Physics 206b

Homework Assignment VI
SOLUTIONS

1. **In your previous assignment, you calculated the total capacitance of a pair of parallel plates separated by air. To repeat: Consider the plates to be circular with a radius of 1 cm. Take the separation between them to be 1/2 mm. What is the maximum charge that could be put on this above capacitor if the dielectric strength of air is $3\frac{kV}{mm}$?**

   Now we have to use the info for air rather than vacuum. It would seem that we could put any charge we want on any capacitor we want just by getting a high enough potential across it. After all, we have the relation $Q = CV$. If we make V big enough, we should be able to get any desired Q. Unfortunately, (or maybe fortunately—I don't think I'd like to have to worry about touching a capacitor with a couple of dozen coulombs on it!) this won't work. When the potential difference between the plates gets too big, the charge will jump across the gap between the plates. Since, for parallel plates, $C = \varepsilon_0 \frac{A}{d}$ we could make the separation very tiny, otherwise, and get a huge capacitance. Because of this tendency to "break down," there is a balance between the capacitance and the maximum charge or voltage.

   Since the separation of the plates on the capacitor under consideration is $d = 5\times10^{-4}\ m$, the maximum potential difference we can have on the capacitor before it breaks down will be $V_{max} = 5\times10^{-4}\ m \times 3\frac{kV}{mm} = 1500V$. Taking $C = 5.5\times10^{-12}\,F$, this gives

   $$Q_{max} = CV_{max} = 5.5\times10^{-12}\,F \times 1500V = 8.25\times10^{-9}\,C.$$

2. **Now, the gap in the capacitor above is filled with paper (see table 19.1 in your text). What is the maximum charge that could be put on the capacitor in this case? What voltage difference would that require?**

   Now we put a piece of paper between the plates. According to the table, paper has a dielectric constant $\kappa = 3$ and I told you in class and on the previous assignment that air has a dielectric strength of $8\frac{kV}{mm}$. This gives a capacitance of $C = \kappa\varepsilon_0 \frac{A}{d} = 1.65\times10^{-11}$. Since our maximum potential difference is now $V_{max} = 5\times10^{-4}\ m \times 8\frac{kV}{mm} = 4000V$. This gives a charge of $Q_{max} = CV_{max} = 1.65\times10^{-11}\ F\times 4000V = 6.6\times10^{-8}\ C$.

   Be careful not to confuse "dielectric strength" with "dielectric constant." It's *terrible* nomenclature! Dielectric strength tells us what maximum electric field a dielectric can tolerate before breaking. The dielectric constant tells us what affect a dielectric has on a system in which it is placed.

3. **A current of 2 amps flows through a wire. How many electrons per second will pass through a given cross section of the wire?**

   This is really a piece of cake. I just wanted to make sure that I dispelled the very common notion that an ampere is 1 electron per second. An ampere is one ***coulomb*** per second. The charge on an electron is $q_e = 1.6\times10^{-19}\ C$. So the number of electrons per second in one ampere is

$$I_e = \frac{1\frac{coulomb}{second}}{q_e} = \frac{1\frac{coulomb}{second}}{1.6\times10^{-19}\ \frac{coulombs}{electron}} = 6.25\times10^{18}\ \frac{electrons}{second}$$. In 2 amperes, we would therefore have $I_e = 1.25\times10^{19}\ \frac{electrons}{second}$.

4. **Copper has a resistivity of $\rho = 1.72\times10^{-8}\Omega\cdot\mathrm{m}$. 22 Gauge wire has a diameter of 0.64516 mm. If a potential difference of 1.5 V is placed across the ends of a length of 22 Gauge copper wire, what length of wire will be needed to dissipate ½ watt of power?**

It is important to remember that everything has some resistance. When there are devices in a circuit that use a lot of power, we tend to ignore the resistance in the pieces of wire that deliver the current to those devices, but this is just an approximation. Even the best piece of wire, designed to have the smallest possible resistance, has *some*. As charges travel in such a wire, they will give up some or all of their energy, which will go to heating up the wire usually.

The resistance of a piece of wire of length L and cross-sectional area A is given by $R = \rho\frac{L}{A}$, where ρ is the "resistivity" of the wire—a quantity that depends only on the material out of which the wire is made. Thus we see that a longer wire will have a larger resistance and a narrower wire will also have a larger resistance.

The power dissipated by any device is given by $P = VI = I^2R = \frac{V^2}{R}$. Note that this does *not* need to be turned to heat: A motor, for example, will have a resistance associated with it. The power expended by that motor lifting a load obeys the exact same relation. Which of the three equations for power one uses really depends on what one happens to know about the system. In our case, we know the potential difference and the resistance, so we can use the final form. (Note that the three forms are linked by Ohm's law. Starting with $P = VI$, which derives simply from the definition of potential change and current, we can get to the other two forms simply by substituting Ohm's law into the equation.) Thus we have $P = \frac{V^2}{R} = \frac{V^2A}{\rho L}$. Solving for the length gives $L = \frac{V^2A}{\rho P}$. Since we're given the diameter of the wire and not the cross-sectional area *per se* we should also substitute for this. This gives $L = \frac{V^2A}{\rho P} = \frac{\pi r^2V^2}{\rho P}$. Inserting numbers gives us $L = 85.5m$.

5. **A battery delivering 12 V is connected to a coil of wire with a resistance of 3 ohms. The coil of wire is immersed in 250 grams of water initially at 20° C. How long will it take for the water to reach the boiling point?**

As I hope you remember from the first part of this semester, the change in temperature of a substance that does no work on its environment (assuming its phase doesn't change—e.g., assuming it doesn't freeze or boil) is related to the heat that flows into our out from the substance via the equation $Q = mc\Delta T$. Here, Q is the heat that flows into the substance (causing a change in internal energy), m is the mass of the sample, ΔT is the temperature change, and c is a parameter related to the substance in its current phase called its "specific heat."

We are given the mass of the sample. The change in temperature is just the difference between 20° C and 100° C. That is, 80° C (which is the same as 80 K). The specific heat of water is $c = 4.186 \dfrac{J}{gm \cdot K}$. Putting these together, we get a total energy change of $83720\,J$.

Now, the total heat dissipated by the resistance in the coil divided by the time over which it is dissipated is, by definition, the power. That is, $P = \dfrac{Q}{t}$. (Be careful: There are two different "t"s in this problem. There is also a *Q* that is *not* a charge! Keep it clear in your head what each symbol refers to what!) Playing around with this, we get $t = \dfrac{Q}{P}$.

All that's left is to figure out the power. Since we know the potential change (voltage) and the resistance, we use $P = \dfrac{V^2}{R}$. This gives $P = \dfrac{V^2}{R} = \dfrac{144\,V^2}{3\Omega} = 48W$.

Combining everything we know, we have $t = \dfrac{\Delta Q}{P} = \dfrac{83720\,J}{48W} = 1744\,s = 29\,minutes$.

Problems #6 and #7 have been moved to the next assignment.