# Electron Charge to Mass Ratio

Matthew Norton, Chris Bush, Brian Atinaja, Becker Steven

**Abstract**

The electron charge to mass ratio was an experiment that was used to calculate the ratio of the electron's charge to its mass. A beam of electrons was used that was subjected to a magnetic field that caused it to shift direction. This experiment was one of the first experiments to attempt to find the charge or mass of an electron. After the charge to mass ratio was found, all someone would have to do was to find the charge or mass of an electron, and the other value would be known. It was found that the electron charge to mass ratio when the voltage was held constant, was $1.715\times10^{11}$ and when the current was held constant, the charge to mass ratio was $1.442\times10^{11}$. Overall, the most accurate charge to mass ratio that was found was $1.71\times10^{11} \pm 5.91\times10^{9}$ C/kg.

**Introduction**

When a particle with a charge $q$ moves through a region that has a magnetic and an electric field, the force on the particle is given by,

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) \tag{1}$$

where **E** is the electric fields and **B** is the magnetic field. In performing this experiment, an electron source, a heated tungsten filament and an electrode, and Helmholtz coils are used to create the magnetic field. The electron beam source and the electrode are in a container that has a small amount of mercury vapor. A near vacuum is necessary because if there were too many ions in the sealed vessel, they would be attracted to the positive and negative terminals and therefore neutralize the terminals. The gas in the sealed vessel is ionized and its electrons achieve an excited state. When these electrons return to their normal energy level, photons are released and thus, producing light. If the electron beam were to be in a complete vacuum, it would not be visible since it needs the other gas in the tube to become visible. Inside the container are five pins that allow the diameter of the electron beam diameter. The values of the diameter of the pins are, 0.065, 0.078, 0.090, 0.103, and 0.115 m.

JJ The electron beam observed is a cathode rays. Cathode rays are streams of electrons that are observed in vacuum tubes. Thomson used cathode rays to find the wave-particle duality of light. JJ Thomson came to his conclusion about the wave-particle duality of cathode rays when he took two cylinders with a slit in them and put them inside of each other with the outer one grounded and the inner one attached to a measuring device. Thomson found that the cathode rays would not enter the slits unless a magnet acted upon them. Once the cathode rays entered the cylinders, a negative charge built up on the inner cylinder.

Helmholz coils produce the magnetic field that is used to control the beam of electrons. Helmholtz coils have a mean radius, *R* and they are separated by a distance of *R*. A magnetic field is formed and it is given by,

$$B = \frac{\mu_0 N I R^2}{\left[R^2 + \left(\frac{d}{2}\right)^2\right]^{3/2}}, \tag{2}$$

which can be simplified to

$$B = \frac{8\mu_0 N I}{R\sqrt{125}} \tag{3}$$

where $N$ is the number of turns in the coils, which is 72 for the setup that was used, and $\mu_0$ is the permeability of free space, $4\pi\times10^{-7}$ WB/A-m. Equation (3) is valid near the center of the coils. On the plane that is halfway between the coils with a small displacement in the z direction, the magnetic field is given by,

$$B(r) = B(0)\left[1 - \frac{54}{124}\frac{r^4}{4} + \cdots\right] \tag{4}$$

where *r* is the distance from the z-axis. The magnetic field through the Helmholtz coils is shown in figure 1[1].

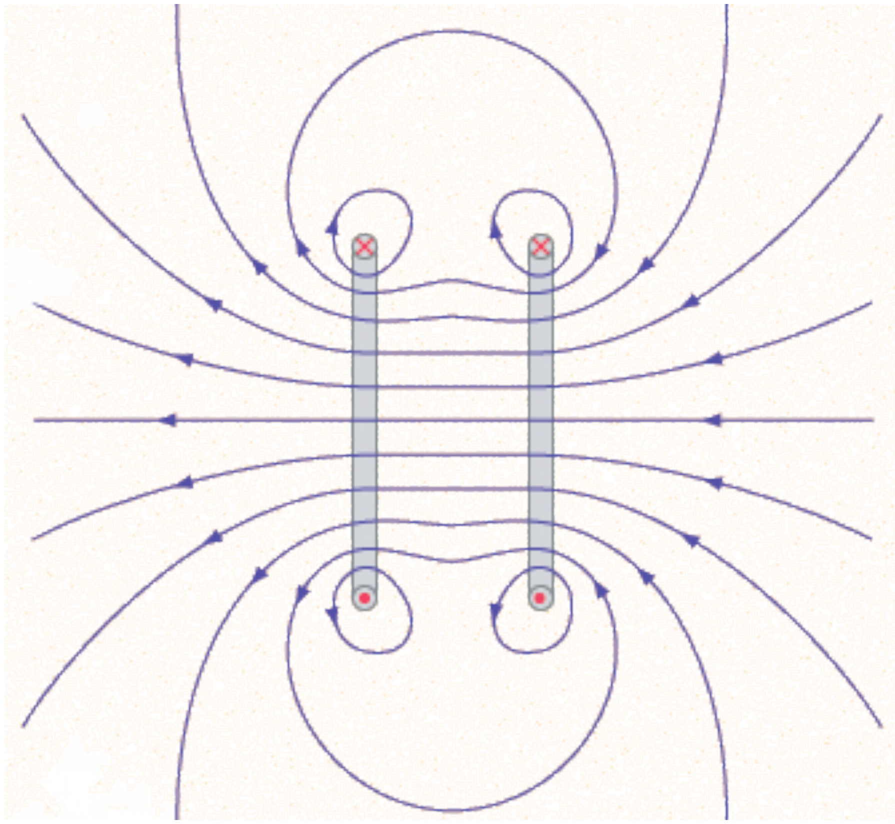

Fig. 1. Magnetic field for Helmholtz coils

In the center of the coils, there is a region that has a nearly uniform magnetic field in all directions. In order to find the equation that will allow for the calculation of *e/m*, one starts with the kinetic field of an electron that is accelerated through an electric potential, *V*, which is given by,

$$eV = \frac{1}{2}mv^2 \tag{5}$$

[1] Taken from http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/helmholtz.html

Where $V$ is the anode potential and $e$ is the charge of an electron. The equation's right-hand side is the familiar kinetic energy of a particle of mass, $m$, and speed, $v$.

The scalar form of equation (1) is needed because there is no electric field

$$F_B = evB \tag{6}$$

Since the electron beam is traveling in a circle, one must also consider the centripetal force for an electron with a mass, $m$, following a circular path with a radius of $r$, with a speed of $v$.

$$F_c = \frac{mv^2}{r} \tag{7}$$

However, equations (6) and (7) are describing the same thing so

$$evB = \frac{mv^2}{r} \tag{8}$$

Equation (8) can be simplified to

$$\frac{e}{m} = \frac{v}{Br} \tag{9}$$

This equation contains the velocity $v$, which one can eliminate by using equation (5). By rewriting equation (5), one finds

$$v = \sqrt{\frac{2eV}{m}} \tag{10}$$

By substituting equation 10 into equation 9, and then by squaring both sides, one obtains

$$\frac{e}{m} = \frac{\sqrt{\frac{2eV}{m}}}{Br} \Rightarrow \frac{e^2}{m^2} = \frac{2eV}{mB^2r^2} \tag{11}$$

By simplifying equation (11), one achieves the desired result, the charge to mass ratio of the electron

$$\frac{e}{m} = \frac{2V}{B^2 r^2} \tag{12}$$

The accepted value for the charge to mass ratio is $1.75882 \times 10^{11}$ C/kg.

**Experimental Procedure**

Initially the direction of the earth's magnetic field, then the apparatus was aligned to the earth's north-south axis. The apparatus was then inclined an angle of 28º in an attempt to cancel the earths magnetic field so that it would have a negligible affect on the experiment. Each group member measured to the diameter of the Helmholtz coils. The mean diameter of the inner diameter was 0.644±0.005m and the mean diameter of the outer diameter was 0.678±0.005. In performing the calculations, the inner diameter was used because the inner diameter describes the distance that the coils were from the electron beam. The outer diameter was not used because it added over 0.03m onto the calculations and added 0.03m on the coils that did not contribute to the magnetic field. The measurement of the diameter was done using a one-meter stick that had an uncertainty of ±0.005m. The meter stick was held perpendicular the rings and the diameter was measured. The Helmholtz coils used in this experiment were not circular. The measurements of the diameter varied as much as 0.017m. Therefore, one must conclude that the Helmholtz rings used in the experiment were not circular.

The distance between the Helmholtz coils was measured in a similar fashion. The mean inner distance was .0310m and the mean outer distance was 0.349±0.005. The inner distance was again used due to the reason stated before in why the inner diameter was used. The distance between the coils varied the same as the diameter of the coils did, 0.017m. The distance between the coils is approximately equal to the radius; however, there is enough variance in the measure of the distance to create some doubt that the distance between the coils equaled the radius of the coils.

Figure 2 is the circuit diagram for the Helmholtz ring, Figure 3 is the circuit diagram for the filament, and Figure 4 is the circuit diagram for the anode.

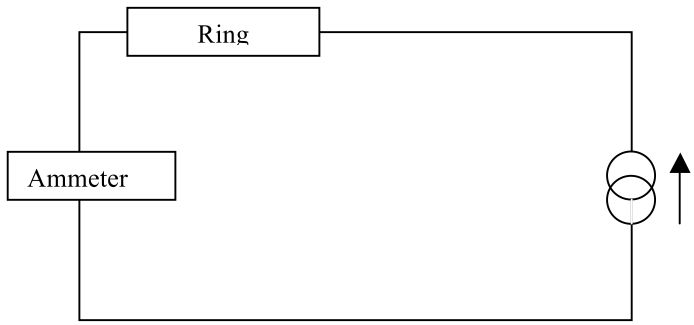


Fig. 2. Circuit diagram for Helmholtz rings

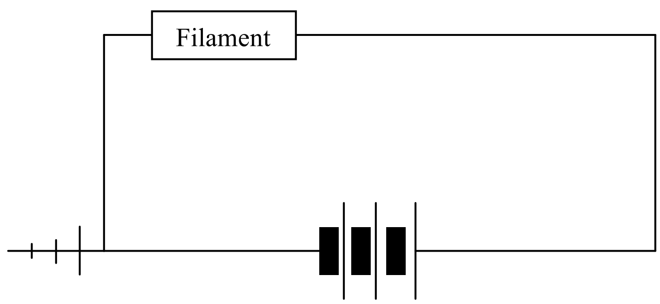


Fig. 3. Circuit diagram for filament

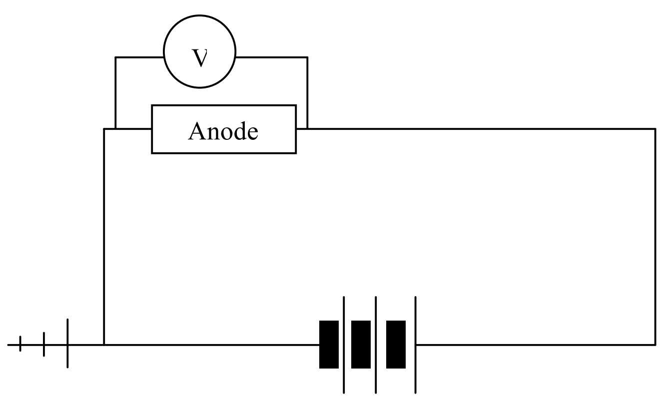


Fig. 4. Circuit diagram for anode

Notice that the circuits in figures 3 and 4 are both grounded; they are grounded to a common ground. The ground is needed to allow for accurate measurements in the anode and filament circuits, because otherwise it would not be possible to take useable measurements because there would be no common reference point between the circuits. When the coil current is varied, the beam of electrons starts out slightly more than 90º from the pins. Then as the current is increased, the beam of electrons curls in towards the pins and eventually intercepts each pin. When the anode voltage was varied, a similar behavior was noticed, however, it was not possible to make the electron beam intercept all the pins for higher amperages in the Helmholtz coils. When the direction of the current in the Helmholtz coils was reversed, the electron beam curled the opposite way.

In an another attempt to find out how the electron beam was affected by the presence of a magnetic field, the electron beam was set such that it intersected the third pin. A bar magnet was used to affect the electron beam. It is possible to use a bar magnet to form a helix with the electron beam. If the north end of the magnet is on the underside of the apparatus, with the magnet held horizontal to the table, the north end of the magnet pulls the electron beam in while the south end of the magnet pulls it away, thus producing a helix.

The apparatus was set to an anode voltage of 30V. The beam was not initially straight; therefore, the current through the coils was increased until the beam was straight. This current was recorded and then was subtracted from the values of the current at each pin to correct for the earth's magnetic field. The current is increased until it intercepts the outermost pin and then the other four pins. This procedure was repeated for voltages of 40, 50, and 60V. Table 1 shows the data for the twenty measurements of the potential.

| V = 30.0 | | V = 40.1 | |
|---|---|---|---|
| $I_0$ | 0.18 | $I_0$ | 0.13 |
| $I_1$ | 1.83 | $I_1$ | 2.03 |
| $I_2$ | 2.03 | $I_2$ | 2.22 |
| $I_3$ | 2.20 | $I_3$ | 2.60 |
| $I_4$ | 2.62 | $I_4$ | 3.00 |
| $I_5$ | 3.06 | $I_5$ | 3.43 |
| V = 50.0 | | V = 60.0 | |
| $I_0$ | 0.10 | $I_0$ | 0.10 |
| $I_1$ | 2.30 | $I_1$ | 2.49 |
| $I_2$ | 2.53 | $I_2$ | 2.76 |
| $I_3$ | 2.88 | $I_3$ | 3.09 |
| $I_4$ | 3.23 | $I_4$ | 3.53 |
| $I_5$ | 3.79 | $I_5$ | 4.13 |

Table 1. Data taken at fixed anode potential

The experiment was then repeated with the current through the Helmholtz currents held constant and the voltage through the anode potential. This method is less reliable because the setup that was used did not allow the voltage to be increased to the point where the electron beam would intercept the inner pins, therefore, that data was not available to be used to calculate the electron charge to mass ratio. Table 2 shows this data

| Amp = 2.5 | | Amp = 3.0 | |
|---|---|---|---|
| $V_1$ | 63.1 | $V_1$ | |
| $V_2$ | 48.3 | $V_2$ | 7.3 |
| $V_3$ | 38.5 | $V_3$ | 57.3 |
| $V_4$ | 27.8 | $V_4$ | 41.7 |
| $V_5$ | 19.9 | $V_5$ | 28.8 |
| Amp = 3.5 | | Amp = 4.5 | |
| $V_1$ | | $V_1$ | |
| $V_2$ | | $V_2$ | |
| $V_3$ | | $V_3$ | |
| $V_4$ | 58.0 | $V_4$ | |
| $V_5$ | 42.1 | $V_5$ | 73.1 |

Table 2. Data taken at fixed Helmholtz current

It is possible to determine the electron charge to mass ratio for each current and voltage value. Table 3 shows the data for the values for the charge to mass ratio for each measured value of voltage or amperage.

| | | |
|---|---|---|
| 1.65E+11 | 1.69E+11 | 1.51E+11 |
| 1.64E+11 | 1.68E+11 | 1.45E+11 |
| 1.8E+11 | 1.75E+11 | 1.5E+11 |
| 1.64E+11 | 1.8E+11 | 1.54E+10 |
| 1.7E+11 | 1.68E+11 | 1.56E+11 |
| 1.76E+11 | 1.68E+11 | 1.51E+11 |
| 1.8E+11 | 1.73E+11 | 1.51E+11 |
| 1.67E+11 | 1.74E+11 | 1.54E+11 |
| 1.64E+11 | 1.8E+11 | 1.62E+11 |
| 1.78E+11 | 1.51E+11 | 1.7E+11 |
| 1.67E+11 | 1.45E+11 | |

Table 3. Electron charge to mass ratio for each measurement made, each in C/kg

Figure 5 shows a histogram of the results for the electron charge to mass ratio.

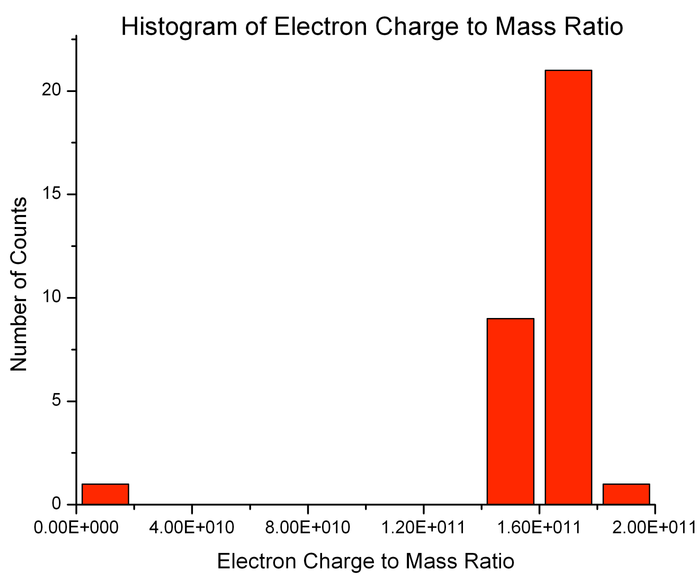


Fig. 5. Histogram of electron charge to mass ratio

The average value for electron charge to mass ratio was $1.60\times10^{11}$ C/kg with a standard deviation of $2.85\times10^{10}$. If the extraneous value, e/m=$1.54\times10^{10}$ C/kg, is removed, the mean electron charge to mass ratio is $1.65\times10^{11}$ C/kg with a standard deviation of $1.09\times10^{10}$. The percent difference between all the values and accepted value is 8.81% and the percent difference with the extraneous value removed is 6.16%. However, the values for the charge to mass ratio with constant current should be excluded from the calculations. Since the Helmholtz coils were not perfectly circular and the distance between the coils was not equal to the radius of the coils, the constant current through the Helmholtz coils did not produce the desired result. The Helmholtz coils were not set up correctly, therefore, the magnetic field created by the coils is not constant. When the voltage was held constant the current was changing and thus the magnetic field was changing, therefore, the changes in the magnetic field were cancelled out over many measurements. Since the assumption is made that the magnetic field is constant when calculating the charge to mass ration using the values for a constant current is false, the values for the charge to mass ratio calculated for those values should be ignored because they are based upon highly unreliable sources.

If the values of charge to mass ratio are used with excluding the values for the constant current, the mean of the values is $1.71\times10^{11}$ C/kg with a standard deviation of $5.91\times10^{9}$. The percent difference between this value and the accepted value is 2.47%. Figure 6 shows the histogram for these values of the charge to mass ratio. Notice how the counts all together when the constant voltage data was used. There is no point that is off by itself that is a factor of ten off from the other ones.

One should also consider the amount that the beam had to be offset to make it appear straight. This amount should be able to be found without measuring it. However, a graph of the data does not appear to give any information about $I_0$. Figure 7 shows the graph of each set of data versus the distance it is from the electron beam source.

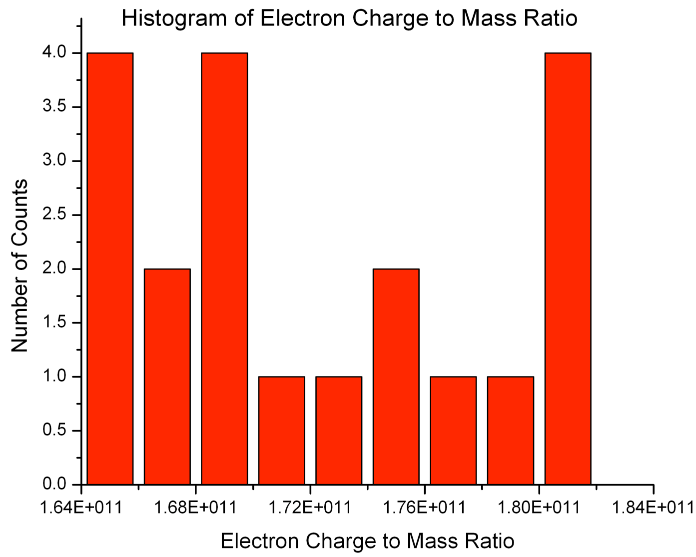


Fig. 6. Histogram for electron charge to mass ratio for only the data from the constant current

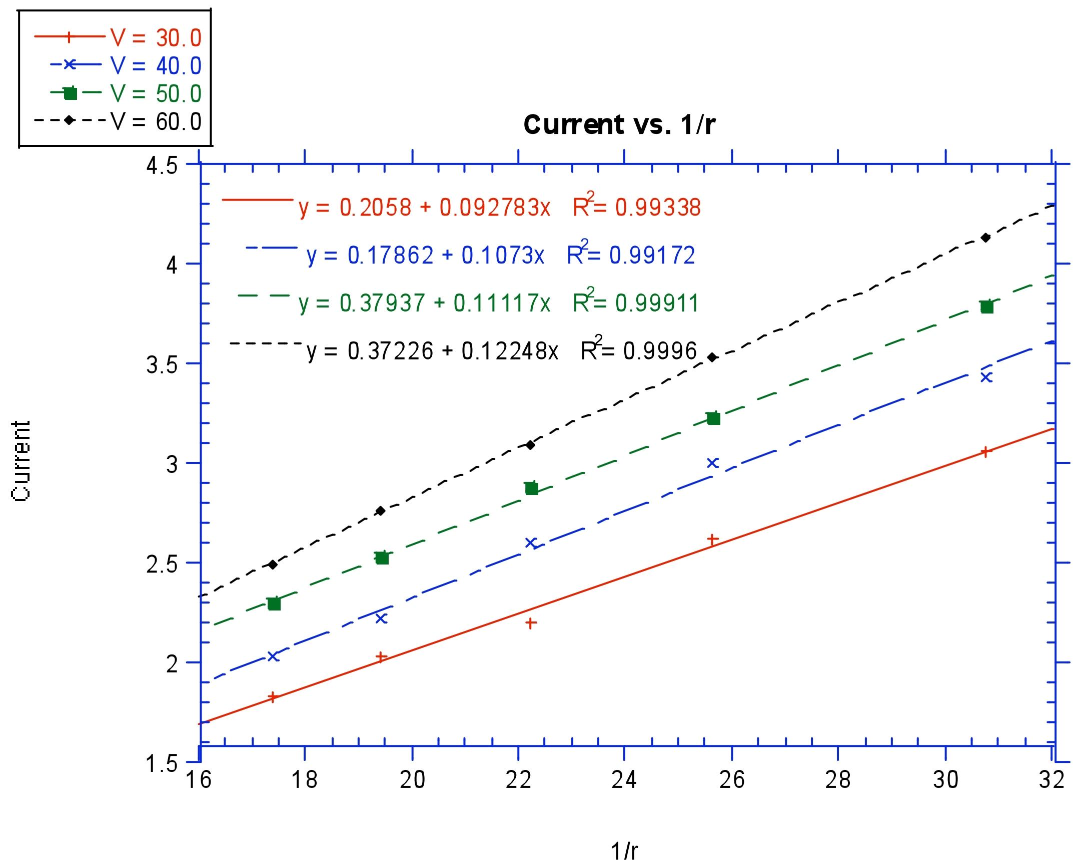


Figure 7. Current vs. 1 / distance from electron beam source

Notice that the y-intercept for a voltage of 30.0V is 0.20 and the measured value is 0.18. This is the only y-intercept that is close to $I_0$ value. Therefore, there should be no correlation between the y-intercept and the $I_0$ values. The $I_0$ value is the correction for imperfections in the Helmholtz coils and the earth's magnetic field that was not cancelled out by tilting the apparatus.

Another way to examine the problem is to look at the graph of the voltage versus the magnetic field multiplied by the pin distance, quantity squared. Figure 8 shows this graph.

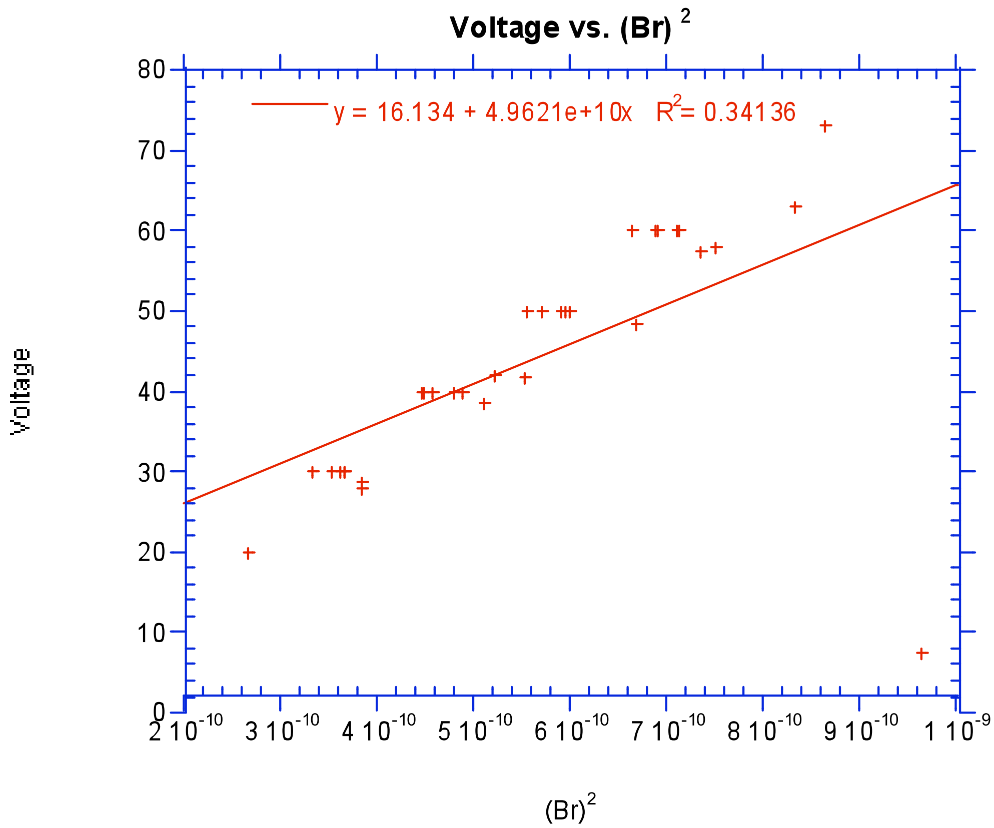


Fig. 8. Voltage vs. $(Br)^2$

Notice how one point is way off and does not fit the general trend. This skews the curve fit of the data. The slope of a linear least squares fit line on this graph, voltage over the magnetic filed multiplied by the pin distance, quantity squared. This is equal to one-half of electron charge to mass ratio, equation (13) shows this,

$$\frac{V}{(Br)^2} = \frac{e}{2m} = \frac{1}{2}\frac{e}{m}_{theoretical} = 8.7941 \times 10^{10} \qquad (13)$$

The slope of the linear least square line of the graph is $4.96 \times 10^{10}$, which is almost half of what the value should be. However, in the earlier, there was one point that was off by a factor of ten from the other values. If this point is again removed from the calculations, a better fit was obtained. Figure 9 shows this graph.

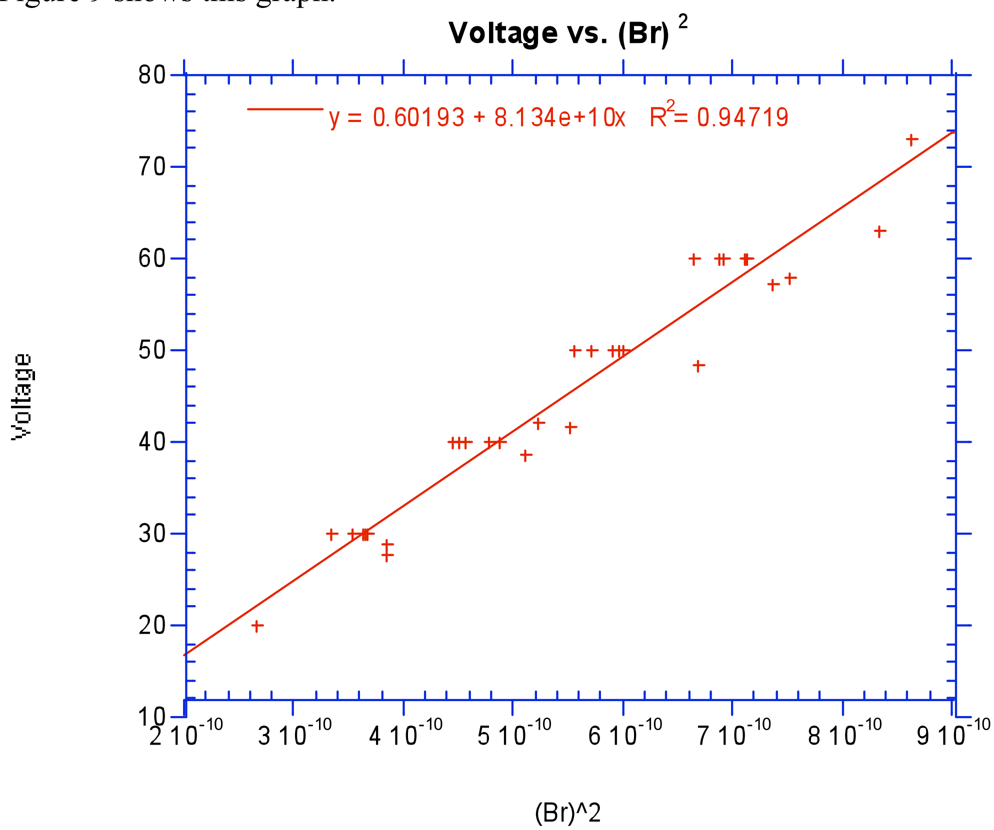


Fig. 9. Voltage vs. $(Br)^2$ with the erroneous point removed

The value for the slop of this line is very close to the expected value of one-half of the electron charge to mass ratio, it was $8.13 \times 10^{10}$, which has a percent error of 7.51%. If one considers only the values from the constant current experiment again, the slope of the line is $8.69 \times 10^{10}$. This has a percent error of 1.13% also, if one were to consider more than the allowed significant digits, percent difference between the two percent errors is, of the order $10^{-4}$, which is beyond the ability of the significant digits that are required. Therefore, there is no difference between the values for the electron charge to mass ratio from the slop of the line are more reliable then calculating a value for the ratio for each value and then taking the mean of the values. This data helps to show that the calculated values for the electron charge to mass ratio are precise and can be considered accurate.

There were several apparent systemic errors apparent in this experiment. It is difficult to account for all the factors that could have produced error in this experiment. There is the error that could have resulted from the earth's magnetic field. The earth's magnetic field may have affected the results from the experiment. There may have also been environmental factors involved with the experiment; it is unknown if the temperature of the room affected the electron beam or if the size of the Helmholtz coils varied slightly from changes in temperature. In addition, it is unknown if the angle that the apparatus was held at was held constant, it may have slipped slightly over the course of the experiment. There is also the parallax that occurred during the measurement of the diameter of the Helmholtz rings and the distance between them. While there were methods took to minimize the parallax errors, it is almost impossible to get rid of it completely. There was also some instrument drift, in this experiment; the value of the voltage would shift between two or three values. The current measured by the ammeter would also increase over time. There was also hysteresis when voltage or current was changed.

There were uncertainties related to every step in the calculations of the electron charge to mass ratio. The uncertainty in the voltmeter was ±0.1V. The uncertainty of the ammeter was ±0.01 Amps. The uncertainty of the meter stick is technically ±0.0005m, but it was considered±0.001m to account for any parallax in the measurements. After performing an uncertainty analysis, it was found that the electron charge to mass ratio is most accurately reported as $1.71\times10^{11} \pm 5.9\times10^{9}$ C/kg.

**Conclusion**

This experiment was designed to discover the electron charge to mass ratio. This experiment was designed to allow for an analysis of the errors involved in performing an experiment. In performing this experiment, the best estimate of the electron charge to mass ratio was $1.71\times10^{11}$ C/kg with a standard deviation of $5.91\times10^{9}$. the best percent error obtained was 2.47%. However, slope of the best-fit line from equation (13) produced the estimate of the electron charge to mass ratio, $1.74\times10^{11}$ C/kg, which has a percent error of 1.19%. This experiment could have been improved by redesigning the apparatus to allow it to follow designs more precisely. It was found that the Helmholtz coils were not circular in addition; the distance between the Helmholtz coils was not equal to the radius of the Helmholtz coils. Moreover, a lecture on the process of performing uncertainty calculations over many steps in a calculation.

**Answers to Questions**

1.) The magnetic field from the apparatus varied from $3.32\times10^{-4}$ T to $9.04\times10^{-4}$ T. This is within an order of ten from the earth's magnetic field. Therefore, the earth's magnetic field, which is approximately $1.\times10^{-4}$ T, is close enough that it could interfere with the experiment. However, several steps were taken to minimize its affects on the apparatus; however, the earth's magnetic field most likely had an affect, negligible as it may be, may have been a factor in the error in this experiment.

2.) Equation (3) uses the difference current because there must be a correction for the fact that the electron beam did not initially come straight out perpendicular the pins of the apparatus. If this corrected value were not used, the calculated value for the electron charge to mass ratio would be a too small, and thus inaccurate.

3.) Since a no-zero y-intercept was observed in figure 7, there must be some improvements made to the experiment. The best way to facilitate this would be to redesign the apparatus to allow for the correct dimensions. In addition, there might have been greater measures taken cancel out the earth's magnetic field, such as increasing the inclination of the apparatus.

4.) The shape of the Helmholtz coils affected the experiment since they were not circular; the magnetic field was not uniform as it was expected to be. Therefore, the fact that the Helmholtz rings were more of an oval than a circle, the result of this experiment was slightly skewed.

5.) J.J. Thomson measured the charge-to-mass ratio of the cathode rays by measuring how much they were deflected by a magnetic field and how much energy they carried. He found that the charge to mass ratio was over a thousand times higher than that of a proton, suggesting either that the particles were very light or very highly charged. Thomson's conclusions were bold: cathode rays were indeed made of particles, which he called "corpuscles", and these corpuscles came from within the atoms of the electrodes, themselves, meaning they were in fact divisible. Thomson imagined the atom as being made up of these corpuscles swarming in a sea of positive charge; this was his plum pudding model of the atom. Another attempt to find e/m can be done by Dunnington's Method, which involves the angular momentum and deflection due to a perpendicular magnetic field. In addition, The Magnetron Method- Using a GRD7 Valve (Ferranti valve), electrons are expelled from a hot tungsten wire filament towards an anode. The electron is then deflected using a solenoid. From the current in the solenoid and the current in the Ferranti Valve, e/m can be calculated. Another method is the fine beam Tube Method- Electrons are accelerated from a cathode to a cap shaped anode. The electron is then expelled into a helium filled ray tube, producing a luminous circle. From the radius of this circle, e/m is calculated. Thomson measured v by using the Schuster's estimate. In addition, in a lecture, Wiechert stated that he tried to find the electron's velocity by comparing the time taken by the cathode rays to pass along the tube with the time of swing of an electrical oscillation of the Hertzian type. The apparatus used in the experiment is similar to Thompson's design, except for that it can be aligned with the earth's magnetic field to minimize its affects on the expect. Therefore, our apparatus is better than Thompson's design.

6.) I have not performed the Millikan oil drop experiment, however, the accepted value for the charge of an electron, $1.602\times10^{-19}$ C can be used to find the mass of the electron.

$$\frac{1.602\times10^{-19}C}{m} = 1.74\times10^{11}\,{}^{C}\!/_{kg} \Rightarrow m = \frac{1.602\times10^{-19}C}{1.74\times10^{11}\,{}^{C}\!/_{kg}} = 9.21\times10^{-31}kg$$

which has a percent error of 1.10% of the theoretical value of $9.1093826\times10^{-31}$ kg?