



Khodr Shamseddine (khodr@physics.umanitoba.ca)
Department of Physics and Astronomy, University of
Manitoba, Winnipeg, Manitoba R3T 2N2, Canada,
*Analysis on a Non-Archimedean Field Extension
of the Real Numbers and Applications.*

ABSTRACT. In this talk, I will review the algebraic and topological properties of the smallest non-Archimedean ordered field extension of the real numbers that is Cauchy-complete and real closed. In fact, the Levi-Civita field is small enough to allow for the calculus on the field to be implemented on a computer and used in applications such as the fast and accurate computation of the derivatives of real functions as "differential quotients" up to very high orders.

I will then give an overview of recent research on the Levi-Civita field. In particular, I will summarize the convergence and analytical properties of power series, showing that they have the same smoothness behavior as real power series; and I will present a Lebesgue-like measure and integration theory on the field. Moreover, based on continuity and differentiability concepts that are stronger than the topological ones, we discuss solutions to one-dimensional and multi-dimensional optimization problems as well as existence and uniqueness of solutions of ordinary differential equations.