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MO, USA, *Positive definite functions and stable
random vectors.*

ABSTRACT. We say that a random vector $X = (X_1, \dots, X_n)$ in \mathbb{R}^n is an n -dimensional version of a random variable Y if for any $a \in \mathbb{R}^n$ the random variables $\sum a_i X_i$ and $\gamma(a)Y$ are identically distributed, where $\gamma : \mathbb{R}^n \rightarrow [0, \infty)$ is called the standard of X . An old problem is to characterize those functions γ that can appear as the standard of an n -dimensional version. In this talk, we prove the conjecture of Lisitsky that every standard must be the norm of a space that embeds in L_0 . This result is almost optimal, as the norm of any finite dimensional subspace of L_p with $p \in (0, 2]$ is the standard of an n -dimensional version (p -stable random vector) by the classical result of P.Lévy. An equivalent formulation is that if a function of the form $f(\|\cdot\|_K)$ is positive definite on \mathbb{R}^n , where K is an origin symmetric star body in \mathbb{R}^n and $f : \mathbb{R} \rightarrow \mathbb{R}$ is an even continuous function, then either the space $(\mathbb{R}^n, \|\cdot\|_K)$ embeds in L_0 or f is a constant function. Combined with known facts about embedding in L_0 , this result leads to several generalizations of the solution of Schoenberg's problem on positive definite functions.