ECE 585 Homework on Discrete System Theory
Due Thursday September 7, 2017

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Problem 1. Z-transform
(a) Compute the z-transform, \(X(z)\), of the time domain series given below
\[x(n) = a^n T.\]
(b) Check your answer using MATLAB.

Problem 2. Z-transform
(a) Compute the z-transform, \(X(z)\), of the time domain series given below
\[x(n) = a^n T \sin(n \omega_0 T)\]
(b) Check your answer using MATLAB.

Problem 3. Inverse z-transform
(a) Find the inverse transform, \(x(n)\), for the \(X(z)\) given below
\[X(z) = \frac{z(z+1)}{(z-1)^3}\]
(b) Check your answer using MATLAB.

Problem 4. Discrete time-domain transfer function
A discrete time system is described by the difference equation shown below
\[y(n) = a_1 y(n-1) + a_2 y(n-2) + b x(n)\]
where \(T = 2, b = 2, a_1 = 1, \text{ and } a_2 = -0.5.\)
(a) Find the transfer function \(H(z) = \frac{Y(z)}{X(z)}\). 
(b) If the system is driven with a step input, then find the time domain series describing the output i.e. \(y(n)\).

Problem 5. Textbook problem 13.12
Upsampling and downsampling are often used for converting between two sample rates. Give a block diagram showing how one might convert a 0-20 kHz band-limited audio signal from a sampling rate of 50 kHz to a sampling rate of 40 kHz.
Problem 6.  Textbook problem 13.15
Consider the transfer function given below
\[ H(z) = \frac{0.05z}{z - 0.95} \]

(a) What is the gain at DC?

(b) What is the gain at half-the sampling frequency?

(c) Derive an expression for the magnitude response of the system.

(d) Derive an expression for the phase response of the system.

(e) Find an expression for the -3 dB frequency of the system.

(f) Use MATLAB to plot the magnitude response. Assume a sampling frequency of 500 kHz.

(g) Is the -3dB point in the plot what you predicted?

Problem 7.  Textbook problem 13.19
Consider the second-order continuous-time transfer function shown below with a maximally-flat passband \( i.e., Q = \sqrt{2} \).
\[ H(s) = \frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2} \]

(a) Using the bilinear approximation, find an \( H(z) \) to realize a second-order transfer function with a DC gain of one and a -3dB frequency at 1 kHz when the sampling rate is 100 kHz.

(b) Compare the response (both magnitude and phase) of the continuous-time circuit with that of the discrete-time circuit using MATLAB.

Problem 8.  S/H Frequency Response
Use MATLAB to plot the frequency response (magnitude) of a S/H operated at 100 kHz.

(a) How much attenuation is provided at \( \frac{1}{4}f_s \)?

(b) How much attenuation is provided at \( \frac{1}{2}f_s \)?

(c) How much attenuation is provided at \( f_s \)?