Your Name

☐

Your Signature

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Student ID

- Give your answers in exact form. Do not give decimal approximations.
- Calculators are not allowed.
- In order to receive credit, you must show your work. Do not do computations in your head. Instead, write them out on the exam paper.
- Place a box around YOUR FINAL ANSWER to each question.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.

Problem	Total Points	Score
1	5	
2	5	
3	12	
4	8	
5	6	
6	9	
7	12	
8	18	
9	12	
10	8	
11	5	
Total	100	

1. [5 points total] Mark each statement below as true or false by circling **T** or **F**.

2. T F The function $f(x) = |x|$ is continuous at $x = 0$.

3. T F The graph of $f(x) = x^{2016}$ has an inflection point at $x = 0$.

4. T F If $x = a$ is a critical point of a function $f(x)$, then $f'(a) = 0$

5. T F If $f(x)$ and $g(x)$ are continuous functions which are defined for all real numbers, then

$$\int_a^b (cf(x) - g(x))dx = c\int_a^b f(x)dx + \int_b^a g(x)dx$$

5. T F If $f(y)$ is a twice differentiable function whose first derivative is continuous, decreasing, and negative for all real numbers y, the $f(y)$ is concave up.

6. [5 points total] Circle the correct answer.

7. Suppose f is a function such that $f'(2) = 5$ and $f(2) = 7$. An equation of the line tangent to the curve $y = f(x)$ at the point $x = 2$ is:
   A. $y = 2x + 7$
   B. $y - 7 = 2(x - 5)$
   C. $y = 5x - 3$
   D. None of the above

8. Suppose f has a local maximum at a. What can you say about $f''(a)$?
   A. $f''(a) > 0$.
   B. $f''(a) < 0$.
   C. $f''(a) = 0$.
   D. You cannot say anything about $f''(a)$ without more information.

9. Suppose that for $a < b < c$, $\int_a^b f(x)dx = 5$, $\int_b^c g(x)dx = 3$ and $\int_a^c g(x)dx = 7$. What is the value of $\int_a^b (f(x) - g(x))dx$?
   A. -5
   B. 0
   C. 1
   D. 9

10. Consider the function $h(x) = e^{-g(x)}$ where the function $g(x)$ is continuous with a continuous first derivative in $(-\infty, \infty)$. If the function $g(x)$ has a local maximum at the point a, then the function $h(x)$
    A. Has local minimum at the point a.
    B. Has local maximum at the point a.
    C. Is negative at the point a.
    D. You cannot say anything about $h(x)$ without more information.

11. Suppose f is a function such that $f'(3) = 0$, and $f''(3) = 0$. What can be said about the function?
    A. The function has local maximum value at $x = 3$.
    B. The function has local minimum value at $x = 3$.
    C. The function has neither a local maximum nor local minimum value at $x = 3$.
    D. You need more information to determine whether f has a local maximum or minimum at $x = 3$.

12. [12 points total] Consider the function $f(x) = \frac{e^x}{x-1}$.
    (a) (3 pts) Find the x -coordinates of the x -intercepts and the y -coordinates of the y -intercept (if any).
    (b) (3 pts) Find the intervals on which f increases and the intervals on which f decreases.

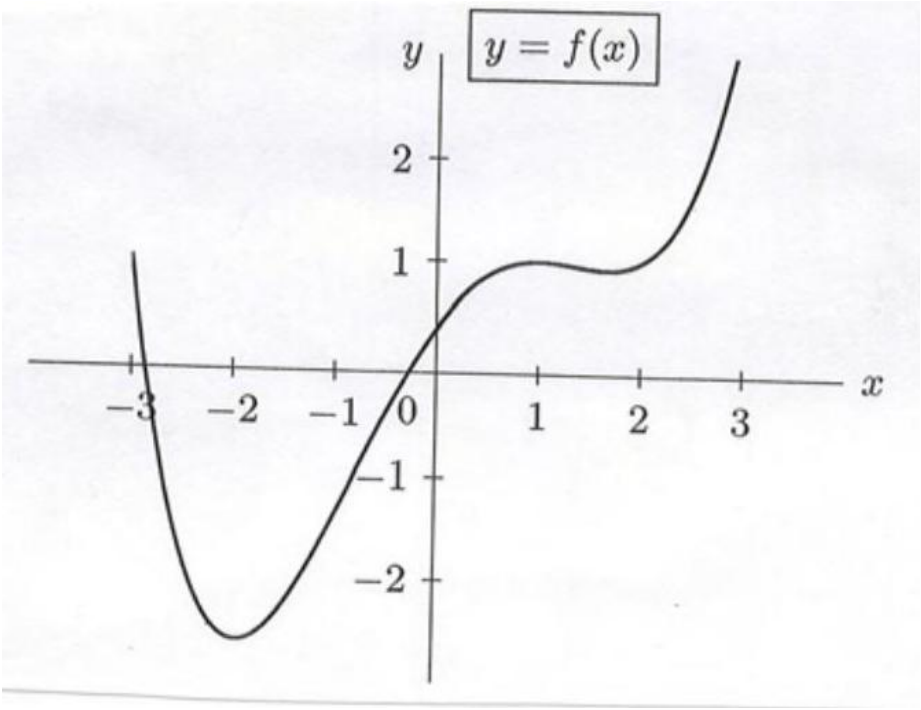
(c) (3 pts) Find the x -coordinates of any local maxima or minima.
(d) (3 pts) Find the intervals on which f is concave up and the intervals on which f is concave down.

13. [8 points total] For what value of c is

$$f(x) = \begin{cases} x^2 - 1, & x < 3 \\ 2cx, & x \geq 3 \end{cases}$$

continuous at every x ?

5. [6 points total] Below is the graph of a function $f(x)$.



Graph its derivative $f'(x)$.

6. [9 points total] Evaluate the following limits. Show work!
(a) $\lim_{x\to 1} \frac{x^2-1}{|x-1|}$
(b) $\lim_{x\to 0^+} x \ln x$
(c) $\lim_{x\to 0} \frac{\int_0^x e^{-t^2} dt}{x}$

7. [12 points total] Find $f'(x)$ (you should simplify and write your final answers without negative exponents) if
(a) $f(x) = x^3 \ln 3x$
(b) $f(x) = \frac{\cos x}{x^4+3}$
(c) $f(x) = \sin e^{\tan x^2}$
(d) $f(x) = x^{\tan x}$

8. [18 points total] Evaluate the following integrals

(a) $\int_0^1 \left(x^2+2\right)\sqrt{x^3+6x+5}dx$

(b) $\int \frac{\sin x}{\cos^2 x}dx$

(c) $\int \frac{6x^5-\sqrt{x}+5x^2}{x^3}dx$

(d) $\int \frac{1}{1+16x^2}dx$

(e) $\int_0^{3\pi/2} |\sin x|dx$

(f) $\int \frac{(\ln x)^2}{x}dx$

9. [12 points total] Air is being pumped into a spherical balloon at the rate of 7 cubic centimeters per second. What is the rate of change of the radius at the instant the volume equals 36π ? The volume of a sphere of radius r is $\frac{4\pi}{3}r^3$

10. [8 points total] Find the equation of the tangent line to the curve $e^y \sin x + x - xy = \pi$ at the point $(\pi, 0)$.

11. [5 points total] Show that the equation $3x + 2\cos x + 5 = 0$ has exactly one real root.