4.10. Limiters

✧ **Limiter** is a nonlinear circuit with an output saturation characteristic
✧ It rejects envelope variations but preserves the phase variations.

\[
v_{in}(t) = R(t)\cos[w_c t + \theta(t)]
\]

\[
v_{out}(t) = KV_L \cos[w_c t + \theta(t)]
\]

Where \( K \) is the level of the fundamental component of the square wave, \( 4/\pi \), multiplied by the gain of the output (bandpass) filter. \( V_L \) is output limits.
4.11. Mixers, Up Converters, Down Converters

- **Ideal mixer** is a mathematical multiplier of two input signals. One of the signals is sinusoidal generated by a local oscillator. Mixing results in frequency translation.

- Mixers are used to obtain frequency translation of the input signal.

where $V_{in}(t)$ is bandpass signal: $v_{in}(t) = \text{Re}\left\{ g_{in}(t)e^{jw_c t} \right\}$
4.11. Mixers, Up Converters, Down Converters

Bandpass Input Signal

\[ v_{in}(t) = \text{Re}\{g_{in}(t)e^{j\omega t}\} \]

Mixer Output

\[ v_{1}(t) = \left[ A_0 \text{Re}\{g_{in}(t)e^{j\omega t}\} \right] \cos \omega_0 t \]

\[ = \frac{A_0}{4} \left[ g_{in}(t)e^{j\omega t} + g^*_{in}(t)e^{-j\omega t} \right] (e^{j\omega_0 t} + e^{-j\omega_0 t}) \]

\[ = \frac{A_0}{4} \left[ g_{in}(t)e^{j(\omega_c+\omega_b)t} + g^*_{in}(t)e^{-j(\omega_c+\omega_b)t} + g_{in}(t)e^{j(\omega_c-\omega_b)t} + g^*_{in}(t)e^{-j(\omega_c-\omega_b)t} \right] \]

\[ \text{Re}\{\cdot\} = \frac{1}{2}\{\cdot\} + \frac{1}{2}\{\cdot\}^* \]

\[ v_{1}(t) = \frac{A_0}{2} \text{Re}\{g_{in}(t)e^{j(\omega_c+\omega_b)t}\} + \frac{A_0}{2} \text{Re}\{g_{in}(t)e^{j(\omega_c-\omega_b)t}\} \]

\[ f_u = f_c + f_0 \quad f_d = f_c - f_0 \]

UPCONVERSION \quad DOWNCONVERSION

BANDPASS FILTER \quad BASEBAND OR BANDPASS FILTER
4.11. Mixers, Up Converters, Down Converters

\[ v_1(t) = \frac{A_0}{2} \text{Re}\left\{ g_{in}(t) e^{j(\omega_c + \omega_0) t} \right\} + \frac{A_0}{2} \text{Re}\left\{ g_{in}(t) e^{j(\omega_c - \omega_0) t} \right\} \]

Up-conversion

\[ f_u = f_c + f_0 \]

Down-conversion

\[ f_d = f_c - f_0 \]

Bandpass Filter

Baseband/bandpass Filter \((f_c - f_0)\)
4.11. Mixers, Up Converters, Down Converters

- If \((f_c - f_0) = 0\) ➞ Low Pass Filter gives baseband spectrum
- If \((f_c - f_0) > 0\) ➞ Bandpass filter ➞ Modulation is preserved

Filter Output:
\[ v_2(t) = \text{Re}\{g_2(t)e^{j(\omega_c - \omega_0)t}\} = \frac{A_0}{2} \text{Re}\{g_{in}(t)e^{j(\omega_c - \omega_0)t}\} \]

- If \(f_c > f_0\) ➞ modulation on the mixer input is preserved

- If \(f_c < f_0\) ➞ 
\[ v_1(t) = \frac{A_0}{2} \text{Re}\{g_{in}(t)e^{j(\omega_c + \omega_0)t}\} + \frac{A_0}{2} \text{Re}\{g_{in}^*(t)e^{j(\omega_0 - \omega_c)t}\} \]

‘\(\omega\)’ needs to be positive
4.11. Mixers, Up Converters, Down Converters

- Complex envelope of an *Up Converter*:

\[ g_2(t) = \frac{A_0}{2} g_{in}(t); \quad f_u = f_c + f_0 > 0 \quad - \text{Amplitude is scaled by } A_0/2 \]

- Complex envelope of a *Down Converter*:

\[ f_d = f_c - f_0 > 0 \quad \text{i.e., } f_0 < f_c \rightarrow \text{down conversion with low-side injection} \]

\[ g_2(t) = \frac{A_0}{2} g_{in}(t) \quad - \text{Amplitude is scaled by } A_0/2 \]

\[ f_d = f_0 - f_c > 0 \quad \text{i.e., } f_0 > f_c \rightarrow \text{down conversion with high-side injection} \]

\[ g_2 = \frac{A_0}{2} g_{in}^*(t) \quad - \text{Amplitude is scaled by } A_0/2 \]

- Sidebands are reversed from those on the input
4.12. Frequency Multipliers

*Frequency Multipliers* consists of a nonlinear device together with a tuned circuit. The frequency of the output is \( n \) times the frequency of the input.

\[
v_{\text{in}}(t) = R(t)\cos(\omega_c t + \theta(t))
\]
\[
v_1(t) = K_n v_{\text{in}}^n(t)
= K_n R^n(t)\cos^n(\omega_c t + \theta(t))
\]
\[
v_1(t) = CR^n(t)\cos(n\omega_c t + n\theta(t)) + \text{other terms}
\]
\[
v_{\text{out}}(t) = CR^n(t)\cos(n\omega_c t + n\theta(t))
\]
4.13. Detector Circuits

- **Detectors** convert input bandpass waveform into an output baseband waveform.
4.13. Detector Circuits

Envelope Detector

➢ Ideal envelope detector: Waveform at the output is a real envelope $R(t)$ of its input

Bandpass input:

$$v_{in}(t) = R(t) \cos[\omega_c t + \theta(t)] \quad R(t) \geq 0$$

Envelope Detector Output:

$$v_{out}(t) = KR(t)$$

$K$ – Proportionality Constant
4.13. Detector Circuits

Envelope Detector

The envelope detector is typically used to detect the modulation on AM signal. The detected DC is used for Automatic Gain Control (AGC).

In this case, the complex envelope of the input signal

\[ g(t) = Ac[1+m(t)]. \]

\[ \nu_{out}(t) = KR(t) \]
\[ = K|g(t)| \]
\[ = KA_c[1 + m(t)] \]
\[ = DC + Message \]
4.13. Detector Circuits

**Product Detector**

**Product Detector** is a Mixer circuit that down converts input to baseband.

\[ v_{in}(t) = R(t) \cos(\omega_c t + \theta(t)) \]

or

\[ v_{in}(t) = \text{Re}[g(t) e^{j\omega_c t}] \]

where \( g(t) = R(t) e^{j\theta(t)} \)

\[ v_0(t) = A_0 \cos(\omega_c t + \theta_0) \]

\[ f_c \text{- Freq. of the oscillator} \]

\[ \theta_0 \text{- Phase of the oscillator} \]
4.13. Detector Circuits

Product Detector

Output of the multiplier:

\[ v_1(t) = R(t) \cos[\omega_c t + \theta(t)] A_0 \cos(\omega_c t + \theta) \]

\[ = \frac{1}{2} A_0 R(t) \cos[\theta(t) - \theta] + \frac{1}{2} A_0 R(t) \cos[2\omega_c t + \theta(t) + \theta] \]

Low-pass Filter passes down conversion component:

\[ v_{out}(t) = \frac{1}{2} A_0 R(t) \cos[\theta(t) - \theta] = \frac{1}{2} A_0 \text{Re}\{g(t)e^{-j\theta}\} \]

Where \( g(t) \) is the complex envelope of the input and \( x(t) \) & \( y(t) \) are the quadrature components of the input:

\[ g(t) = R(t)e^{j\theta(t)} = x(t) + jy(t) \]
4.13. Detector Circuits

Applications of Product Detector

- **INPHASE DETECTOR**
  
  \[
  \text{if } \theta_0 = 0 : \quad \Rightarrow v_{out}(t) = \frac{1}{2} A_0 x(t)
  \]

- **QUADRATURE PHASE DETECTOR**
  
  \[
  \text{if } \theta_0 = 90 \quad \Rightarrow v_{out} = \frac{1}{2} A_0 y(t)
  \]

- **ENVELOPE DETECTOR**
  
  \[
  \text{if } \theta(t) = 0 \quad \Rightarrow v_{out} = \frac{1}{2} A_0 R(t)
  \]

- **PHASE DETECTOR** If an angle modulated signal is present at the input and reference phase \( \theta_0 = 90^\circ \)
  
  \[
  \text{if } \theta_0 = 90^\circ \quad v_{in}(t) = A_c \cos \left[ \omega_c t + \theta(t) \right]
  \]
4.13. Detector Circuits

Frequency Modulation Detector

- A ideal FM Detector is a device that produces an output that is proportional to the instantaneous frequency of the input.
4.13. Detector Circuits

Frequency Modulation Detector

Frequency demodulation using slope detection.

\[ v_{in}(t) = A(t) \cos[\omega_c t + \theta(t)] \quad \theta(t) = K_f \int_{-\infty}^{t} m(\tau) d\tau \]

\[ v_1(t) = V_L \cos[\omega_c t + \theta(t)] \quad v_2(t) = -V_L \left[ \omega_c + \frac{d\theta(t)}{dt} \right] \sin[\omega_c t + \theta(t)] \]

\[ v_{out}(t) = -V_L \left[ \omega_c + \frac{d\theta(t)}{dt} \right] = V_L \left[ \omega_c + \frac{d\theta(t)}{dt} \right] = \]

\[ V_L \omega_c + V_L K_f m(t) = DC + AC \text{ (Proportional to } m(t)) \]

- The DC output can easily be blocked