Anthony G. O'Farrell (anthonyg.ofarrell@gmail.com), Department of Mathematics, NUI, Maynooth, Co. Kildare, Ireland, ***Pervasive and maximal function algebras.***

ABSTRACT. Let X be a compact Hausdorff topological space and $C(X;\mathbb{C})$ (respectively, $C(X;\mathbb{R})$) the Banach algebra of all continuous complex-valued (respectively, real-valued) functions on X endowed with the uniform norm. A function space S on X is a closed subspace of $C(X;C)$. We denote by $clos_{C(E;\mathbb{C})}S$ the closure in $C(E;\mathbb{C})$ of the function space S, where E is a closed subset of X. Similarly, we denote by $clos_{C(E;\mathbb{R})}S$ the closure in $C(E;\mathbb{R})$ of the real subspace S of $C(X;\mathbb{R})$.

A function space S of $C(X;\mathbb{C})$ is said to be *complex pervasive* if $clos_{C(E;\mathbb{C})}S = C(E;\mathbb{C})$ whenever E is a proper non-empty closed subset of X. Similarly, a real subspace S of $C(X;\mathbb{R})$ is said to be real pervasive if $clos_{C(E;\mathbb{R})}S = C(E;\mathbb{R})$.

The term pervasive was introduced by Hoffman and Singer in 1960. They studied (complex) pervasive uniform algebras, motivated by the relationship with maximal uniform algebras. They obtained some preliminary results about algebras of analytic functions on connected open subsets of the Riemann sphere.

In 1971, work by Gamelin and Garnett on the so-called Dirichlet algebras revealed connections with analytic capacity, and with the abstract Farrell-Rubel-Shields Theorem.

The real pervasiveness of spaces of harmonic functions on Euclidean spaces was studied by Netuka in 1987. He showed that if the open set $U \subset \mathbb{R}^d$ is bounded and connected, and $bdyU = bdy\ clos\ U$, then the space of functions continuous on $clos\ U$ and harmonic on U is real pervasive on $bdy\ U$.

More recent work by the speaker, Netuka and Sanabria led to rather satisfactory results in spaces of analytic functions of one complex variable (and, separately, spaces of real parts of such functions) on arbitrary open sets and on Riemann surfaces. The results are quite deep, relying on the theory of uniform algebras as well as Vitushkin-type techniques using capacities.

In this talk we review the history of this concept, and report on recent work with Pamela Gorkin, on the relationship between pervasiveness and maximality.