx$

(c) $\displaystyle\int_1^2 \frac{x^2 - x + 1}{x}\, dx$

(d) $\displaystyle\int_0^{2\pi} |\sin x|\, dx$

8. [**8 points total**] We need to enclose a field with a rectangular fence. We have 500 ft of fencing material and a building is on one side of the field and so won't need any fencing. Determine the dimensions of the field that will enclose the largest area.

9. [**8 points total**] Air is being pumped into a spherical balloon at the rate of $7\,cm^3/sec$. What is the rate of change of the radius at the in stant the volume equals $36\pi\,cm^3$? The volume of the sphere of radius r is $\dfrac{4\pi}{3}r^3$.

10. [**7 points total**] Find the equation of the tangent line to the curve $(y+1)^2 = x - 2$ at the point $(6, 1)$. Write your answer in slope-intercept form.