

MATH 150
Sample Problems

Answer the following problems completely. You are **not permitted** to use **any calculator**.

1. For the given functions, determine the interval(s) in which the function is increasing or decreasing. Then, determine the intervals in which $f(x)$ is concave up or concave down.

(a) $f(x) = x(x - 5)^4$

Answer: $f(x)$ is increasing on $(-\infty, 1)$ and $(5, \infty)$; $f(x)$ is decreasing on $(1, 5)$; $f(x)$ is concave down on $(-\infty, 2)$; $f(x)$ is concave up on $(2, 5) \cup (5, \infty)$

(b) $g(x) = \sin x - \cos x$ on $[0, 2\pi]$

Answer: $g(x)$ is increasing on $[0, 3\pi/4) \cup (7\pi/4, 2\pi]$; $g(x)$ is decreasing on $(3\pi/4, 7\pi/4)$; $g(x)$ is concave down on $(\pi/4, 5\pi/4)$; $g(x)$ is concave up on $[0, \pi/4) \cup (5\pi/4, 2\pi]$

2. Differentiate the following functions.

(a) $f(x) = \sin^4(x) + \sin(x^4) + \sin(4x)$

Answer: $f'(x) = 4[\sin^3(x)][\cos(x)] + [\cos(x^4)][4x^3] + [\cos(4x)][4]$

(b) $h(x) = \sqrt{x^4 + x^2}$

Answer: $h'(x) = (1/2)(x^4 + x^2)^{-1/2}(4x^3 + 2x) = x(2x^2 + 1)(x^4 + x^2)^{-1/2}$

(c) $y = \tan(\cos(2x))$

Answer: $y' = [\sec^2(\cos(2x))][-\sin(2x)](2)$

3. Find the slope of the tangent line to the given curve at the given point.

(a) $x^4 + 16y^4 = 32$, $(x, y) = (2, 1)$

Answer: $m = -1/2$

(b) $2xy + x^2y^2 = (x + y)^2 + 8$, $(x, y) = (2, 2)$

Answer: $m = -1$

(c) $y = \sqrt{4x^2 + 1}$, $(x, y) = (0, 1)$

Answer: $m = 0$

4. Using differentials, approximate the value of $\sqrt[3]{27.27}$.

Answer: $\sqrt[3]{(27.27)} \approx 3.01$

5. The distance traveled by an object (measured in feet) at time t (measured in seconds) is given by the function

$$s(t) = \frac{1}{12}t^4 - t^3 + \frac{9}{2}t^2 + 20.$$

Find the velocity of the object when the acceleration is zero.

Answer: Velocity is equal to 9 feet/second.

6. A rectangular field is to be fenced along the bank of the river. Due to possible water hazards, the amount of fencing on the side along the river is double the amount needed to fence the side parallel to the river. If 300 yards of fencing is available, find the dimensions of the field with the largest area.

Answer: 50 yards by 75 yards, where the length of the side along the river is 50 yards