Timur Oikhberg (qbu@olemiss.edu), Department of Mathematics, University of California - Irvine, CA 92617, USA and Department of Mathematics, University of Illinois at Urbana-Champaign, Urbana, IL 61801, USA, ***The rate of best approximation.***

ABSTRACT. An increasing chain (A_n) of homogeneous subsets of a quasi-Banach space X is called an approximation scheme if the union of the sets A_n is dense in X, and $A_n + A_n$ belongs to $A_{K(n)}$, for some increasing function K. Examples of approximation schemes include, for instance, the sets of polynomials of degree n, or the sets of free node splines of given order. Describing the behavior of the sequence of best approximation errors of an element x of the space X by the sets A_n is one is one of the central problems of approximtion theory. We study the "slowest possible" rate of approximtion of an element x of the space X by the sets A_n. An approximation scheme (A_n) is said to statisfy Shapiro's Theorem if, for any sequence (c_n) decreasing to 0, there exists an element x of X such that $dist(x, A_n) > c_n$ for infinitely many values of n. If X is a Banach space, this is equivalent to the existence of x s.t. $dist(x, A_n) > c_n$ for all values of n. We show that many "naturally occuring" approximation schemes satisfying Shapiro's Theorem. In the non-commutative setting, we show that, whenever X and Y are infinite dimensional Banach spaces, and the sequence (c_n) decreases to 0, there exists an operator T in $B(X, Y)$ whose sequence of approximation numbers behaves like (c_n). This is a joint work with J.Almira.