A Brief Introduction to Scientific Notation

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Physical scientists frequently need to deal with numbers which are either very large or very small. In order to make this easier, we have created a system of writing down numbers that makes this more convenient. We call this system "scientific notation." (There is a variation to this called "engineering notation." The fundamentals are the same, just the way it's done is a little different. We won't worry about this alternative system.)

(A quick note on the way I do things: I like to start a development of an idea at the very bottom. So I tend to include some technical details that are not strictly necessary. You can skip the next three paragraphs and go right to the "recipe" if you'd rather just get right into it.)

From the standpoint of number theory, scientific notation is made possible by the fact that we use a "positional notation" for writing down numbers. To see what I mean by this, consider a real number, say 123.4. We start with the "radix"—in this case, the decimal point. (Many Europeans use a comma as a radix where Americans use a decimal point. This is a source of confusion!) We know, in this case, that the "1" means "one hundred" because it is in the third position to the right of the radix (we start counting with zero at the first location—this is a bit confusing). We know that the "2" is "twenty" because it is in the second place. And so on. In a positional number system, when we write down a number, what we're really doing is writing down a recipe (an algorithm) that says "multiply the integer in each slot by the power of ten associated with that slot." (As an aside, think about the role of zero in such a system: Zero becomes just a placeholder. Something to tell us that a particular slot is empty. People *have* used systems in which empty slots were simply left empty. Zero was just too spooky a concept for them! Of course, this is very cumbersome and prone to severe errors.)

Now, note that, in base 10, multiplying by 10 results in the entire number being shifted to the left by one space. So we would have $123.4 \times 10 = 1234$. Similarly, dividing by 10 shifts the whole number to the right. We would have, in this case, $123.4 \div 10 = 12.34$. Dividing again, we would have $123.4 \div 100 = 12.34 \div 10 = 1.234$.

This gives us the key to scientific notation: If we have a large number, we divide it by 10 enough times so that there is only one nonzero digit to the left of the decimal point. Of course, we don't *really* want to divide, so we then multiply this new number by the power of 10 that we need to get the original number back. Using the example above, we could say $123.4 = 1.234 \times 100 = 1.234 \times 10^2$. We will do this whenever we have a number that is inconveniently large.

The recipe we will follow is this: For any number, we write that number down as a factor that is greater than or equal to 1 (or less than or equal to –1 for negative numbers) and multiply this by 10 raised to an integer power. Written as an equation, this is $x = a \times 10^b$ where $a \geq 1$ (or $a \leq -1$ for negative x) and b is an integer (positive or negative). The factor a is called the "mantissa" and b is called the "exponent" (the exponent is sometimes called the "characteristic").

This sounds more difficult than it is. It's actually quite easy. When given a big number, just rewrite it by sticking the decimal point right after the first digit. So, if you have 123.4, just rewrite it as 1.234—this is the mantissa. Now you'll need the exponent. Go back to the original number. How many places did you have to move the decimal point? Just count. It got slid over two places when we did this. So our exponent is 2. We write this number as 1.234×10^2.

What about small numbers? No problem: We just do the same thing, but with a *negative* exponent. So, let's say we have 0.001234. We would write this as 1.234×10^{-3}. Where did I get that –3? With the original number, I started at the decimal point and counted how many places to the right I had to go to hit the first number that isn't zero. Since I went to the right, this is a negative power of ten.

This system is a bit scary, at first, but with a little practice, it becomes very easy and totally natural.

Using scientific notation, we can write down ridiculously large numbers, such as "Avogadro's Number," with ease. Without scientific notation, this would be $N_A = 602000000000000000000000$. With scientific notation, this becomes $N_A = 6.02 \times 10^{23}$. We can also write down ridiculously small numbers with similar ease, for example "Planck's constant." Without scientific notation, this is $h = 0.0000000000000000000000000000000006626$. With scientific notation, it becomes 6.626×10^{-34} (those of you who are paying attention will notice that I've left off the units from this—normally an unforgivable offense!). Clearly, this system helps us to write numbers down more easily. But its power goes beyond that. Scientific notation allows us to calculate very easily.

Once we have numbers in scientific notation, multiplication, division, raising-to-powers, and root-taking become very much easier. Here are the rules:

- When multiplying, multiply the mantissas and *add* the exponents.

- When dividing, divide the mantissas and *subtract* the exponent of the denominator from the exponent of the numerator.

- When raising to a power, raise the mantissa to the power and *multiply* the exponent by the power.

- When taking a root, take the root of the mantissa and *divide* the exponent by the "root."

Here are some examples:

a) $(4.1 \times 10^2) \times (2.3 \times 10^3) = (4.1 \times 2.3) \times 10^{2+3} = 9.43 \times 10^5$

b) $(4.1 \times 10^2) \div (2.3 \times 10^3) = \dfrac{4.1}{2.3} \times 10^{(2-3)} = 1.7826 \times 10^{-1}$

c) $10^2 + 10^3 = 100 + 1000 = 1100 = 1.1 \times 10^3$

d) $\dfrac{10^{35} \times 10^{80}}{10^{12} \times 10^{-3}} = \dfrac{10^{(35+80)}}{10^{(12-3)}} = \dfrac{10^{115}}{10^9} = 10^{(115-9)} = 10^{106}$

e) $\dfrac{(6 \times 10^{23}) \times (1 \times 10^{-19})}{2 \times 10^3} = \dfrac{6 \times 1}{2} \times \dfrac{10^{23} \times 10^{-19}}{10^3} = 3 \times \dfrac{10^{(23-19)}}{10^3}$

$= 3 \times \dfrac{10^4}{10^3} = 3 \times 10^{(4-3)} = 3 \times 10^1 = 3 \times 10 = 30$

Notice that scientific notation buys us *nothing* when it comes to adding and subtracting.

One of the simplest and most powerful features of scientific notation, however, is that it allows us to compare the "order of magnitude" of two numbers rapidly. The order of magnitude of a number is its general size. This is usually embodied by its exponent. Some people (I'm one of them) round the mantissa as well. So I would say that the order of magnitude of 8.3×10^9 is 10^{10}. Since we don't have to spend time and effort counting out decimal places every time we look at a number, we can determine how important that number is to whatever we're doing just by looking at it. This can be crucial!

One final note: Yes, it is true that there are "rules" for creating numbers in scientific notation. While good form dictates that one stick with the rule, as long as the overall rules of Mathematics are obeyed, there is no problem with deviating from the rules of scientific notation from time to time. This is a tool and, as such, it should be used to make your life easier. If you have a reason for writing, say, $123456 = 123.456 \times 10^3$ rather than as $123456 = 1.23456 \times 10^5$, by all means do so! As with all tools, you use *it*. Don't permit it to use *you*. I promise, the Number Cops won't come and get you if you do this.