

Your Name

Your Signature

Student ID

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- Give your answers in exact form. Do not give decimal approximations.
- Calculators are not allowed.
- In order to receive credit, you must show your work. Do not do computations in your head. Instead, write them out on the exam paper.
- Place a box around YOUR FINAL ANSWER to each question.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.

Problem	Total Points	Score
1	5	
2	5	
3	12	
4	8	
5	6	
6	9	
7	12	
8	18	
9	12	
10	8	
11	5	
Total	100	

1. [5 points total] Mark each statement below as true or false by circling **T** or **F**.
2. T F If a function  $f$  is continuous at the point  $a$  then it is differentiable there.
3. T F Since the function  $f(x) = \frac{(x+3)(x+5)}{(x-2)(x-6)}$  is equal to  $\frac{15}{12}$  when  $x = 0$ , and is equal to  $-16$  when  $x = 3$ , the Intermediate Value Theorem can be used to conclude that  $f(a) = 0$  for some  $a$  between 0 and 3 .
4. T **F** If  $c$  is a critical number of a function  $f$  and also  $f''(c) = 0$ , then by the Second Derivative Test, it follows that  $f$  achieves neither a local maximum nor a local minimum at  $x = c$ .
5. T F If  $f(x)$  and  $g(x)$  are continuous functions which are defined for all real numbers, then

$$\int_{-2015}^{2015} (x^4 + x^6 \sin x + 12) dx = \int_{-2015}^{2015} (x^4 + 12) dx$$

5. T F The absolute minimum value of a continuous function  $f(x)$  defined on a closed interval  $[a, b]$  can only be realized at an endpoint ( $x = a$  or  $x = b$ ) or at a point where the graph of  $f$  has a horizontal tangent.
6. [5 points total] Circle the correct answer.
7. If  $f''(x) = 3^{-x}(x - 5)(x - 14)^{2014}$ , then  $f(x)$  has inflection point(s) at
  - A.  $x = 5$  and  $x = 14$
  - B.  $x = 5$  only.
  - C.  $x = 14$  only.
  - D.  $x = 0$  only.
8. The absolute minimum value of  $f(x) = 1 - x^2$  on  $[-1, 2]$  is
  - A.  $-3$  .
  - B.  $0$ .
  - C.  $2$ .
  - D.  $3/2$ .
9. The minimum value of the slope of the tangent line to  $h(x) = 2x^3 - 3x^2 - 12x + 5$  occurs at
  - A.  $x = 2$ .
  - B.  $x = -1$ .
  - C.  $x = 1/2$ .
  - D. There is no such value for the slope

10. Consider the function  $h(x) = \ln(g(x))^3$  and assume that  $g(2) = 5$  and  $g'(2) = -3$ . The  $h'(2)$  equals
- $\frac{3}{5}$ .
  - $-\frac{3}{5}$ .
  - $-\frac{9}{5}$ .
  - $\frac{5}{9}$ .
11. Suppose  $f$  is a function such that  $f'(3) = 0$ , and  $f''(3) < 0$ . What can be said about the function?
- The function has local maximum value at  $x = 3$ .
  - The function has local minimum value at  $x = 3$ .
  - The function has neither a local maximum nor local minimum value at  $x = 3$ .
  - You need more information to determine whether  $f$  has a local maximum or minimum at  $x = 3$ .
12. [12 points total] Consider the function  $f(x) = xe^{-x}$ .
- (4 pts) Find the intervals on which  $f$  increases and the intervals on which  $f$  decreases.
  - (4 pts) Find the  $x$ -coordinates of any local maxima or minima.
  - (4 pts) Find the intervals on which  $f$  is concave up and the intervals on which  $f$  is concave down.
13. [10 points total] Suppose  $f(x) = \frac{1}{3x}$ . Using the definition of the derivative, find  $f'(2)$ . (You will receive NO credit for finding the derivative using a different method.)
14. [6 points total] Sketch a well-labeled graph of a continuous function,  $g$ , which satisfies all of the following properties.
- $g(0) = -1$ .
  - $g(1) = 0$ .
  - $g'(x) = 2$  for  $1 < x < 2$ .
  - $g'(x) = -2$  for  $2 < x < 3$ .
  - $g$  is decreasing for  $x > 3$ .
  - $g''(x) < 0$  for  $x > 3$
  - $g$  is concave down for  $x < 1$ .
6. [9 points total] Evaluate the following limits. Show work!
- $\lim_{x \rightarrow 1^-} \frac{x^2 - 1}{|x - 1|}$

- (b)  $\lim_{x \rightarrow \infty} x^2 e^{-x}$   
 (c)  $\lim_{x \rightarrow 0} \frac{(1 - \cos x)^2}{x \sin x}$
7. [12 points total] Find  $f'(x)$  (you should simplify and write your final answers without negative exponents) if
- (a)  $f(x) = x^3 e^{-3x}$   
 (b)  $f(x) = \frac{e^{\sec^2 x}}{\ln x}$   
 (c)  $f(x) = \int_1^{\sqrt{\cos x}} (t^{2016} + 2015)^{2016} dx$   
 (d)  $f(x) = (\cos x)^x$
8. [18 points total] Evaluate the following integrals
- (a)  $\int x^3 \sqrt{x^2 - 1} dx$   
 (b)  $\int \frac{\sin x}{\cos^2 x} dx$   
 (c)  $\int_2^3 \frac{3x^2 + 2x + 1}{x} dx$   
 (d)  $\int \frac{\sin x}{1 + \cos^2 x} dx$   
 (e)  $\int_{-1}^1 |x| dx$   
 (f)  $\int \frac{(\cos(\tan x))}{\cos^2 x} dx$
9. [12 points total] A landscape architect plans to enclose a 4000 - square-foot rectangular region in a botanical garden. She will use shrubs costing \$20 per foot along three sides and fencing costing \$5 per foot along the fourth side. What is the minimum total cost?
10. [ 8 points total] The equation  $x^2 - xy + y^2 = 1$  describes an ellipse. Find the coordinates  $(x, y)$  of all points on the curve where the tangent to the curve is horizontal.
11. [ 5 points total] The acceleration, in  $\text{m/sec}^2$ , of a particle moving along a line is given as a function of time  $t$  (in sec) by the formula

$$a(t) = 2e^t + 3 \sin t - t$$

The initial velocity (at time  $t = 0$ ) is 2 m/sec. What is the particle's velocity at time  $t = 4$ ?