Your Name

□

Your Signature

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Student ID

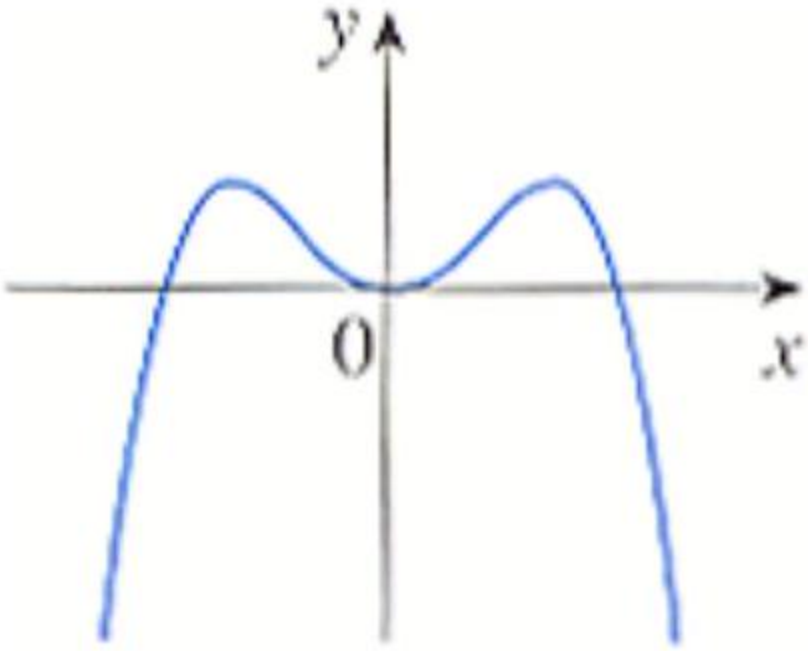
- Give your answers in exact form. Do not give decimal approximations.
- Calculators are not allowed.
- In order to receive credit, you must show your work. Do not do computations in your head. Instead, write them out on the exam paper.
- Place a box around YOUR FINAL ANSWER to each question.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.

Problem	Total Points	Score
1	5	
2	5	
3	13	
4	5	
5	9	
6	20	
7	20	
8	8	
9	8	
10	7	
Total	100	

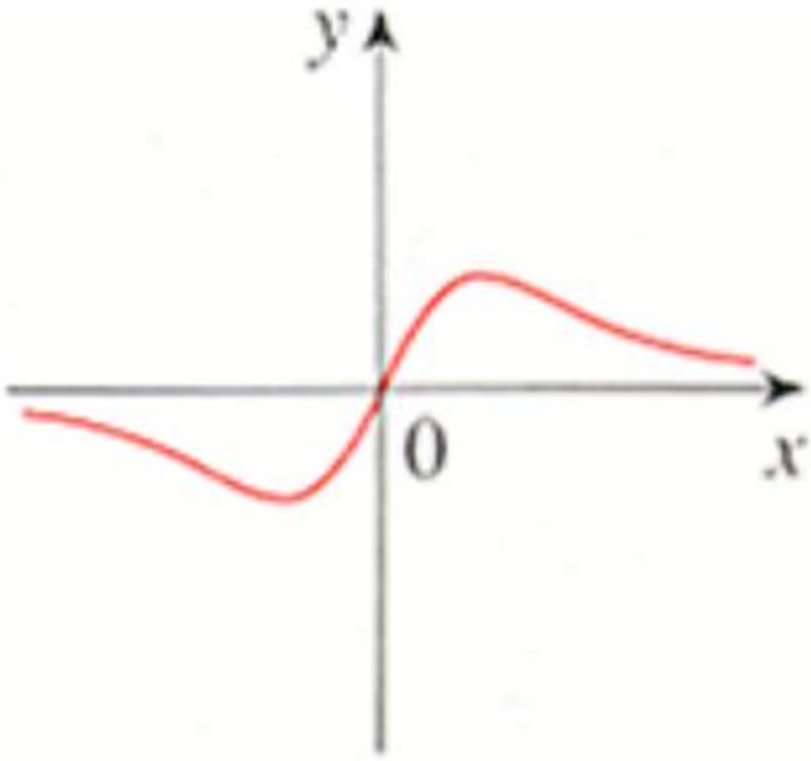
1. [5 points total] Mark each statement below as true or false by circling T or F. No partial credit.
2. T F The function $f(x) = |x - 1|$ is continuous at $x = 1$.
3. T F $\frac{d}{dx}\ln(\pi) = 0$
4. T F If $f''(a) = 0$, then f has an inflection point at a.
5. T F If f and g are continuous on $[a, b]$ and c is a real number, then

$$\int_a^b cf(x)g(x)dx = c\int_a^b f(x)dx \cdot \int_a^b g(x)dx$$

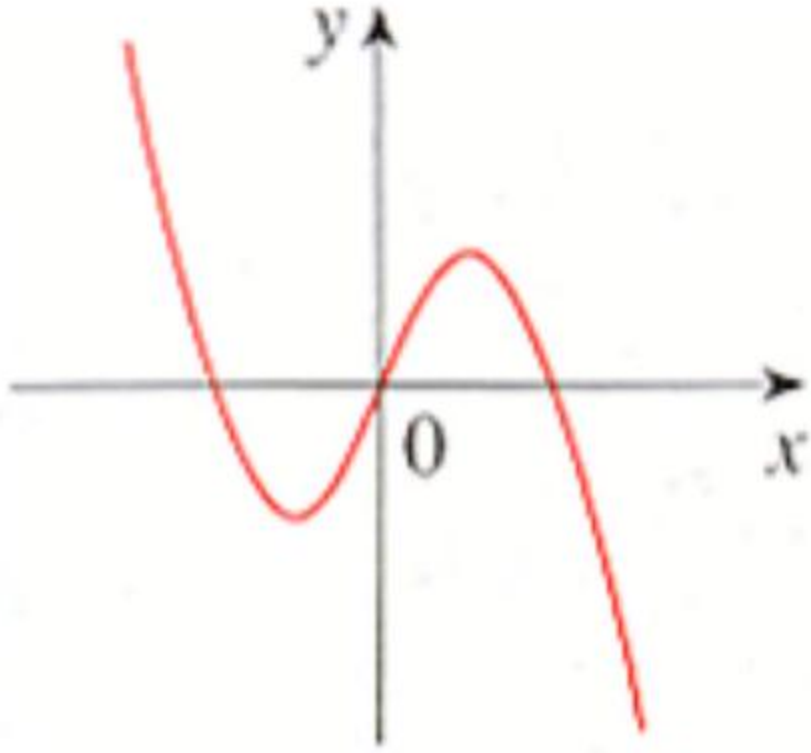
5. T F If $f(x)$ is continuous on a closed interval, then it is enough to look at the points where $f'(x) = 0$ in order to find its absolute maxima and minima.
6. [5 points total] Circle the correct answer. No partial credit.
7. Consider the graph of $y = f(x)$:



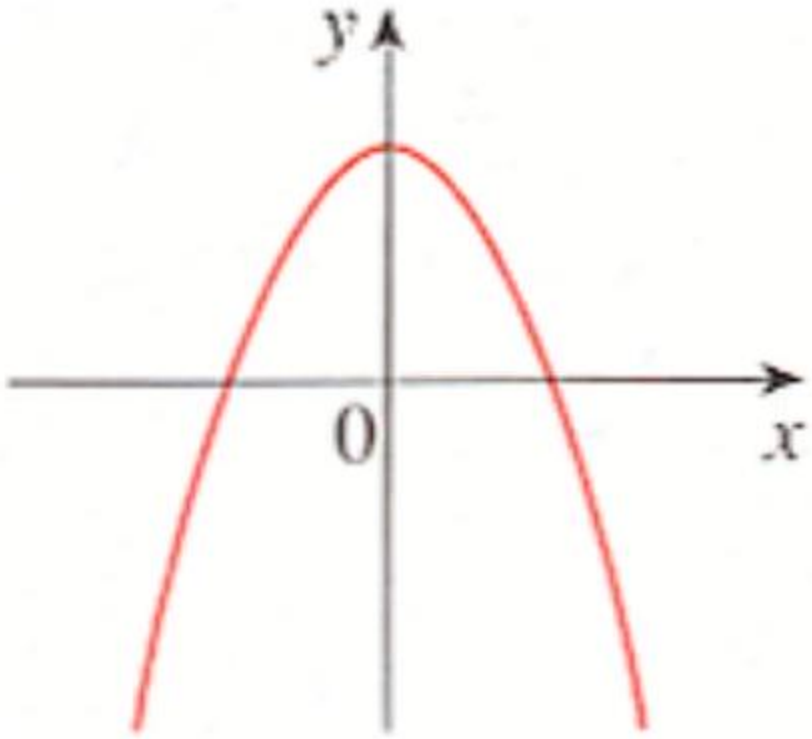
Which is the graph of $y = f'(x)$?
A.



B.



C.



2. Suppose f has a local minimum at a. What can you say about $f''(a)$?
A. $f''(a) > 0$.
B. $f''(a) < 0$.
C. $f''(a) = 0$.
D. You cannot say anything about $f''(a)$ without more information.
3. A slow freight train chugs along a straight track. The distance it has traveled after x hours is given by a function $f(x)$. An engineer is walking along the top of the box cars at the rate of 3 miles per hour in the same direction as the train is moving. The speed of the man relative to the ground is
A. $f(x) + 3$.
B. $f'(x) + 3$.
C. $f(x) - 3$.
D. $f'(x) - 3$.
4. If $f'(a)$ exists, $\lim_{x \to a} f(x)$
A. Equals $f'(a)$.
B. Equal $f(a)$.

C. It must exist, but there is not enough information to determine it exactly.
D. It may not exist.
5. If f is an antiderivative of g, and g is an antiderivative of h, then
A. h is an antiderivative of f.
B. h is the second derivative of f.
C. h is the derivative of f''.
D. None of the above.
3. [13 points total] Consider the function $f(x) = xe^{-x}$.
(a) (3 pts) Determine if f has any asymptotes (horizontal and vertical).
(b) (3 pts) Find the intervals on which f increases and the intervals on which f decreases.
(c) (2 pts) Provide the (x, y) coordinates of any local extrema if any.
(d) (3 pts) Find the intervals on which f is concave up and the intervals on which f is concave down.
(e) (2 pts) Provide the (x, y) coordinates of any inflection points if any.
4. [5 points total] For what values of a and b is

$$f(x) = \begin{cases} x + 4, & x \leq 1 \\ ax + b, & 1 < x \leq 3 \\ 3x - 8, & x > 3 \end{cases}$$

continuous at every x ?
5. [9 points total] Evaluate the following limits with justification. If the limit does not exist, explain why. If there is an infinite limit, then explain whether it is $+\infty$ or $-\infty$.
(a) $\lim_{x \to 1^-} \frac{x^2 - 1}{|1 - x|}$
(b) $\lim_{x \to \infty} \frac{\ln x}{x^3}$
(c) $\lim_{x \to 0} \frac{\int_0^x \ln(t+1) dt}{x^2}$
6. [20 points total] Compute the indicated derivative of each of the following functions. (Do not simplify)
(a) $f(x) = x^2 \ln\left(\frac{x}{3}\right)$. Find $f'(x)$.
(b) $y = (\cos x)^{\sin x}$. Find $\frac{dy}{dx}$
(c) $f(x) = \int_2^{\sqrt{x}} t \ln t dt$. Find $f'\left(e^8\right)$.
(d) $f(\theta) = \tan^2(\sin \theta) + \arctan\left(\frac{1}{\theta}\right)$. Find $f'(\theta)$.
7. [20 points total] Evaluate the following integrals

(a) $\int \left[e^{-\frac{x}{2015}} + \cos(2016x)\right] dx$
(b) $\int \frac{\sin x}{1+\cos^2 x} dx$
(c) $\int_1^2 \frac{x^2-x+1}{x} dx$
(d) $\int_0^{2\pi} |\sin x| dx$
8. [8 points total] We need to enclose a field with a rectangular fence. We have 500 ft of fencing material and a building is on one side of the field and so won't need any fencing. Determine the dimensions of the field that will enclose the largest area.
9. [**8** points total] Air is being pumped into a spherical balloon at the rate of $7\ \text{cm}^3/\text{sec}$. What is the rate of change of the radius at the in stant the volume equals $36\pi cm^3$? The volume of the sphere of radius r is $\frac{4\pi}{3}r^3$.
10. [**7** points total] Find the equation of the tangent line to the curve $(y+1)^2 = x-2$ at the point (6, 1). Write your answer in slope-intercept form.