

Millikan Oil Drop Experiment

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Abstract

The charge of an electron can be experimentally measured by observing an oil drop falling in the presence of an electric field. By using the rise and fall velocity of the oil drop, a multiple of the charge of an electron falls out of the equation. This experiment was one of the early experiments that were designed to find the secrets of the atom. The charge to mass ratio of the electron has already been found using cathode rays. In performing this experiment, it was found that charge of the electron is $1.5950 \times 10^{-19} \pm 3.44 \times 10^{-21}$ C.

Introduction

The Millikan oil-drop experiment was the first compelling experiment that measured the charge of an electron. It was first performed in a 1909 by Robert A. Millikan. The force on an electric charge in an electric field is equal to the product of the charge with the electric field. Millikan was able to measure both the amount of the electric force and the magnitude of the electric field on a charged oil drop and by analyzing the data, retrieve the charge on the oil drop. The oil drops become charged when they are influenced by the electric field and interact with the air. The Air molecules ionize the oil drops and cause them to become charged.

Millikan's early observations showed that the charge of an electron was a multiple of an elementary charge, but his initial findings were not convincing enough. Millikan also observed that the electric force and gravitational force were almost equivalent, therefore, knowing that the electron has a negative charge, the positive plate would have to be on top so that it cancels out the force of gravity, otherwise it add to it and would make it impossible to find the charge of the electron. In order to account for there being multiple charges on the oil drop, an ionization source was in the equipment that allowed the charge to be changed.

Assuming that the total force acting upon a falling particle is equal to $m (dv/dt)$, the differential equation for the total force acting upon the particle in the absence of the electric field is

$$\frac{dv}{dt} = m\ddot{x} - k\dot{x} \quad (1)$$

the differential equation for the particle rising in the presence of the electric field is given by the differential equation

$$\frac{dv}{dt} = m\ddot{x} - kv_r + Eqx \quad (2)$$

In both this situations, there is a small force exerted by air on the droplets, however, the density of the air is much less than the density of the oil drop. To determine the mass of the drop, the equation for the volume of a sphere and the definition of density to give,

$$m = \frac{4}{3}\pi a^3 \rho \quad (3)$$

where a is the radius of the drop and ρ is its density. To determine a , one uses Stokes' law which relate the radius of a spherical body to its velocity in a viscous medium. If η is the coefficient of viscosity of the medium, then

$$a = \sqrt{\frac{9\eta v_f}{2g\rho}} \quad (4)$$

However, there were some errors in Millikan's initial assumptions, and a corrected value for η is needed. This corrected value can be found by

$$\eta_{eff} = \frac{\eta}{\left(1 + \frac{b}{pa}\right)} \quad (5)$$

where b is a constant equal to 8.20×10^{-3} Pa·m.

The value for η can be experimentally calculated by finding the fall and rise velocity, the potential across the plates, and the barometric pressure, and assuming the theoretical value for the charge on an electron, and solving the equations for η , one can find η .

In performing this experiment, the total amount of charge, $q=ne$ is equal to

$$q = ne = \frac{mg(v_f - v_r)}{Ev_f} \quad (6)$$

By substituting equation (5) into equation (4), it can be found that,

$$a = \frac{3}{2} \sqrt{\frac{2a\eta p v_f}{(ap + b)g\rho}} \quad (7)$$

and then by solving for a , it is seen that,

$$a = \sqrt{\left(\frac{b}{2p}\right)^2 + \frac{9\eta v_f}{2g\rho}} - \frac{b}{2p} \quad (8)$$

By substituting this result in equation (3), it can be seen that,

$$m = \frac{4}{3} \pi \left[\sqrt{\left(\frac{b}{2p}\right)^2 + \frac{9\eta v_f}{2g\rho}} - \frac{b}{2p} \right]^3 \rho \quad (9)$$

By substituting this result into equation (6), one finds that the charge of the oil drop is given by

$$q = ne = \frac{4}{3} \pi \rho g \left(\frac{v_f + v_r}{Ev_f} \right) \left[\sqrt{\left(\frac{b}{2p}\right)^2 + \frac{9\eta v_f}{2g\rho}} - \frac{b}{2p} \right]^3 \quad (10)$$

In the Millikan oil drop experiment, the plates are separated by a distance d , are charged with a potential difference, V . There electric field between the plates us given by,

$$E = \frac{V}{d} \quad (11)$$

in equation (10), ρ is the density of the mineral oil in kg/m^3 , g is the acceleration due to gravity, η is the viscosity of the air in $\text{N}\cdot\text{s/m}^2$, p is the barometric pressure in Pa, v_f is the fall velocity in m/s, v_r is the rise velocity, and V is the plate potential in volts. Table 1 shows the value used for these values.

b	$8.20 \times 10^{-3} \text{ Pa}\cdot\text{m}$
d	$7.57 \times 10^{-3} \text{ m}$
V	501 V
E	$6.62 \times 10^4 \text{ V/m}$
ρ	886 kg/m^3
p	98712 Pa

Table 1. Values for the constants in equation (10)

The accepted value for e is $1.602 \times 10^{-19} \text{ C}$.

Experimental Procedures

In performing this experiment, first the apparatus was tested to make sure that it was working properly. A grid was marked off in 0.5 mm major intervals and 0.1 mm minor intervals. A stopwatch was used to find the rise and fall velocity of the oil drops. Six different drops were observed in this experiment. Table 2 shows the mean rise and fall velocities for each drop. These values were placed in equation (10) to find the charge of the drop. Table 3 shows the values for the charge of the drops. Since n has to be an integer, I took the floor and ceiling function on each value to find the integer value above and below the real value, this allows for a more accurate sampling because the value for n was not always near an integer.

Drop & Dir.	Velocity (m/s)
1U	3.445×10^{-4}
1D	3.436×10^{-4}
2U	4.032×10^{-4}
2D	4.038×10^{-4}
3U	3.139×10^{-4}
3D	3.031×10^{-4}
4U	2.048×10^{-4}
4D	2.036×10^{-4}
5U	3.272×10^{-4}
5D	3.227×10^{-4}
6U	2.124×10^{-4}
6D	1.886×10^{-4}

Table 2. Mean velocities for the six oil drops

Charge (in C)	n value
1.5887×10^{-19}	39
1.6305×10^{-19}	38
1.5805×10^{-19}	50
1.6128×10^{-19}	49
1.5550×10^{-19}	34
1.6021×10^{-19}	33
1.5439×10^{-19}	18
1.6347×10^{-19}	17
1.5818×10^{-19}	36
1.6269×10^{-19}	35
1.5458×10^{-19}	18
1.6367×10^{-19}	17

Table 3. Electron charge and n value

Table 3 shows the mean values along with the standard deviations and the percent error relative to the accepted value.

	Charge	Std. Dev.	% Error
ceiling	1.57×10^{-19}	2.00×10^{-21}	2.25%
floor	1.62×10^{-19}	1.37×10^{-21}	1.37%
both	1.59×10^{-19}	3.44×10^{-21}	0.44%

Table 4. Mean charge of an electron, standard deviation, and percent error.

The best results are from when both the floor and the ceiling values for n are averaged together. They only have a 0.44 percent error with the accepted value for the charge of an electron.

In observing the drops, sometimes a drop would move quickly upward when the plates were charged. It is most likely that the drop has a relatively large number of electrons and thus is heavily attracted to the positive plate and thus, the electric field greatly outweighs the gravitational force. On the other hand, if a drop drifts slowly downward between the charged plates, it has a relatively small charge and the gravitational force is almost balanced out by the electric field. If the drop drifts slowly downward between the uncharged plates, a residual charge in the plates is affecting fall rate in the oil drops with a small charge.

It appears that the main source of error in this experiment was random, that the errors were mostly cancelled out by taking the mean of several values. Taking the floor and the ceiling of the n values also helped in eliminating the random errors. Since the air pressure was an integral of this experiment, the assumption was made that the air pressure remained constant. This is not entirely the case because the air pressure is fluctuating. There is also the resolution of the equipment. The stopwatch was only accurate to ± 0.1 s and the grid had an accuracy of ± 0.1 mm. The stopwatch also made the user guess the hundredths place of the time. There was also the problem of starting and stopping the stopwatch at the precise moment that the drop crossed a minor grid line. Whether the observer held it or if someone else held it, there was human reaction time that made the time always be slightly off. There was also parallax involved in reading the time and observing the drops. Our apparatus is smaller than the originally used by Millikan. Figure 1 shows the original Millikan apparatus compared to the apparatus that was used in this experiment. The apparatus that was used in this experiment also most likely has better optical

equipment and has a more precise measurements of voltage used to charge the plates that in Millikan's apparatus.

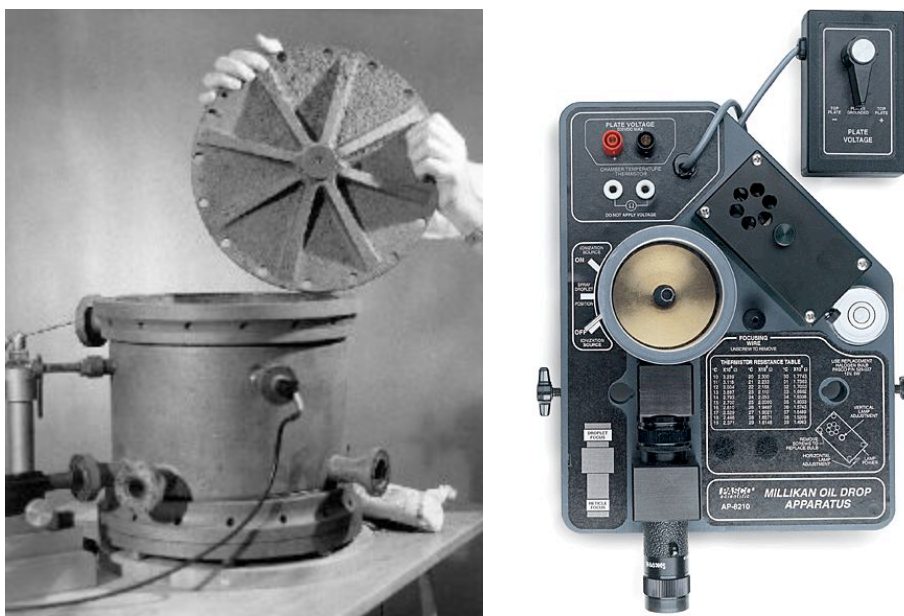


Fig. 1. Millikan's apparatus and the apparatus used in this experiment.¹

Conclusion

This experiment was designed to discover the charge of an electron by observing the motion on a charged oil drop in the presence of an electric field. By using several constants, the charge on each drop falls out easily. The best average value for the charge on the electron was $1.59 \times 10^{-19} \text{ C}$ with a standard deviation of $3.44 \times 10^{-21} \text{ C}$. This has a percent error of 0.44%. The best single calculation of the charge of the electron was $1.602 \times 10^{-19} \pm 1.3 \times 10^{-21} \text{ C}$, which has a percent error of 0.01%. This experiment could have been improved by performing repair on the equipment. There were major difficulties in trying to get the oil drops into the chamber. The rig

¹ Millikan apparatus taken from http://en.wikipedia.org/wiki/Image:Millikan%E2%80%99s_oil-drop_apparatus_2.jpg, PASCO apparatus taken from <http://store.pasco.com/pasco/wizards/largerimage.cfm?PN=AP-8210>

most likely needs a good cleaning and a recalibration. The atomizer seemed to be clogged and should be unclogged or replaced in order to make the experiment more reliable. Another improvement that could be made is that a digital stopwatch would prevent the parallax from reading the analog stopwatch.

In the interest of find the mass of the electron, the electron charge to mass ratio was found earlier to be 1.74×10^{11} C/kg; the mass of the electron can be found

$$\frac{1.59 \times 10^{-19} \text{C}}{m} = 1.74 \times 10^{11} \text{ C/kg} \Rightarrow m = \frac{1.59 \times 10^{-19} \text{C}}{1.74 \times 10^{11} \text{ C/kg}} = 9.14 \times 10^{-31} \text{kg}$$

This has a percent error of 0.34% with the accepted value of $9.10938188 \times 10^{-31}$ kg.