Physics 206b

Homework Assignment IV
SOLUTIONS

1. If a refrigerator has a coefficient of performance ("efficiency") of 2.5, how long will it take your refrigerator to cool $1\,\mathrm{L}$ of boiling water to one degree above freezing if the refrigerator draws 1 kW?

We must first remember the definition of the coefficient of performance of a refrigerator: $\beta = \frac{Q}{W}$ where Q is the energy removed from the contents of the refrigerator and W is the work done by the refrigerator in accomplishing this task. Since the work done by the refrigerator is the power it draws (1 kW) multiplied by the time over which it draws this (using the definition of power, $P = \frac{W}{t}$), we can easily solve for the time to get $\beta = \frac{Q}{W} = \frac{Q}{Pt} \Rightarrow t = \frac{Q}{P\beta}$.

To cool 1L (1 kg) of water from 100° C to 1° C requires the removal of $Q = 4186\frac{J}{K} \times 99K = 4.14 \times 10^5 J$ from the water. This gives $t = \frac{Q}{P\beta} = \frac{4.14 \times 10^5 J}{1000W \times 2.5} = 166s$.

Clearly there's something wrong here: The numbers I gave you are not unrealistic. But we know that it would take a whole lot more than the ~3 minutes found here to accomplish the cooling we've described. So what's wrong? Well, your refrigerator *doesn't* cool the water directly. Your refrigerator must cool some coils filled with refrigerant first. It then blows air from within the refrigerator over those coils to fill the refrigerator with cold air. The air then cools the water via a combination of convection and conduction (and a little bit of radiation, but not much). So our example made a false assumption: That refrigerators cool things put into them directly. Refrigerators cool the things put into them indirectly, so the process is much more complex than what we've laid out here.

2. You see advertised a motor that claims that for every Joule of energy it gets out of burning fuel at $700\,^{\circ}\mathrm{C}$, given the world average temperature of 15 °C, it can on average do 0.85 Joules of work. Is such a motor feasible?

The *maximum* amount of work that can be gotten from any heat engine is given by $W = \varepsilon Q$ where Q is the energy derived from the engine and ε is the efficiency of the process. But the maximum efficiency is given by $\varepsilon = \frac{T_H - T_C}{T_H}$ where T_H and T_C are the temperature of the hottest segment of the engine's cycle and the temperature of the environment, respectively.

In the system presented, the purported engine's efficiency is $\varepsilon = 0.85$. But the maximum efficiency such a motor can have, given the specifications cited, is $\varepsilon = \frac{(700+273)-(15+273)}{700+273} = \frac{685}{973} = 0.704$, where I've added 273 to each of the temperatures given to convert them to absolute (Kelvin) temperatures. So the best that our engine can do is about a 70% efficiency—well below the 85% claimed. This motor is clearly *not* feasible.

3. Two charged objects are separated by 1 meter. The net force between them is 2×10^{-3} Newtons and is attractive. The net charge on one of the objects is 7.11×10^{-4} C. What is the other charge?

Here we use Coulomb's law directly: $F = k\frac{q_1 q_2}{r^2}$. We're told the size of the force and that it is attractive. Since it is attractive, we can immediately conclude that the signs on the two charges are opposite. The one charge (let's call it q_1) is given to be positive with a size of 7.11×10^{-4} C. Thus the other charge (q_2) must be negative. Solving for the size is just a simple piece of algebra, giving $q_2 = \frac{Fr^2}{kq_1}$. All that's left is to plug in numbers (recalling that $k = 9\times10^{9}\ \frac{N\cdot m^2}{C^2}$) which gives

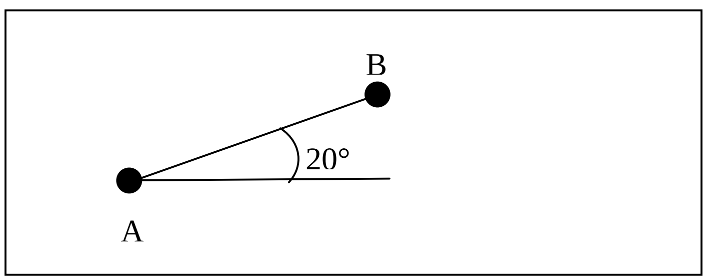
$$q_2 = -\frac{Fr^2}{kq_1} = -\frac{2\times10^{-3}\ N\times1\,m^2}{9\times10^9\ \frac{N\cdot m^2}{C^2}\times7.11\times10^{-4}\ C} = -3.1\times10^{-10}\ C$$

(Notice the minus sign, which I inserted by hand based on knowledge of the sign of the final charge.)

4. **Consider again the two charges in the previous problem. If the distance between the charges is doubled, what then will be the net force between them?**

No direct work is necessary here: If the distance between the charges is doubled, the force will go down by a factor of ¼. Since the original force is 2×10^{-3} Newtons, this means $F_{final} = \frac{2\times10^{-3}\ Newtons}{4} = 5\times10^{-4}\ Newtons$.

5. **Object A has charge of 1.7 C and object B has a charge of 3.1 C. The objects are separated by 2.9 meters. A line drawn between the two makes an angle of 20° relative to the horizontal, as shown. Expressed as a vector, what is the force experienced by object B due to object A?**

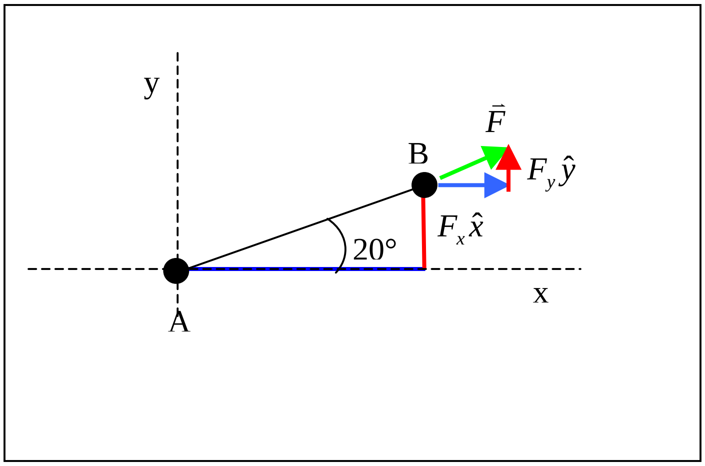


We now introduce the vector nature of Coulomb’s law. Recall the statement of Coulomb’s law: “The force between two point charges separated by a distance *r* will be given by $F = k\frac{q_1 q_2}{r^2}$ and will be directed along the line connecting the two charges.” Let’s begin by calculating the size of the force.

Directly plugging in the numbers for the charges and their separation, we get

$F = k\frac{q_1 q_2}{r^2} = 9\times10^9\ \frac{N\cdot m^2}{C^2}\times\frac{1.7C\times3.1C}{(2.9\,m)^2} = 5.6\times10^9\ Newtons$. (Notice how huge this is. A coulomb is a whopping big charge. It would be very rare to find an object that actually had a net charge on the order of a Coulomb.)

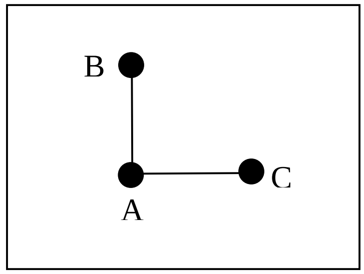
Now for the hard part (for those who are rusty working with vectors). We must represent this as a vector. Let’s begin by making a little sketch with our coordinate system in place:



Here I’ve included the force exerted on B by A. Since this force is in the same direction as the line connecting A and B, the *x* and *y* components of the force will be in the same ratio to the total force as the legs of the triangle formed by A and B, as shown. Now we can do just a tiny bit of trigonometry to find the answer. Calling the line connecting A and B $\overline{AB}$ and the perpendicular legs of the triangle parallel to the *x* and *y* axes $\overline{AB}_x$ and $\overline{AB}_y$ respectively (the blue and red lines in the picture), we can say $\sin(20°) = \frac{\overline{AB}_y}{\overline{AB}}$ and $\cos(20°) = \frac{\overline{AB}_x}{\overline{AB}}$. Extending this to the triangle formed by the force and its components, we can say $\sin(20°) = \frac{F_y}{F}$ and $\cos(20°) = \frac{F_x}{F}$. Thus, $F_x = F\cos(20°)$ and $F_y = F\sin(20°)$.

Combining all of the above, we can write $\vec{F} = F_x\hat{x} + F_y\hat{y} = F\cos(20°)\hat{x} + F\sin(20°)\hat{y}$. Plugging in numbers, this becomes $\vec{F} = 5.6\times10^9\ Newtons(0.94\hat{x} + 0.34\hat{y})$.

6. **Objects A, B, and C sit at the corners of a 45° right triangle as shown. B and C each have a charge of 0.61 C while object A has a charge of —0.7 C. The distance between A and C (which is the same as the distance between A and B) is 13 mm. What is the net force (expressed as a vector!) experienced by object A?**



Here we follow the same procedure as before (I won't do it in quite such gory detail, however). But there's a twist: Now, we have two charges each exerting forces on the "target" charge. This is no problem: We simply add the forces, remembering to do so vectorially. Since each of the forces are exerted along the lines connecting the "source" charge to the "target" charge (I'm using the quotes because Newton's third law tells us that there is no real difference between these roles, they just help us keep things clear in our heads) and since those lines make a right angle with respect to each other, this addition is particularly easy, in this case. (Note that the fact that I placed those lines along the conventional coordinate axes is irrelevant: Let's say that I *didn't* do this but that the lines were still perpendicular to each other. You would just *construct* a pair of coordinate axes parallel to the lines. As long as they make a right angle, you can use them as the basis of a coordinate system without any problem. Don't fall into the trap of believing that your *x* and *y* [and *z*] axes always have to be oriented in the same directions!)

Calling the forces of B on A and C on A $\vec{F}_{BA}$ and $\vec{F}_{CA}$, respectively, we have

$$\vec{F}_{BA} = k\frac{q_A q_B}{r^2} = 9\times10^9 \ \frac{N\cdot m^2}{C^2}\times\frac{(0.7C)\times(0.61C)}{(0.013m)^2}\hat{y} = 2.3\times10^{13} \ Newtons\,\hat{y}$$

and

$$\vec{F}_{CA} = k\frac{q_A q_C}{r^2} = 9\times10^9 \ \frac{N\cdot m^2}{C^2}\times\frac{(0.7C)\times(0.61C)}{(0.013m)^2}\hat{x} = 2.3\times10^{13} \ Newtons\,\hat{x}\ .$$

Therefore, the total force on A is

$$\vec{F}_A = \vec{F}_{CA} + \vec{F}_{BA} = 2.3\times10^{13}\ Newtons\ (\hat{x}+\hat{y})\ .$$

7. **Consider once again the charge configuration described in problem #6. What is the total force experienced by object C? What is the total force experienced by object B?**

Now things are getting a bit messy! The key here is to break the problem down into bite-sized pieces. Here's the strategy: Recognize that the total force on each of the charges is a sum of two forces. Solve for each of the vector components of each of those two forces. Add together the *x* components to find the final *x* component of the total force. Likewise for the *y* components. That's it. Now let's do it.

For object C, we can write $\vec{F}_C = \vec{F}_{AC} + \vec{F}_{BC}$. Now, from Newton's third law, we can conclude that the force of A on C is the opposite of the force of C on A. So we can immediately write $\vec{F}_{AC} = k\frac{q_A q_C}{r^2} = -2.3\times10^{13}\ Newtons\,\hat{x}$. (Some people prefer to bundle the sign with the unit vector and write $(-\hat{x})$. This is perfectly acceptable.) Finding the force of B on C is a bit more problematic. Without going into the level of detail I did in problem #5, we use the same procedure we used in that problem. We will need to find the magnitude of the force first, however. This will be found, once again, using Coulomb's law. First we'll need to know the distance between the charges. Using the Pythagorean theorem, this is $r_{BC}^2 = r_{AB}^2 + r_{AC}^2 = 2\times(.013\,m)^2$. (Note that we *could* take the square root, but why? We're going to use the distance squared in our final answer, so there's no need to waste the step.) This gives us

$$F_{BC} = k\frac{q_B q_C}{r_{BC}^2} = 9\times10^9\ \frac{N\cdot m^2}{C^2}\times\frac{0.61C\times0.61C}{2\times(0.013m)^2} = 9.9\times10^{12}\ Newtons\ .$$

Now we can find the components: $F_{BCx} = F_{BC}\sin(45°)$ and $F_{BCy} = F_{BC}\cos(45°)$. (Of course, in this case, these are both the same. But that isn't true in general, so let's not take too many shortcuts.) Note that F_{BCy} will act in the $-\hat{y}$ direction. And the two forces can be added component-wise to give, finally

$$\vec{F}_C = \left(F_{BCx} + F_{AC}\right)\hat{x} + F_{BCy}\hat{y}$$

$$= \left(9.9\times10^{12}\ N\sin(45°) - 2.3\times10^{13}\ N\right)\hat{x} - 9.9\times10^{12}\ N\cos(45°)\hat{y}$$

$$= -1.6\times10^{13}\ N\ \hat{x} - 7\times10^{12}\ N\ \hat{y}$$

(Note that there are a variety of equally valid ways you might write this.)

For the force on B, all the above methods hold except that the forces are in slightly different directions. The force on B due to A is in the $-\hat{y}$ direction. For the force on B due to C, we can just invoke Newton’s third law and our solution found previously to write $\vec{F}_{CB} = -\vec{F}_{BC}$. Combining these, we have

$$\vec{F}_B = F_{BCx}\hat{x} + \left(F_{BCy} + F_{AB}\right)\hat{y}$$

$$= -9.9\times10^{12}\ N\sin(45°)\ \hat{x} + \left(9.9\times10^{12}\ N\cos(45°) - 2.3\times10^{13}\ N\right)\hat{y}$$

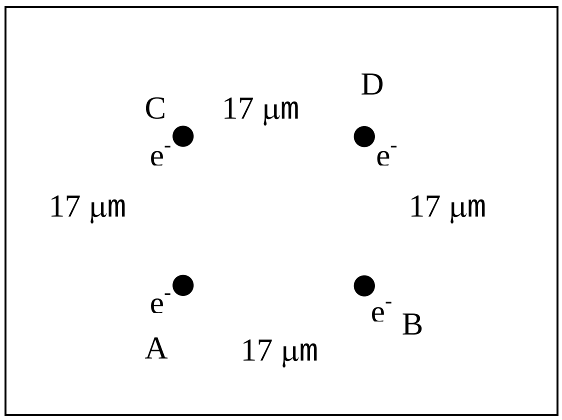
$$= -7\times10^{12}\ N\ \hat{x} - 1.6\times10^{13}\ N\ \hat{y}$$

Just one more note on this question: This problem is a nice example of a problem that should be done qualitatively before beginning it quantitatively. You should get in the habit of thinking through the generalities of what you expect the answer to be before plunging into the math. When you’re done with the precise analysis, look back on your prediction. If the two agree, great! If they do not agree, then one of two things is happening: Either you made a mistake in the math (likely) and your qualitative, intuitive answer might be able to guide you to the source of the blunder. Or your intuition was faulty, in which case you should take the opportunity to “reprogram” yourself. Figure out where your intuition went wrong. Did you just overlook something when you originally thought about it? Or did you actually misunderstand something? Get in the habit of doing this with *all* problems. The ultimate goal of your training in Science is to give you a deeper understanding of how the universe works. That understanding manifests itself in the refinement of your intuition.

8. **An electron sits at each of the corners of a square whose side-length is 17 μm $\left(1\,\mu m = 1\times 10^{-6}\,m\right)$. If no forces other than the Coulomb force act on this system, what is the acceleration experienced by the electron at the top right corner?**

Let's apply the advice I gave you in the last paragraph of the previous problem: They're all electrons, so the force will be repulsive. By symmetry, I expect the resultant to lie directly on the diagonal of the square. Now let's see if this is correct and get a quantitative answer for it.

We have this situation:



I've named each of the charges, for convenience. Note that the charge in which we are interested is "D." Now, for a little strategy: We note that the problem asks for the acceleration. That should set off fireworks in your head by now. There is a force (the Coulomb force) and you are asked for the acceleration. This should make you instantly think of Newton's second law, $\vec{F} = m\vec{a}$. You'll need the mass, of course, but since you were told that the objects were electrons, you effectively know this—just look in the front cover of your textbook or any of thousands of other possible references to find that $m_e = 9.11\times 10^{-31}\,kg$. So the problem really boils down to finding the force.

From here on, it's just geometry and algebra. We do the same thing as in the previous two problems (you do see that this is basically the same problem, I hope). The total force on D will be $\vec{F}_D = \vec{F}_{AD} + \vec{F}_{BD} + \vec{F}_{CD}$. (Get used to the notation I've used here as I'll use it often: *Interaction*$_{(something)(something\ else)}$ means "the effect of *something else* on *something*. So $\vec{F}_{AD}$ means "the force of *D* on *A*. While $\vec{F}_D$ would just mean "the force on *D* without regard to source. As always, if in doubt, *ask*.)

By reference to the previous problem, we can do $\vec{F}_{BD}$ and $\vec{F}_{CD}$ directly:

$$\vec{F}_{BD} = k\frac{q_B q_D}{r^2}\hat{y}$$

$$= 9\times 10^9 \ \frac{N\cdot m^2}{C^2}\times\frac{\left(1.6\times 10^{-19}\ C\right)\times\left(1.6\times 10^{-19}\ C\right)}{\left(1.7\times 10^{-5}\ m\right)^2}\hat{y} = 7.97\times 10^{-19}\ \ Newtons\,\hat{y}$$

$$\vec{F}_{CD} = k\frac{q_B q_D}{r^2}\hat{x}$$

$$= 9\times 10^9 \ \frac{N\cdot m^2}{C^2}\times\frac{\left(1.6\times 10^{-19}\ C\right)\times\left(1.6\times 10^{-19}\ C\right)}{\left(1.7\times 10^{-5}\ m\right)^2}\hat{x} = 7.97\times 10^{-19}\ \ Newtons\,\hat{x}$$

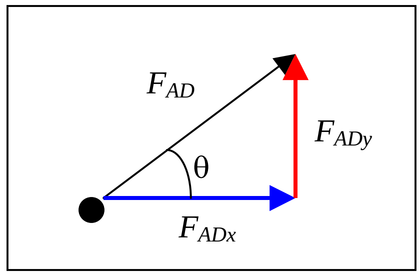
Notice that I've put the direction in "by hand" and ignored the signs on the charges. Be careful with this: I could have left the signs in the charges to determine the final direction of these forces (I'd still have had to put in that they were in the *x* or *y* directions by hand—this can be done mathematically, but it's a bit complex) and just "played dumb" about not knowing the direction. In that case, my answer would simply have told me a final direction and I would have had to look at it in the context of the diagram to determine whether it's attractive or repulsive. This would have been fine. The big thing to remember is to do it *either* one way or the other—either by hand or by leaving all the signs in. Don't do it both ways or you'll get into trouble!

Likewise, we can do $\vec{F}_{AD}$ directly: This is, once again, the force between two charges that lie separated by the hypotenuse of a right triangle. Even better (but don't get too comfortable with it!), it is a 45° right-triangle. This is absolutely identical to the $\vec{F}_{BC}$ of problem #5 except for the direction. Let's do the size (magnitude) of the force first and then break it into its vector components. We have

$$F_{AD} = k\frac{q_A q_D}{r_{AD}^2}$$

$$= 9\times 10^9 \ \frac{N\cdot m^2}{C^2}\times\frac{1.6\times 10^{-19}\ C\times 1.6\times 10^{-19}\ C}{2\times\left(1.7\times 10^{-5}\ m\right)^2} = 3.99\times 10^{-19}\ Newtons$$

Now, this force is going to be split up into an $\hat{x}$ component and a $\hat{y}$ component. Since the force is repulsive, the situation is that shown below:



In this case, $\theta = \frac{\pi}{4}\ rad$ (nasty of me not to put it in degrees, I know—I'll break you of that prejudice sooner or later!). We can write $F_{ADx} = F_{AD}\cos\left(\frac{\pi}{4}\right)$ and $F_{ADy} = F_{AD}\sin\left(\frac{\pi}{4}\right)$. Since, in this case, $\sin\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$, we have $F_{AD\,x} = F_{ADy} = \frac{1}{\sqrt{2}}F_{AD} = .707\times 3.99\times 10^{-19}\ Newtons = 2.82\times 10^{-19}\ Newtons$.

Putting all these pieces together, we have $\vec{F}_D = \vec{F}_{AD} + \vec{F}_{BD} + \vec{F}_{CD} = \left(F_{ADx} + F_{CD}\right)\hat{x} + \left(F_{ADy} + F_{BD}\right)\hat{y}$

$$= \left(2.82\times 10^{-19}\ N + 7.97\times 10^{-19}\ N\right)\hat{x} + \left(2.82\times 10^{-19}\ N + 7.97\times 10^{-19}\ N\right)\hat{y}$$

$$= 1.08\times 10^{-18}\,\hat{x} + 1.08\times 10^{-18}\,\hat{y}$$

From here, finding the acceleration is just a matter of dividing by the mass (almost anticlimactic, isn't it?). This gives

$$\vec{a}_D = \frac{1.08\times 10^{-18}\ N\hat{x} + 1.08\times 10^{-18}\ N\hat{y}}{9.11\times 10^{-31}\ kg} = 1.19\times 10^{12}\ \frac{meters}{second^2}\left(\hat{x}+\hat{y}\right).$$

This is a *huge* acceleration due to a pair of very small charges a relatively large distance apart! This is, in part, because electrons are very light. Mostly, however, it's simply because the Coulomb force is just so very large.

As a variation you might want to work on on your own: Place the charges at the corners of a rectangle so that you lose the high level of symmetry we had in this problem. This would be a particularly profitable exercise for those of you who are still uncomfortable with vector decomposition and vector addition.

9. **Consider again the configuration in the previous problem. Replace the electron in the top right corner with a proton. What acceleration does the proton experience?**

Piece of cake! Since the proton has the exact same *size* charge as the electron but of opposite sign, just change the signs on all the forces. This gives $\vec{F}_D = \vec{F}_{AD} + \vec{F}_{BD} + \vec{F}_{CD} = -1.08\times10^{-18}\,\hat{x} - 1.08\times10^{-18}\,\hat{y}$. Now, for the acceleration we must use the mass of the proton $m_p = 1.673\times10^{-27}\ kg$. This gives $\vec{a}_D = \dfrac{-1.08\times10^{-18}\ N\hat{x} - 1.08\times10^{-18}\ N\hat{y}}{1.673\times10^{-27}\ kg} = -6.46\times10^{8}\ \dfrac{meters}{second^2}\left(\hat{x}+\hat{y}\right)$.

10. **What is the electric field (expressed as a vector!) resulting from a 0.78 C charge at a point 0.9 meters from that charge in the $\hat{x}$ direction?**

Let's recall where the notion of "electric field" comes from: The electric field at a point in space is the force that a positive, 1 Coulomb charge would experience at that point if it were placed there.

To make this a bit more concrete, consider the force exerted on a point charge, let's call it the "target," by another point charge, let's call it the "source." The force between the two is given by Coulomb's law, $F = k\dfrac{q_{source}q_{target}}{r^2}$ where *r* is the distance between the two charges. The direction of this force is along the line connecting the two charges and it is either attractive or repulsive depending on whether the charges have the same or opposite signs.

We would like to think the interaction between the charges doesn't just "turn on" out of nowhere. The source doesn't know about the existence of the target, after all. So we have the sense that the source doesn't exert its force on the target directly, but rather it creates a field in all the space around it. When a target charge is placed at some point in space, the interaction between the charge and the *field* causes the target to experience a force. (Of course, the target charge creates a field of its own. It is this field that the source charge experiences. **IT IS ABSOLUTELY ESSENTIAL THAT YOU REMEMBER THAT THE SOURCE DOES NOT EXPERIENCE THE FIELD THAT IT CREATES!**) Of course, Coulomb's law gives us the size of the force, so we associate part of Coulomb's law with the field:

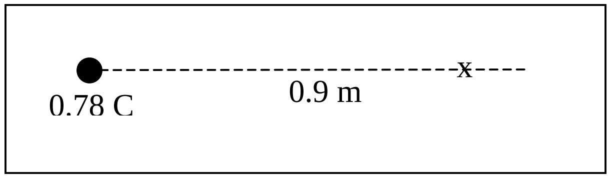
$$F = \underbrace{k\frac{q_{source}}{r^2}}_{\mathrm{E}} q_{target}$$

$$F = Eq_{target}$$

(In these equations I have left out the vector nature for clarity. This is a dreadful abuse! Both the electric field and, of course, the Coulomb force are vectors. Do as I say, not as I do: If you leave the vector character off of a quantity like this on an exam, you will lose all points on that question.)

Notice that we'd get the exact same result if we defined, as above, the field as the force experienced by an imaginary 1 Coulomb test charge.

The biggest source of error I've seen on this topic is confusing the source with the target charges. The source charge *creates* the field. The target charge *experiences* the force. At the risk of muddying the waters, I do need to remind you that the roles of target and source are *arbitrary*! One charge creates a field that another one experiences and the other one creates a field that the first one experiences. Since the size of the field is proportional to the source charge and the size of the force is proportional to the target charge, the forces work out to be the same, as predicted by Coulomb's law and demanded by Newton's third law.



In the situation at hand, pictured above, our source charge is .78 Coulombs and the distance to the point in space, indicated with an "x" in the figure, at which we are interested in the field is 0.9 m. So we have $E = k\frac{q_{source}}{r^2} = 9\times10^9\ \frac{N\cdot m^2}{C^2}\times\frac{.78\ C}{(.9\ m)^2} = 8.7\times10^9\ \frac{N}{C}$. (We will learn later that the proper unit for this is $\frac{volts}{meter}$, but $1\frac{volt}{meter} = 1\frac{N}{C}$, so we're fine using this.)

Now, we're not done yet: The electric field is a vector. We need the direction. The direction is the direction of the force that a 1 coulomb positive test charge would experience if it were placed at that point in space. In this case, the force is repulsive so the force would be in the positive $\hat{x}$ direction. This gives, finally, $\vec{E} = 8.7\times10^9\ \frac{N}{C}\hat{x}$.

11. **Consider the charge in the previous problem. What is the electric field resulting from that charge at a point 0.9 meters from it in the $\hat{y}$ direction?**

No work needed here: The distance is the same. The charge is the same. All that's changed is the direction. Now, our field points in the positive $\hat{y}$ direction. So we have $\vec{E} = 8.7\times10^9\ \frac{N}{C}\hat{y}$. This seems utterly trivial, but it makes all those pictures of radially-directed arrows make more sense. Make sure you understand this!

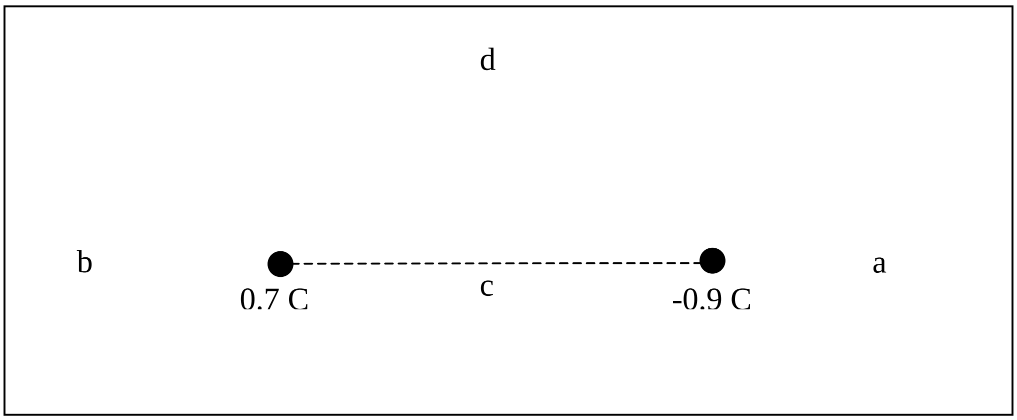
12. **Two charges lie on the *x* axis, as shown. The one on the right is -0.9 C and the one on the left is 0.7 C. They are separated by 37 cm. What is the electric field (expressed, as always, as a vector!) at the following points:**

a. **10 cm to the right of the charge on the right?**

b. **10 cm to the left of the charge on the left?**

c. **The point on the *x* axis midway between the two charges?**

d. **The point 12 cm in the $\hat{y}$ direction directly "above" the midpoint between the charges?**



What I really want you to get from this problem is that this is nothing more or less than the previous two problems once we break it down into bite-sized pieces. Electric fields simply add as vectors. That's all there is to it. Just calculate the field due to each of the source charges at the various points and add the results together. Just remember to add them as vectors.

The magnitude of the fields due to each of the charges is (saving the directional aspect for later) $E = k\frac{q_{source}}{r^2}$. Let's make things a bit more convenient and call the charge on the left L and the one on the right R. We have $E_L = k\frac{.7C}{r^2}$ and $E_R = -k\frac{.9C}{r^2}$.

Now, point "a" is 10 cm from R and 47 cm from L so we have $E_a = E_{La} + E_{Ra} = k\frac{.7C}{(.47\,m)^2} - k\frac{.9C}{(.1\,m)^2} = -7.81\times10^{11}\,\frac{N}{C}$. This is in the $\hat{x}$ direction (to the left, because of the negative sign), so $\vec{E}_a = -7.81\times10^{11}\,\frac{N}{C}\,\hat{x}$. (Be sure you understand the direction of this: If I placed a positive charge at “a,” it would be attracted to R and repelled from L. But the attraction would win both because R is larger than L and because R is closer to “a.” This is all carried in the negative sign of the answer.)

For “b,” we do the same thing except now R is 47 cm from the point in question and L is 10 cm from it. This gives $E_b = E_{Lb} + E_{Rb} = k\frac{.7C}{(.10\,m)^2} - k\frac{.9C}{(.47\,m)^2} = 5.93\times10^{11}\,\frac{N}{C}$.

Now, here’s a tricky bit: This is *also* to the left, like the previous field. In this case, if we put a positive test charge at the point in question, the repulsion would win. We can tell this by the fact that the answer has a positive sign—a positive sign means repulsion. A negative sign means attraction. But, in the case of “b,” repulsion means “to the left”—i.e., in the $-\hat{x}$, just like before. Be careful! The sign on the answer tells you the direction of the field relative to the source point—toward or away. Since the target point might be in the positive or negative direction relative to the source, the sign does *not*, by itself, tell you anything about the direction of the field. You need to know the relative positions of the source and target.

In the case of “b,” the field is $\vec{E}_b = -5.93\times10^{11}\,\frac{N}{C}\,\hat{x}$.

For “c,” we do the same thing, but now, the direction is going to start being important, so I’m going to introduce it at an earlier stage (we could have done this for the others without any problem, it just would have been a smidge more work). We have $\vec{E}_c = \vec{E}_{Lc} + \vec{E}_{Rc}$. Note that I’m just going to treat the individual fields as vectors from the beginning. Now, charge L would repel a positive test charge at “c” while charge R would attract one. Thus *both* fields point to the right. This gives

$$\vec{E}_c = \vec{E}_{Lc} + \vec{E}_{Rc} = k\frac{.7\,C}{(.185\,m)^2}\,\hat{x} + k\frac{.9\,C}{(.185\,m)^2}\,\hat{x} = 4.2\times10^{11}\,\frac{N}{C}\,\hat{x}\,.$$

Again, make sure you understand where that came from!

Finally, let’s do “d.” As with “c,” we’ll just add the fields directly, without passing through the phase of not using the directions. But this means finding each field as a full vector right from the beginning. Let’s do them one at a time.

The field due to L is repulsive. It will have $\hat{x}$ and $\hat{y}$ components given by the full magnitude of that field times, respectively, the cosine and sine of the angle made by the field with the horizontal. This is because the overall direction of the field is along the line connecting the point with charge L. Thus we have $\vec{E}_{Ld} = E_{Ldx}\hat{x} + E_{Ldy}\hat{y} = k\frac{q_L}{r_{Ld}^2}\{\cos(\theta)\hat{x} + \sin(\theta)\hat{y}\}$. Now, before you get all freaky because I didn’t tell you the angle (that was intentional!), note that you don’t need to know the angle! You know the lengths of the sides of the triangle (formed by dropping a perpendicular from “d” to the horizontal), and that’s enough to find the sine and cosine that you need. The *x* component is 18.5 cm and the *y* component is 12 cm, as stated in the problem. The hypotenuse can be found from these using the theorem of Pythagoras: $h = \sqrt{(.12\ m)^2 + (.185\ m)^2} = .22\ m$. Thus $\sin(\theta) = \frac{12}{22}$ and $\cos(\theta) = \frac{18.5}{22}$. Inserting numbers, we have

$$\vec{E}_{Ld} = k\frac{q_L}{r_{Ld}^2}\{\cos(\theta)\hat{x} + \sin(\theta)\hat{y}\} = 9\times10^9\ \frac{N\cdot m^2}{C^2}\times\frac{.7\ C}{(.22\ m)^2}\times\left\{\frac{18.5}{22}\hat{x} + \frac{12}{22}\hat{y}\right\}$$

$$= 1.3\times10^{11}\times\left\{\frac{18.5}{22}\hat{x} + \frac{12}{22}\hat{y}\right\}$$

We follow the same procedure for charge R to get (noticing that the sine and cosine are going to be the same as in the previous problem)

$$\vec{E}_{Rd} = k\frac{q_R}{r_{Rd}^2}\{\cos(\theta)\hat{x} + \sin(\theta)\hat{y}\} = 9\times10^9\ \frac{N\cdot m^2}{C^2}\times\frac{.9\ C}{(.22\ m)^2}\times\left\{\frac{18.5}{22}\hat{x} - \frac{12}{22}\hat{y}\right\}$$

$$= 1.67\times10^{11}\times\left\{\frac{18.5}{22}\frac{N}{C}\hat{x} - \frac{12}{22}\frac{N}{C}\hat{y}\right\}$$

Again, be careful with the signs: The force experienced by a positive test charge placed at “d” would be toward R. This gives the signs on the components. I’ve already taken the sign on charge R into account with this, so I do *not* include the minus sign in the charge. The systematic way to do this, without including things implicitly like I've done, would be to express each point as a vector and do careful vector algebra. This would give the right answer without us needing to invoke the concepts of "attraction" and "repulsion." It's actually a lot easier that way, but, I'm sorry to say, the level of math needed to do it that way is beyond this course. So you really do need to do these problems partly graphically to get the directions of the vectors. Sorry!

Finally, to find the total field at "d," we just add each of the components of each of the two fields to get

$$\vec{E}_d = \vec{E}_{Ld} + \vec{E}_{Rd} = 1.3\times10^{11}\times\left\{\frac{18.5}{22}\hat{x}+\frac{12}{22}\hat{y}\right\}+1.67\times10^{11}\times\left\{\frac{18.5}{22}\hat{x}-\frac{12}{22}\hat{y}\right\}$$

$$= 2.5\times10^{11}\ \frac{N}{C}\,\hat{x} - 2.0\times10^{10}\ \frac{N}{C}\,\hat{y}$$

Notice the difference in size between the *x* and *y* components of this. Compare this with the picture of the field for an electric dipole shown on page 562 of your text. Why is the *y* component not equal to zero in this case while it is zero in the case shown in your book?

13. **For each of the points identified in the previous problem, what is the acceleration which would be experienced by a proton placed at that point?**

Here we just invoke $\vec{F} = \vec{E}q$ along with $\vec{F} = m\vec{a}$ to get $\vec{a} = \dfrac{\vec{E}q}{m}$. The ratio of the charge on a proton to its mass is $\dfrac{q_p}{m_p} = \dfrac{1.6\times10^{-19}\ C}{1.67\times10^{-27}\ kg} = 9.58\times10^{7}\ \dfrac{C}{kg}$ which gives

a) $\vec{a}_a = -7.81\times10^{11}\ \dfrac{N}{C}\times 9.58\times10^{7}\ \dfrac{C}{kg}\,\hat{x} = -7.48\times10^{19}\ \dfrac{m}{s^2}\,\hat{x}$ (If this look rather large to you, *good*! Yes, this is a ridiculously high acceleration. With charges on the order of a Coulomb and something as light as a proton getting accelerated, things are going to get outrageous. Note that this still *could* happen. The proton would leave the vicinity of the charges very quickly, long before the acceleration had a chance to increase the speed more than a little. We'll do that problem soon.)

b) $\vec{a}_b = -5.93\times10^{11}\ \frac{N}{C}\times9.58\times10^{7}\ \frac{C}{kg}\ \hat{x} = -5.68\times10^{19}\ \frac{m}{s^2}\ \hat{x}$

c) $\vec{a}_c = 4.2\times10^{11}\ \frac{N}{C}\ \times9.58\times10^{7}\ \frac{C}{kg}\ \hat{x} = 4.0\times10^{19}\ \frac{m}{s^2}\ \hat{x}$

d)
$$\vec{a}_d = \left\{2.5\times10^{11}\ \frac{N}{C}\ \hat{x} - 2.0\times10^{10}\ \frac{N}{C}\ \hat{y}\right\}9.58\times10^{7}\ \frac{C}{kg}$$
$$= 2.4\times10^{19}\ \frac{m}{s^2}\ \hat{x} - 1.9\times10^{18}\ \frac{m}{s^2}\ \hat{y}$$