MATH567 Project 5: due on Thursday, Aug 2, 2018

Problem 1 (10 pts): Lax-Wendroff and leapfrog schemes.
The m-file `advection_LW_pbc.m` implements the Lax-Wendroff method for the advection equation on $0 \le x \le 1$ with periodic boundary conditions.

(a) Modify the m-file to create a version `advection_lf_pbc.m` implementing the leapfrog method and verify that this is second order accurate. Note that you will have to specify two levels of initial data. For the convergence test set $U_j^1 = u(x_j, k)$, the true solution at time k.

(b) Modify `advection_lf_pbc.m` so that the initial data consists of a wave packet

$$\eta(x) = \exp(-\beta(x-0.5)^2)\sin(\xi x) \tag{Ex0.0a}$$

Work out the true solution $u(x,t)$ for this data. Using $\beta = 100$, $\xi = 80$ and $U_j^1 = u(x_j, k)$, test that your leapfrog code still exhibits second order accuracy for k and h sufficiently small.

Problem 2 (10 pts): Convergence of SOR.
The m-file `iter_bvp_Asplit.m` implements the Jacobi, Gauss-Seidel, and SOR matrix splitting methods on the linear system arising from the boundary value problem $u''(x) = f(x)$ in one space dimension.

(a) Run this program for each method and produce a plot similar to Figure 4.2.

(b) The convergence behavior of SOR is very sensitive to the choice of ω (`omega` in the code). Try changing from the optimal ω to $\omega = 1.8$ or 1.95.

(c) Let $g(\omega) = \rho(G(\omega))$ be the spectral radius of the iteration matrix G for a given value of ω. Write a program to produce a plot of $g(\omega)$ for $0 \le \omega \le 2$.

(d) From equations (4.22) one might be tempted to try to implement SOR as

```
for iter=1:maxiter
   uGS = (DA - LA) \ (UA*u + rhs);
   u = u + omega * (uGS - u);
   end
```

where the matrices have been defined as in `iter_bvp_Asplit.m`. Try this computationally and observe that it does not work well. Explain what is wrong with this and derive the correct expression (4.24).