MATH567 Project 1: due in class on Monday, July 9, 2018

Problem 1 (5 pts): accurate approximation of π (`pi` in MATLAB).
Write a MATLAB function (use either `for` or `while` loop) to compute an approximation value of π with a given accuracy tol. You may only use basic arithmetic $(+, -, *, /)$ and logic operations in codes. (Hint: use $\tan(\pi/4) = 1$ and truncate the Taylor series of $\arctan(x)$ at $x = 1$.)

Problem 2 (5 pts): solve a SIR model with MATLAB's ODE solvers (`ode45,ode15s,etc...`)
Write a MATLAB script file (call ODE solvers) to solve a SIR (susceptible (S), infected (I), and resistant (R)) model explained in the following webpage: `https://www.maa.org/book/export/html/115609`
Using the same parameters, your solution plot in MATLAB should match with the given plot.

Another online demo: `http://www.public.asu.edu/~hnesse/classes/sir.html`
More background: `http://mat.uab.cat/matmat/PDFv2013/v2013n03.pdf`
Advanced: `http://leonidzhukov.net/hse/2014/socialnetworks/papers/2000SiamRev.pdf`

Problem 3 (10 pts): (use of `fdstencil.m` from [TB] Chapter 1)

(a) Use the method of undetermined coefficients to set up the 5×5 Vandermonde system that would determine a fourth-order accurate finite difference approximation to $u''(x)$ based on 5 equally spaced points,

$$u''(x) = c_{-2}u(x-2h) + c_{-1}u(x-h) + c_0 u(x) + c_1 u(x+h) + c_2 u(x+2h) + O(h^4).$$

(b) Compute the coefficients using the MATLAB code `fdstencil.m` available from the book website, and check that they satisfy the system you determined in part (a).

(c) Test this finite difference formula to approximate $u''(1)$ for $u(x) = \sin(2x)$ with values of h from the array `hvals = logspace(-1, -4, 13)`. Make a table of the error vs. h for several values of h and compare against the predicted error from the leading term of the expression printed by `fdstencil`. You may want to look at the m-file `chap1example1.m` for guidance on how to make such a table.

Also produce a log-log plot of the absolute value of the error vs. h.

You should observe the predicted accuracy for larger values of h. For smaller values, numerical cancellation errors in computing the linear combination of u values impacts the accuracy observed.