Physics 206b

Homework Assignment VII
SOLUTIONS

1. **A man with a mass of 80 kg is being lifted by a winch. The winch is powered by a 12 V battery and has a resistance of 7 Ω. How long does it take the winch to lift the man 17 meters?**

    The resistance of the winch is created by the fact that it is doing work. The charges passing through the motor have some potential energy when they enter it. The motor converts that potential energy to mechanical energy. That mechanical energy does work lifting the man. The work shows up as a change in potential energy of the man, at the end.

    So, the man's final potential energy (relative to his starting point) is $P.E. = mgh$. This is the work done by the motor so we can say $W = mgh$. Assuming that this work was done at a constant rate over some interval of time, t, we can relate the work done to the power, which is defined as $P = \dfrac{W}{t}$. Thus $W = Pt$ and we can substitute this into the energy conservation expression above to yield $W = Pt = mgh$. So the time it took for the man to be lifted is $t = \dfrac{mgh}{P}$. We're left with having to find the power. But this can be determined from the electronic features of the motor. We know that the charges experience a potential change of $V = 12V$ and that the resistance of the motor is $R = 12\Omega$, so we can use the equation relating power to potential change and resistance, $P = \dfrac{V^2}{R}$. Substituting this gives $t = \dfrac{mgh}{P} = \dfrac{mgh}{\left(\dfrac{V^2}{R}\right)} = \dfrac{mghR}{V^2}$. Finally, we substitute numbers to get

$$t = \frac{mghR}{V^2} = \frac{80kg \times 9.8\dfrac{m}{s^2} \times 17m \times 7\Omega}{144V^2} = 647.9s.$$

That's just shy of 11 minutes to travel about the height of a five-storey building.

2. **Find the current that passes through each of the resistors in the circuit below. Also, find the total resistance of the circuit. Take the resistances to be as follows:** $R_1 = 3\Omega, R_2 = 5\Omega, R_3 = 7\Omega, R_4 = 11\Omega, R_5 = 13\Omega$. **Take V=9 V. (Note the problem below. You may want to answer both of these problems simultaneously, in the form of a table.)**
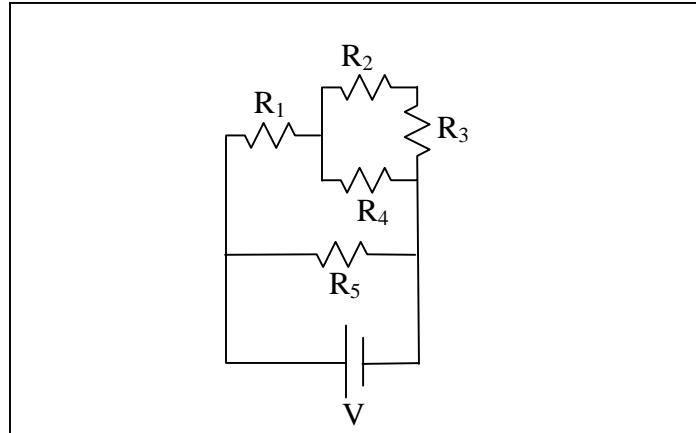
Ah, everybody's favorite! Because we'll wind up finding the potentials as we find the currents, just as a matter of course, I'll do this problem and problem #3 simultaneously. There are two "tricks" to doing this problem: First, break it down into chunks as small as possible. Don't try to get fancy and cut corners or you're doomed! Once you get some experience in this sort of thing, you can take shortcuts. Second, use Ohm's law again and again. Sometimes you'll know the potential drop across something and sometimes you'll know the current through it. In either case, take the thing you know and use it to find the thing you don't know. Here we go:
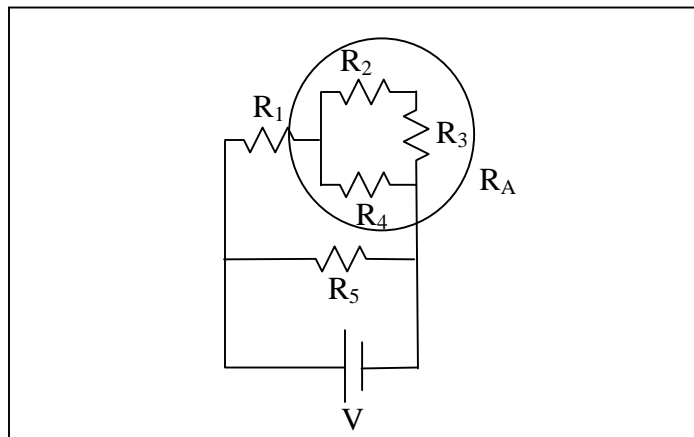
Let's start by finding the resistance of the full circuit. Notice that this is two resistors in parallel: R_5 and the total resistance of the network at the top of the

circuit, I'll call it R$_{top}$. Thus we have $\dfrac{1}{R_{total}} = \dfrac{1}{R_5} + \dfrac{1}{R_{top}}$. Well, in order to find this, we'll need to find R$_{top}$. Notice that R$_{top}$ is just R$_1$ in series with another network of resistors which I have labeled R$_A$ in the drawing above. So $R_{top} = R_1 + R_A$. But R$_A$ is just a combination of R$_2$, R$_3$, and R$_4$. Now, to add them together we need to see that R$_2$ is in series with R$_3$ and that the two of them are in parallel with R$_4$. Make sure you can see that! If not, come and see me—it's a bit to tough to put onto paper.

Thus, we have $\dfrac{1}{R_A} = \dfrac{1}{R_4} + \dfrac{1}{R_2 + R_3}$ (notice that I can do a simple sum of R$_2$ and R$_3$ since they are in series). Substituting numbers, we have $\dfrac{1}{R_A} = \dfrac{1}{R_4} + \dfrac{1}{R_2 + R_3} = \dfrac{1}{11\Omega} + \dfrac{1}{5\Omega + 7\Omega}$. This gives $R_A = 5.74\Omega$. Adding this to R$_1$, we get $R_{top} = R_1 + R_A = 3\Omega + 5.74\Omega = 8.74\Omega$. Now, we can find the total resistance $\dfrac{1}{R_{total}} = \dfrac{1}{R_5} + \dfrac{1}{R_{top}} = \dfrac{1}{13\Omega} + \dfrac{1}{8.74\Omega}$ which gives $R_{total} = 5.23\Omega$. (Remember: When dealing with resistors in parallel, the total resistance is *less than* the lowest resistance in the parallel set.) Using this, we can find the total current leaving (and returning to) the EMF source: $V_{tot} = I_{tot} R_{tot}$ so $I_{tot} = \dfrac{V_{tot}}{R_{tot}} = \dfrac{9V}{5.23\Omega} = 1.72A$.

Let's also find the current through R$_5$ now. This is the easiest since we know that the potential after it is zero. Ohm's law gives us $I_5 = \dfrac{V_{tot}}{R_5} = \dfrac{9V}{13\Omega} = .692A$.

Next, let's find the current through R$_1$. We've just found R$_{top}$. Now, take a close look at the circuit and note that the total potential difference between the beginning of R$_{top}$ and its end is just the full potential of the EMF source, 9V. Also, notice that all of the current that goes into R$_{top}$ goes through R$_1$. Thus, if we find the current through R$_{top}$ we will simultaneously find the current through R$_1$. Using Ohm's law, we have $I_{top} = I_1 = \dfrac{V}{R_{top}} = \dfrac{9V}{8.74\Omega} = 1.03A$.

Having found the current through R$_1$, we can use Ohm's law again to find the potential drop across R$_1$. $V_1 = I_1 R_1 = 1.03A \times 3\Omega = 3.09\Omega$. Now, this is the potential change in

passing through the resistor. Since we start out at 9V at the "entrance" to the resistor, our potential (relative to zero) will be $V_A = 9V - 3.09V = 5.91V$ at the "exit" from R_1. (Note that we could also have found this by applying Ohm's law to R_A with equal justification.)

With this potential, we can calculate the current through R_4, noting that the potential at its exit is zero. Thus, we have, once again using Ohm's law $I_4 = \dfrac{V_A}{R_4} = .537A$.

Note that the current through R_2 and R_3 is the same, since they are in series with each other. Since we have the current flowing into the full set R_A and have now found the current flowing through part of that set R_4, we would be perfectly justified in simply subtracting to get $I_2 = I_3 = I_{top} - I_4 = .493A$. However, let's not do that (smirk). Let's, instead, use Ohm's law again. The reason for this is that doing the simple subtraction works if there's only one current unknown out of the network. Using Ohm's law will work even if there are several parallel resistors with unknown currents flowing through them. This gives $I_2 = I_3 = \dfrac{V_A}{R_2 + R_3} = \dfrac{5.91V}{12\Omega} = .493A$, as expected.

We know the potential after R_3 and R_4: Zero in both cases. Now that we know the current through R_2 we can find the voltage change across it, as well. This again uses Ohm's law and we have $V_2 = I_2 R_2 = .493A \times 5\Omega = 2.465V$. Again, this is the voltage *change* across the resistor. The potential, relative to zero, on the "exit" from the resistor is the "entrance" potential minus this. So we have $5.91V - 2.465V = 3.445V$.

A brief discussion is in order at this point: Potentials, like potential energies, only make sense in the context of a *difference*. There is no such thing as a "potential," strictly speaking, but only of a "potential difference." This means that we have to know what the difference is between. It is a common (but sloppy!) convention to discuss the potential at a point in a circuit. What is meant by this is the difference in potential between that point and the "ground" of the particular circuit—the defined zero point of potential. On the other hand, Ohm's law tells us the relationship between the potential difference between one side of a resistor and the other and the current flowing through the resistor. Sometimes, we apply a potential difference and Ohm's law tells us the current that this will cause to flow. Other times the current is

flowing and Ohm's law tells us how much of the available potential is used in the process. We must be very careful to distinguish between these uses of the word "potential."

Summarizing the results of the problem, here is a table containing the current through each resistor, the potential change across the resistor, and the potential at the point immediately following the resistors:

Resistor	Current (A)	ΔV (Volts)	V_{after} (Volts)
R_1	1.03	3.09	5.91
R_2	.493	2.465	3.445
R_3	.493	3.445	0
R_4	.537	5.91	0
R_5	.692	9	0

3. For each of the resistors in the above circuit, find the potential (relative to zero) at the point immediately following each resistor.

As stated above, the potentials were found in the process of the previous problem. See above.