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PHY 220 Computation Assignment 1
Velocity Dependent Forces and Terminal Velocity
September 27, 2006

Abstract
This assignment, velocity dependent forces and terminal velocities, we learned how to model a ball falling in one dimension with both linear and quadratic drag. We used the Runge-Kutta routine provided for us to solve the differential equations that describe this motion. Using this program, I found that the terminal velocity of a particle with a radius of 0.1 *μm* was 1.186 *mm/s*. I also found that the terminal velocity of a particle with a radius of 0.1 *m* was 204.4 *m/s*.

Introduction
This assignment was designed to coincide with the class discussion on both linear and quadratic drag and solving differential equations. The Runge-Kutta routine can solve differential equations in the form of

$$\frac{dQ_n}{ds} = f_n(s, Q_1, Q_2 ...), \tag{1}$$

where s is the independent variable and the Q_n are the dependent variables. Since in this assignment, we are dealing with strictly one dimensional motion, our first two coupled differential equations of motion are

$$\frac{dv}{dt} = a(x, v, t) \quad ; \quad \frac{dx}{dt} = v. \tag{2}$$

In terms of Q_n and s, we chose $Q_1 = x$, $Q_2 = v$, and $s = t$, thus producing,

$$\frac{dQ_2}{ds} = a(Q_1, Q_2, s) \quad ; \quad \frac{dQ_1}{ds} = Q_2. \tag{3}$$

We will investigate the effects of the size of a sphere on its terminal velocity. We assume that we are dropping spherical ball with a density (ρ) of 8960 kg/m^3 of varying radii, r, out of a ballon of height h. Therefore, the force of gravity on the sphere is given by

$$F_g = -m \cdot g = -\frac{4}{3} \cdot \pi \cdot r^3 \cdot \rho \cdot g, \tag{4}$$

and the drag force is given by

$$F_d = -C_1 \cdot v - C_2 \cdot |v| \cdot v, \tag{5}$$

where $C_1 = 3.1\times10^{-4} \cdot r$ and $C_2 = 0.88 \cdot r^2$. Therefore, the equations of motion are

$$\frac{dx}{dt} = v, \tag{6}$$

and

$$\frac{dv}{dt} = -g - \left(\frac{C_1}{m}\right) \cdot v - \left(\frac{C_2}{m}\right) \cdot |v| \cdot v, \tag{7}$$

where

$$m = \frac{4}{3} \cdot \pi \cdot r^3 \cdot \rho. \tag{8}$$

In class, we found that the quadratic drag dominated when the radius was reasonable large, like larger than a bb. Therefore, by ignoring the linear drag term and balancing the force of gravity with the quadratic drag term, we find that the terminal velocity is given by

$$v_t = 646 \cdot \sqrt{r} \; {}^{m}\!/_{s}, \tag{9}$$

where we substituted in ρ =8960 kg/m^3 and g = 9.8 m/s^2. If on the other hand, the particles are extremely small, like dust particles or smaller, the linear drag term dominates. Therefore, by balancing the linear drag term, we find that the terminal velocity is given by

$$v_t = 1.19 \times 10^9 \cdot r^2 \; {}^{m}\!/_{s}. \tag{10}$$

Results and Discussion

Table 1 shows the data for the particle's terminal velocity based upon its radius. I took the absolute value for the velocity for all the values, because it is impossible to take the logarithm of negative number.

Radius (m)	Terminal Velocity (m/s)
1.00E-07	1.1864E-05
3.00E-07	1.0677E-04
1.00E-06	1.1864E-03
3.00E-06	1.0676E-02
1.00E-05	1.1824E-01
3.00E-05	9.8509E-01
1.00E-03	4.9390E+00
3.00E-04	1.0625E+01
1.00E-03	2.0268E+01
3.00E-03	3.5350E+01
1.00E-02	6.4629E+01
3.00E-02	1.1197E+02
1.00E-01	2.0443E+02

Table 1. Radius of sphere and its terminal velocity

Figure 1 shows the graph of the velocity versus time for the sphere of varying radii with both axes being on a logarithmic scale. Notice how that as radius increases, the terminal velocity increases.

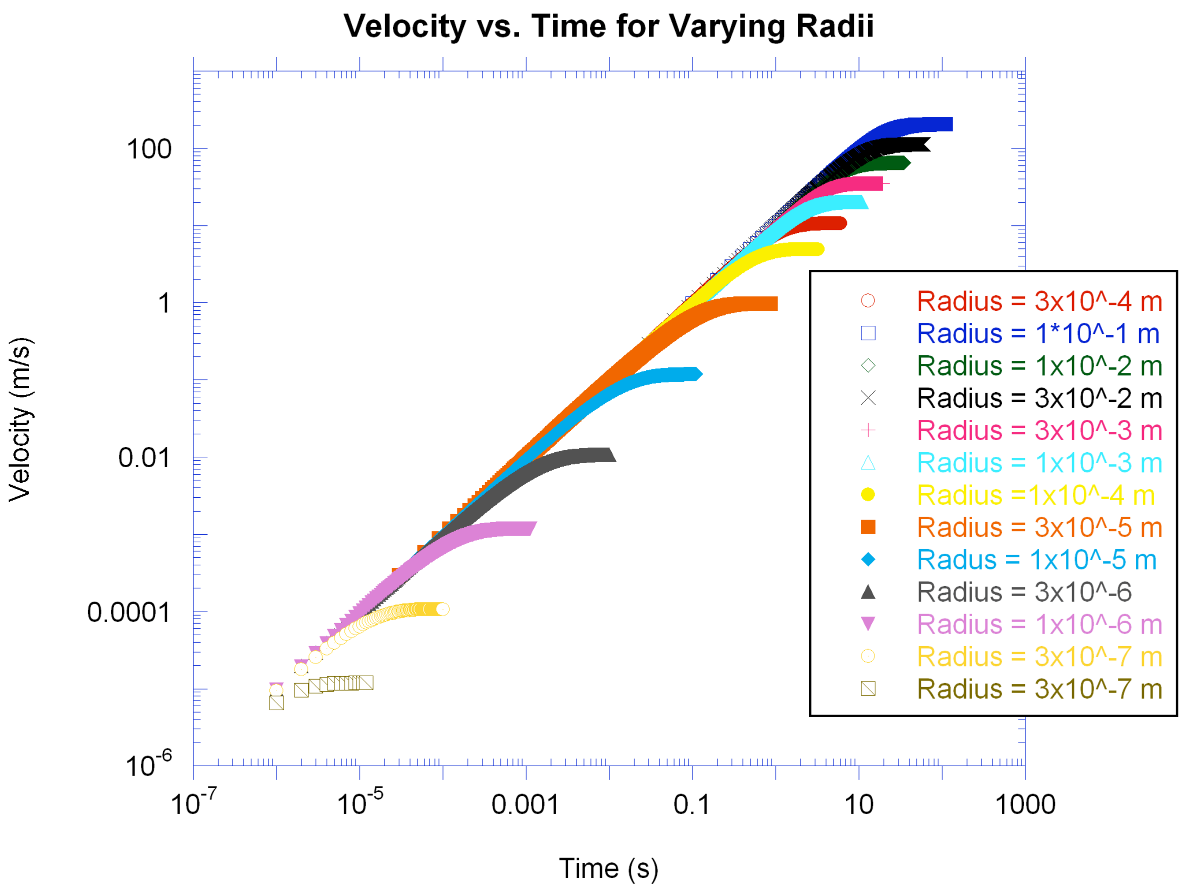

Fig. 1. Velocity vs. Time for varying radii

Figure 2 depicts the terminal velocity versus the radius for radii less than 30 μm. Notice how the curve fit for the data is $v_t = 1\times10^9\cdot r^{1.99}$ m/s. This is very close to the calculated value of $v_t = 1.19\times10^9\cdot r^2$ m/s.

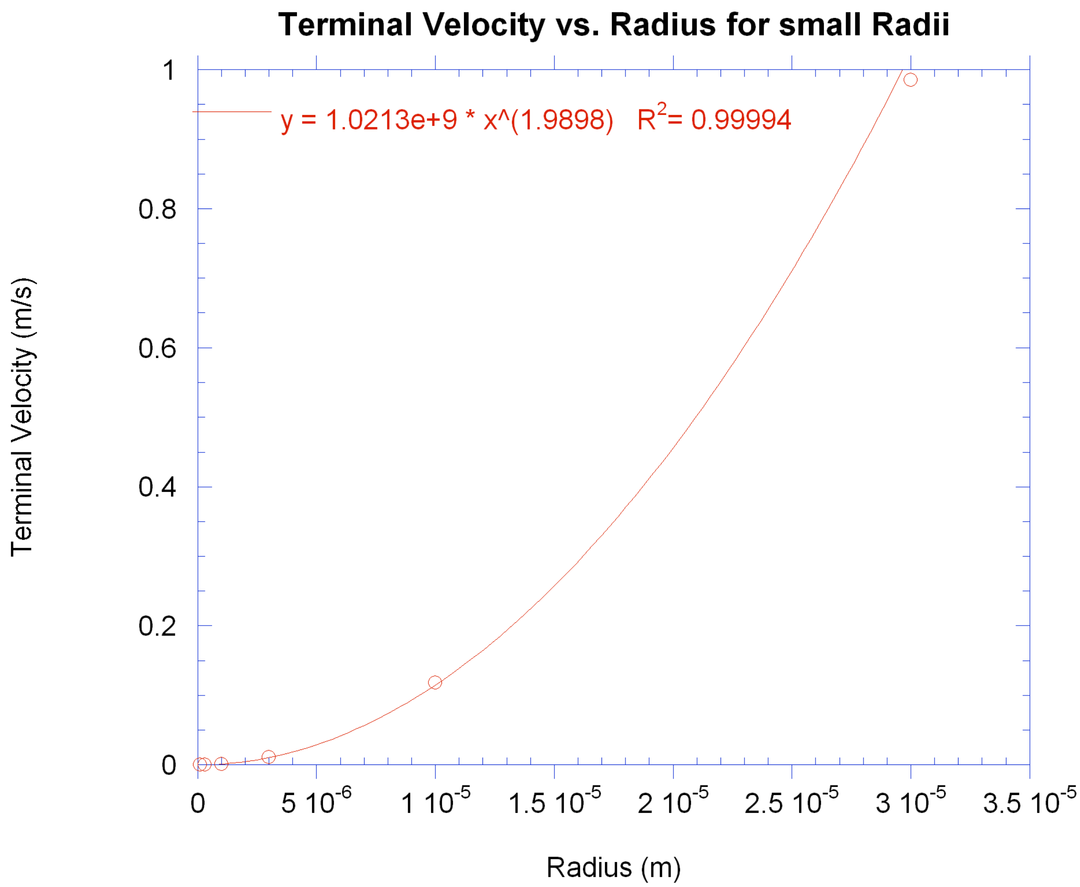

Fig. 2. Terminal velocity vs. radius for relatively small radii

On the other hand, when one studies the radii greater than 30 μm, one finds similar results. The curve fit for this graph is $v_t = 649.97 \cdot r^{0.5}$, which is very close to the calculated value of is $v_t = 646 \cdot r^{0.5}$. Fiure 3 show the graph of this situation.

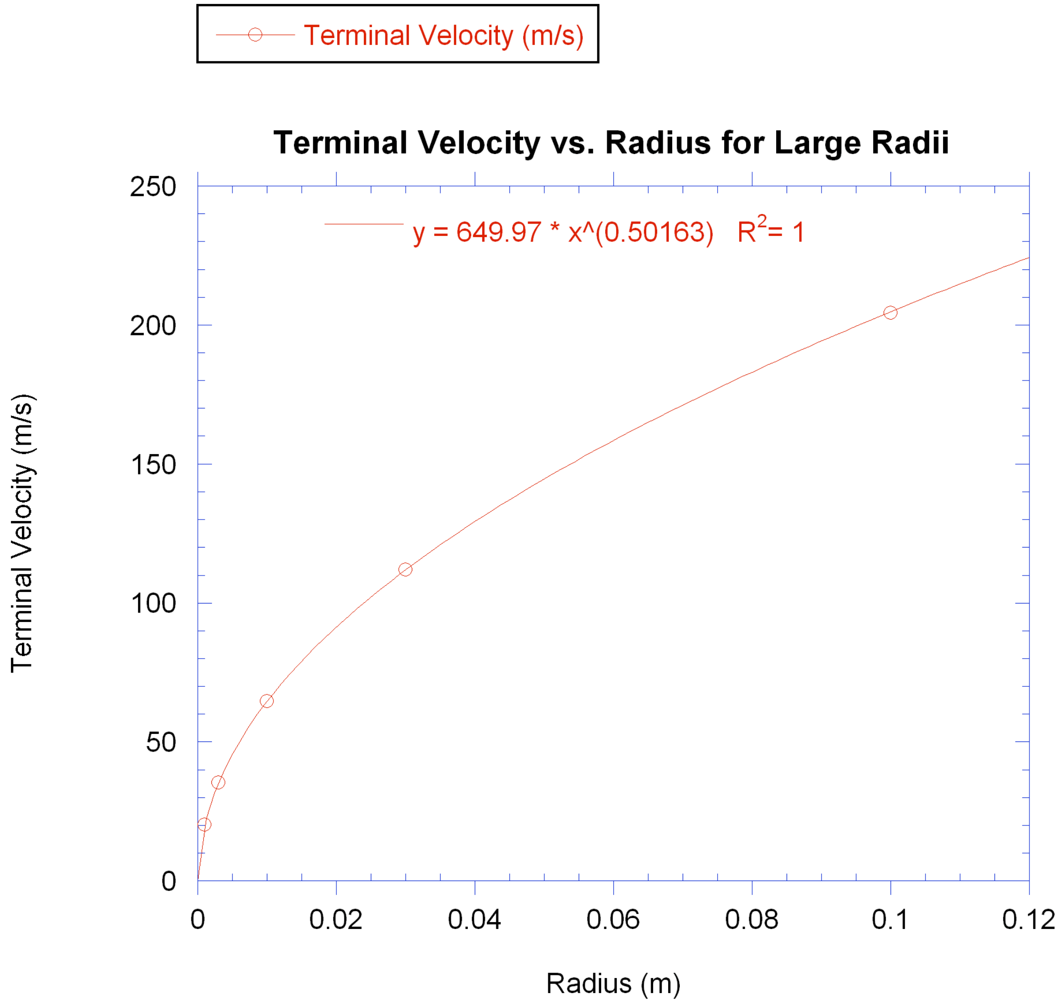

Fig. 3. Terminal velocity vs. radius for relatively large radii

Figure 4 shows the graph of the terminal velocity versus the radius of the sphere. The curve fit for this graph does not match the data as well as the other fits did, especially for the smaller radii. I believe that this is due to the fact that the quadratic drag term is very small. Figure 5 shows the graph of the terminal velocity function. Notice how they are appear to be similar.

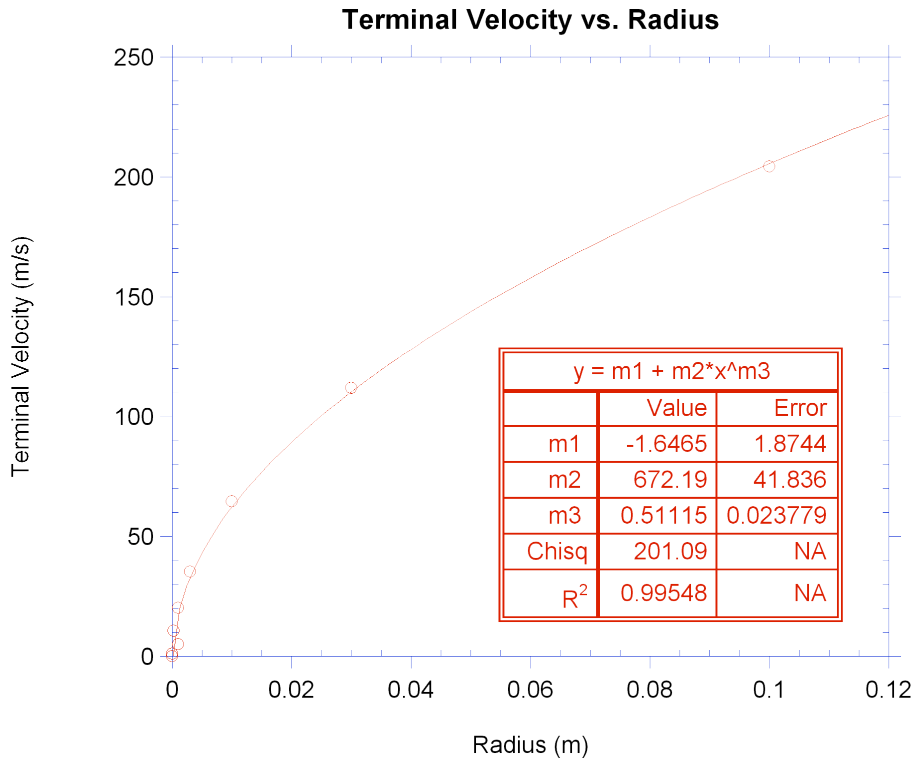

Fig. 5. Terminal velocity vs. varying radii

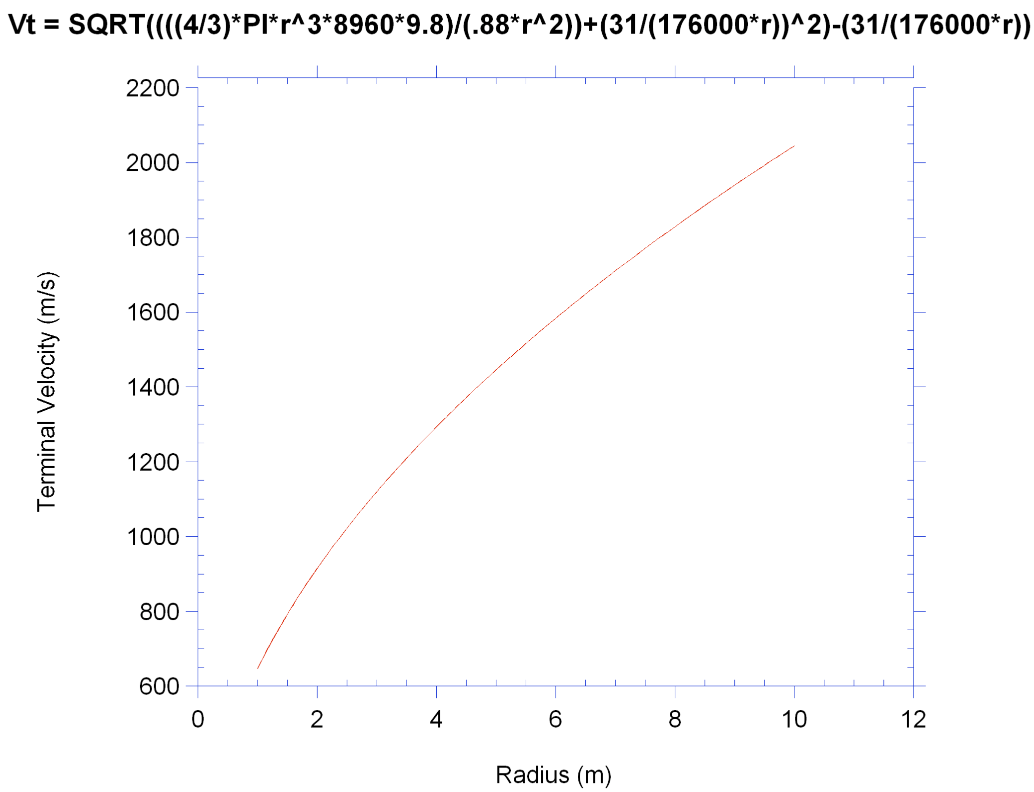

Fig. 6. Graph of the terminal velocity function

Conclusion

This assignment taught me how to use differential equations to solve the free fall in one dimension. I found that the terminal velocity increased as the radius of the sphere increased. I also found that for a relatively small radius, one could ignore quadratic drag. In addition, I found that for a relatively large radius, one could ignore linear drag.