Physics 206b

Homework Assignment II

SOLUTIONS

1. **A pendulum is constructed out of a very thin rod of copper with a bob at the end with a mass of 2 kg. When the pendulum is initially built, its temperature is 290 K and it has a period of one second. If it is warmed up to a temperature of 310 K, what will its period be?**

What a fine way to kick off the semester! I intentionally threw a problem at you that made explicit reference to a topic we dealt with last semester, pendula, to remind you of a critical feature of this course: You don't get to forget! Physics, by its nature, is cumulative. The phenomena, laws, and principles we studied last semester did not stop being true when you turned in your final exams. You are responsible for all the material studied up to this point! Yes, this does imply a somewhat uncomfortable burden for my "stepkids"—if you didn't take Physics 206a with me, then keeping track of just what material you should know will require a bit of work for you. To aid you in this, I've left the web site from last semester in place for a little while. I will take it down on September 3rd, so make sure you go there soon and grab all the background material.

Let's take a look at this problem. We are told that the pendulum's shaft is made of copper. We are also told that it is thin. This is a hint of how to approach the problem. The fact that the shaft is thin means that almost all of the mass is concentrated in the bob. Is all of it located in the bob? No! But we can legitimately approximate this as being the case here. When the mass of a pendulum can be considered all to be located in its bob, the pendulum is called "simple."

We know, from our past analysis, that the period of a simple pendulum is given by $T = 2\pi\sqrt{\frac{L}{g}}$, where L is the length of the shaft and $g = 9.8\frac{m}{s^2}$ is the strength of gravity at the surface of the earth. (This equation actually works anywhere in the universe. You just have to replace the g with whatever acceleration a freely falling object experiences wherever the pendulum is.) (I often forget whether the g goes in the denominator or the numerator of this equation. This is where a dimensional analysis will save the day! Since the period has the dimension of time, a quick piece of algebra tells us that the g must be in the denominator. This is a useful trick that you should get in the habit of using!) So the period will vary if the length of the shaft varies. Since the shaft is a rod of copper, changing its temperature will cause its length to change, so the period will depend on the temperature. Now all we have to do is figure out how much.

The change in the length of an object when its temperature changes is given by

$$\Delta L = L_0 \alpha \Delta T \ .$$

Here L_0 is the length of the object at some initial temperature, ΔT is the change in temperature of the object, and α is the (1-d) coefficient of thermal expansion for the substance out of which the object is made. So the length of the object *after* its temperature is changed will be the initial length plus the change in length (which may be positive or negative—be careful!). This means that the final length of the shaft will be

$$L_f = L_0 + \Delta L = L_0 + L_0 \alpha \Delta T = L_0 (1 + \alpha \Delta T)$$

So the period of the pendulum after the temperature changes will be

$$T_f = 2\pi \sqrt{\frac{L_f}{g}} = 2\pi \sqrt{\frac{L_0 (1 + \alpha \Delta T)}{g}} \ .$$

CAUTION!: Notice that there are two different quantities in this expression that are denoted by the letter "T". One is the temperature and the other is the period. If you fall into the trap of formula-hunting rather than thinking about what you're doing, you ***will*** confuse these two and get into very big trouble. Do not let your brain turn off when you are solving a problem.

Now, at this point we can do this the hard way or the easy way. The hard way is to use the fact that the period at the original temperature is 1 second to solve for the original length of the rod and then to stick that number into the equation that we have found to work out the final period. That will work. But I like the easy way better—if I want more work, I'll solve a harder problem, not solve a problem harder. The easy way is this: Note that we can rewrite the equation for the final period as

$$T_f = 2\pi\sqrt{\frac{L_0}{g}}\sqrt{(1+\alpha\Delta T)}\ .$$

Well, now it should be obvious what to do: The quantity $2\pi\sqrt{\frac{L_0}{g}}$ is precisely the original period. That is $2\pi\sqrt{\frac{L_0}{g}} = 1s$. So we can just write $T_f = 2\pi\sqrt{\frac{L_0}{g}}\sqrt{(1+\alpha\Delta T)} = 1s\times\sqrt{(1+\alpha\Delta T)}$. Piece of cake! We'll need to look up the coefficient of thermal expansion for copper, however. This is found in your text in Table 12.1 (page 365) to be $\alpha = 1.7\times 10^{-5}\left(C°\right)^{-1}$. Since the change in temperature is 20° C (remember that one degree *change* on the Celsius scale is the same as one Kelvin), we have

$$\begin{aligned} T_f &= 1s\times\sqrt{(1+\alpha\Delta T)} = 1s\times\sqrt{\left(1+1.7\times 10^{-5}\left(C°\right)^{-1}\times 20°C\right)} \\ &= 1s\times\sqrt{\left(1+3.4\times 10^{-4}\right)} = 1.00017s \end{aligned}\ .$$

That's a small change, but, as we'll see in the next problem, the impact can be huge.

2. **For the pendulum described in the previous problem, how many cycles will the pendulum go through in 24 hours at the higher temperature? If the pendulum is used as the heart of a clock, what time will the clock display after 24 hours at the higher temperature if it read 3:00 p.m. at the instant the temperature was increased?**

This is a very straightforward piece of number crunching. I just wanted you to refresh your memories about what a period is. Also, the results are interesting in their own right.

The "period" of an oscillating system is the total amount of time needed for it to go through one complete cycle of its oscillation. In this case, that time is 1.00017 seconds. We can write $24\,\text{hours} = n\times 1.00017s$ where n is the number of oscillations (not necessarily an integer). A teensy bit of algebra gives

$$n = \frac{24\,\text{hours}}{1.00017s} = \frac{86400s}{1.00017s} = 86385.3 .$$

So the pendulum will go through 86385.3 cycles in one day instead of 86400 cycles, as it was designed to do.

The repercussion of the variation that we've just calculated is that a clock built to use this pendulum will not keep correct time. Notice that it's off by about 14.7 cycles per day—that is, about 14.7 seconds. So, the clock will read 2:59:44.3 instead of 3:00. A clock that slips by 15 seconds each day rapidly accumulates an error that is quite severe! The implications of this for navigation and its solution will be discussed in class.

3. **A cube of aluminum that is 5 cm on each side is placed in 1 liter of water at 10° C in a graduated cylinder. The water is warmed up slowly, so that the aluminum remains in equilibrium with it, to a temperature of 90° C. What volume is indicated on the graduated cylinder at the higher temperature? (Ignore thermal expansion of the cylinder.)**

This really is pretty much just plug-and-chug, but it's always a good idea to do some thinking first. Let's begin by figuring out what's going on here. We have 1 liter of water and a 5 cm cube of aluminum. The cube and the water warm up. Both of these objects will expand when their temperature increases, so the total volume will increase. Since the aluminum cube is inside the water, the volume measured by the graduated cylinder will be the sum of the two volumes—i.e., the total volume of the system. We need to figure out what this is. Let's do it.

The initial volume of the cube is $V_c = (5cm)^3 = 125cm^3$. The initial volume of the water is $V_w = 1l = 1000cm^3$. The equation for the change in volume of a substance or object when its temperature changes is essentially identical to the one for linear expansion

$$\Delta V = V_0 \beta \Delta T .$$

Notice that the only formal difference is that we've replaced α with β—the 3-d coefficient of thermal expansion. (In almost all cases, an excellent approximation is $\beta = 3\alpha$, as discussed in class.) So the final volumes of the cube and the water, respectively, are

$$V_{fc} = V_c + \Delta V_c = V_c\left(1+\beta_{Al}\Delta T\right)$$
$$V_{fw} = V_w + \Delta V_w = V_w\left(1+\beta_{H_2O}\Delta T\right).$$

Again consulting Table 12.1 in your text, we find

$$\beta_{Al} = 6.9\times10^{-5}\left(C°\right)^{-1}$$
$$\beta_{H_2O} = 2.07\times10^{-4}\left(C°\right)^{-1}.$$

So, inserting numbers, we get

$$V_{fc} = V_c\left(1+\beta_{Al}\Delta T\right) = 125\,cm^3\left(1+80°C\times6.9\times10^{-5}\left(C°\right)^{-1}\right) = 125.7cm^3$$
$$V_{fw} = V_w\left(1+\beta_{H_2O}\Delta T\right) = 1000\,cm^3\left(1+80°C\times2.07\times10^{-4}\left(C°\right)^{-1}\right) = 1016.6cm^3.$$

Thus the final total volume is

$$V_t = V_{fc} + V_{fw} = 125.7cm^3 + 1016.6cm^3 = 1142.3cm^3.$$

4. **At 24° C a steel nut is threaded onto a brass bolt. The bolt has a diameter of ¼ inch. The nut is slightly loose, with a diameter 10 microns larger than that of the bolt. Assuming the nut and bolt always have the same temperature as each other, at what temperature will the nut be tight?**

Just remember: A hole in a solid will change its size in any direction as though it were filled with that solid. So, as the temperature of the nut and bolt is changed, both the hole in the nut and the diameter of the bolt will expand or contract. The amount of the change will be different because the two are made of different materials, even though the temperature change will be the same for both.

We'll do this algebraically in a moment, but let's think about the answer first: Will we have to heat the pair up or cool them down? Let's look at the coefficients of linear thermal expansion for the two materials. As before, this is found in your text. We have

$$\alpha_{steel} = 1.2\times10^{-5}\left(C°\right)^{-1}$$
$$\alpha_{brass} = 1.9\times10^{-5}\left(C°\right)^{-1}.$$

Notice that the nut is slightly loose. If we heat the pair up, the bolt will expand more than the nut will and so the nut will become less loose as the temperature increases. So we expect our change in temperature to be positive. Now let's confirm this with some math.

For either the nut or the bolt, we can write the final diameter as

$$D_f = D_0(1+\alpha\Delta T).$$

Although the initial diameters are different for each of the objects, their *final* diameters will be the same—that's the condition stated in the problem. So, we can just set these equal to each other

$$D_{0nut}(1+\alpha_{nut}\Delta T) = D_{0bolt}(1+\alpha_{bolt}\Delta T).$$

Recall that we *do* know the difference between the two initial diameters—let's call that $\Delta D_0 = D_{0nut} - D_{0bolt}$ just to keep things neat. So a bit of algebra gives

$$\Delta D_0 = (D_{0bolt}\alpha_{bolt} - D_{0nut}\alpha_{nut})\Delta T$$

and, finally

$$\Delta T = \frac{\Delta D_0}{(D_{0bolt}\alpha_{bolt} - D_{0nut}\alpha_{nut})}.$$

Now, it would be perfectly acceptable to stick in numbers at this point. But we can make a very good approximation and save a bit of work: The initial diameters of the nut and bolt differ by only a tiny bit. In the denominator, we can just take $D_{0bolt} = D_{0nut} = 1/4\,\text{inch}$. I'll do this both with and without the approximation to show you.

Using the approximation given above, we have

$$\begin{aligned}\Delta T &= \frac{\Delta D_0}{D_0(\alpha_{bolt} - \alpha_{nut})} = \frac{10\,microns}{\tfrac{1}{4}\,inch\left(1.9\times10^{-5}\,(C^\circ)^{-1} - 1.2\times10^{-5}\,(C^\circ)^{-1}\right)} \\ &= \frac{1\times10^{-5}\,m}{6.35\times10^{-3}\,m\times7\times10^{-6}\,(C^\circ)^{-1}} = 225^\circ C\end{aligned}.$$

If we insist on doing it without the approximation, we must use

$$D_{0bolt} = 1/4\,\text{inch} = 6.35\times10^{-3}\,m$$

while

$$D_{0nut} = 1/4\,\text{inch} + 10\text{ microns} = 6.36\times10^{-3}\,m.$$

These give

$$\Delta T = \frac{\Delta D_0}{\left(D_{0bolt}\alpha_{bolt} - D_{0nut}\alpha_{nut}\right)} = \frac{10\,microns}{\left(6.35\times10^{-3}m\times1.9\times10^{-5}\left(C^\circ\right)^{-1} - 6.36\times10^{-3}m\times1.2\times10^{-5}\left(C^\circ\right)^{-1}\right)}$$
$$= 225.6^\circ C$$

.

This is certainly not identical to the approximate answer we found above, but it's mighty close and it was a lot more work. You are encouraged, strongly, to attempt approximations like this in your problem solving! Getting good at seeing these opportunities can really save you effort down the road.

Note that we've found the *change in* the temperature needed to get the nut to snug onto the bolt. The question asked for the actual, final temperature. So we must add 24° C to these answers to get the final temperature: $T_f = 225^\circ C + 24^\circ C = 249^\circ C$.

5. How much energy is needed to raise the temperature of a piece of aluminum with a mass of 38 grams by 6 degrees C? (See the table in your textbook on page 373.)

Although we could just plug into a formula for this, I prefer to go the long way around so that you can get some idea of how this relates to the first law of thermodynamics. We hadn't yet seen the First Law when you did this problem, but you know it now. This is $\Delta U = Q - W$. Stated in words: The change in internal energy of a system is equal to the heat put into the system minus the work done ***by*** the system. In this case, no work is being done by the system, so we can say $\Delta U = Q$. The change in internal energy (ΔU) shows up as a change in temperature. Note that this is not the only way that a change in internal energy can be manifested. For example, we could induce a chemical change in the system or a phase change. A change in temperature is the easiest one to see and to understand, however.

So heat flows into the piece of aluminum and changes its internal energy and that change shows up as a change in temperature. The relationship between the change in internal energy and the change in temperature is called the "specific heat" of the substance and is denoted by the letter *c*. This must be scaled by the quantity of matter—the mass of the sample under consideration.

Putting these together, we get the formula $Q = mc\Delta T$. Contrary to our usual policy, we often express the mass in a calculation like this in grams rather than kilograms. This is because the specific heat is frequently given in joules per gram-kelvin (if it is given in kilojoules per kilogram-kelvin the number will be the same; in this latter case, we can express the mass in kilograms, a standard unit, but then we must remember to express the energy in kilojoules rather than joules). Checking the table in your text, you will see that the specific heat of aluminum is $c = 0.9\frac{\text{joule}}{\text{gram}\cdot\text{kelvin}}$. So, in our specific case, we have $Q = mc\Delta T = 38\,\text{grams}\times 0.9\frac{\text{joule}}{\text{gram}\cdot\text{kelvin}}\times 6\,\text{kelvin} = 205.2\,\text{joules.}$

6. A Snickers® bar contains 280 Calories (1 Calorie=1000 calories) of chemical potential energy. If such a bar is burned, converting all of its chemical potential energy to heat, by how much could it raise the temperature of a 500 gram sample of water?

Usually, I'd recommend converting the kilocalories into joules and then just working with them. However, since we are working with water in this case, the specific heat is particularly easy: A kilocalorie is *defined* to be the amount of heat needed to raise the temperature of one kilogram of water by one degree Celsius. So let's stick with kcals. Now we have

$$Q = mc\Delta T = 500\,\text{grams}\times 1\frac{\text{calorie}}{\text{gram}\cdot\text{kelvin}}\times \Delta\text{T} = 2.8\times 10^5\,\text{calories.}$$

Dividing to solve for the temperature increase, we have

$$\Delta\text{T} = \frac{2.8\times 10^5\,\text{calories}}{500\,\text{grams}\times 1\frac{\text{calorie}}{\text{gram}\cdot\text{kelvin}}} = 560\,\text{K.}$$

Clearly this is enough to raise the temperature to the boiling point of the water and then to convert much of it to steam! (I flubbed a decimal when I wrote the problem or

I would have specified a larger sample of water. My apologies for any confusion this caused!)

Although it wasn’t asked for in the problem, let’s take this the next step: How much of the water will be turned to steam? Let’s think about what’s happening to figure out how to handle it: Heat (from the burning candy bar) flows into the water. This raises its temperature. But once the water hits approximately 100° C (it’s approximate because the precise boiling temperature depends on the atmospheric pressure), the addition of heat does not raise the water temperature any further. Rather, it is used in creating the phase change to turn the water into vapor. A *huge* amount of energy is needed to accomplish this: Each gram of water converted to vapor requires 2260 joules of extra energy!

So, to figure out how much water gets vaporized, we’ll first need to figure out how much of the available energy is needed just to raise the water’s temperature to the boiling point. Since I didn’t tell you the starting temperature of the water, we’ll have to make an assumption. (This is always a valid thing to do: If I don’t give you a quantity that you think is needed for a calculation, you have two option. The *best* thing to do is simply use a symbol to represent the quantity. Often, you’ll discover that this quantity cancels out later in the calculation. If it doesn’t cancel and you actually want a numerical solution [almost always, I’m perfectly satisfied with a symbolic result], then simply make up a reasonable number. Just communicate with me and your grader that you’ve made this substitution and give a couple of words of justification for it.) Let’s just assume that the water started at about room temperature—let’s say 24° C. To reach the boiling temperature of 100° C, we’ll need to raise the temperature by 76° C, this is our ΔT. So the total energy needed to bring the water to the boiling point is

$$Q_{boil} = mc\Delta T = 500\,\text{grams} \times 1\frac{\text{calorie}}{\text{gram}\cdot\text{kelvin}} \times 76\,\text{K} = 3.8\times 10^{4}\,\text{calories.}$$

Now, we know that we have $Q_{tot} = 2.8\times 10^{5}\,c$, so after we’ve got the water boiling, whatever’s left over will go to creating vapor. This is $Q_{vapor} = 2.8\times 10^{5} - 3.8\times 10^{4} = 2.42\times 10^{5}\,\text{calories}$. The “latent heat of vaporization” of a substance is the amount of energy needed to turn a quantity of it into vapor assuming that it’s already at its boiling temperature. For water, this is $L_v = 2260\frac{joules}{gram} = 539.9\frac{calories}{gram}$. So, taking m to be the mass of water turned into vapor, we have $Q_{vapor} = 2.42\times10^5\,\text{calories} = m\times L_v$. This can easily be solved to give $m = \dfrac{2.42\times10^5\,\text{calories}}{539.9\frac{\text{calories}}{\text{gram}}} = 448.2\,\text{grams}$. So our candy bar could boil away almost our entire 500 gram sample of water!

(I won't do the extra math here, but now consider the possibility if we found that we had *more* than enough energy to boil the water. What then? Well, assuming that we still put that energy into the water, which is now vaporized, we'd be able to increase the vapor's temperature above the 100° point. Using whatever energy was left, we'd just use $Q = mc\Delta T$ again to find the final temperature of the vapor. We'd have to be careful to use the specific heat for water vapor, not water, however.)

7. A bartender wishes to cool down a shot of pure ethanol with a mass of 35 grams by adding a cold chunk of glass to it. (Again, see the table in your text.) If a 5 gram piece of glass is transferred from a bath of liquid nitrogen (T=77 K) into the ethanol (T=300 K), what will be the final temperature of the ethanol?

The Physics in this problem is simple, but the algebra can be tricky, so let's do it one step at a time. If we treat the ethanol and the chunk of glass as distinct systems, in each case we have $\Delta U = Q$. Now, in the case of the glass, since heat will flow *into* the glass ΔU will be positive. In the case of the ethanol, the ethanol will cool off (get colder), so ΔU will be negative—energy will flow *from* the ethanol. Now, any energy that enters the glass must have come from the ethanol, so we can say $\Delta U_{glass} = -\Delta U_{ethanol}$. Thus $m_{glass}c_{glass}\Delta T_{glass} = -m_{ethanol}c_{ethanol}\Delta T_{ethanol}$.

Recall that "Δ" means final value minus initial value. So $\Delta T = T_{final} - T_{initial}$ but we have a minus sign for the ethanol's change in temperature, so let's write $-\Delta T_{ethanol} = T_{initial} - T_{final}$. In other words, just flip the order.

Finally, realize that the *final* temperature of both the ethanol and the glass will be the same—if there were any difference between the temperatures of the substances, heat would flow from one to the other until the temperatures were the same. This is what temperature *is*, after all: Temperature is the thing that stops changing when two things are in equilibrium. So we can write $m_{glass}c_{glass}\left(T_{final}-T_{initial}^{(glass)}\right)=m_{ethanol}c_{ethanol}\left(T_{initial}^{(ethanol)}-T_{final}\right)$. Collecting terms with the final temperature in them on the left, we have $T_{final}\left(m_{glass}c_{glass}+m_{ethanol}c_{ethanol}\right)=m_{ethanol}c_{ethanol}T_{initial}^{(ethanol)}+m_{glass}c_{glass}T_{initial}^{(glass)}$. Before proceeding, look at what we've got: On the left is the total thermal energy in the system at the end of the process. On the right is the total thermal energy in the system at the beginning of the process. Since nothing enters or leaves the system, these two are equivalent.

Now, we solve for the final temperature by dividing and substituting numbers (some of which can be found in the table in your book):

$$T_{final}=\frac{m_{ethanol}c_{ethanol}T_{initial}^{(ethanol)}+m_{glass}c_{glass}T_{initial}^{(glass)}}{\left(m_{glass}c_{glass}+m_{ethanol}c_{ethanol}\right)}$$

$$=\frac{35\,gm\times 2.45\frac{J}{gm\cdot K}\times 300\,K+5\,gm\times 0.84\frac{J}{gm\cdot K}\times 77K}{5\,gm\times 0.84\frac{J}{gm\cdot K}+35\,gm\times 2.45\frac{J}{gm\cdot K}}$$

$$=\frac{2.605\times 10^4}{89.95}=289.6\,K$$

So the drink will be cooled down by about 10.4 degrees. The bartender should have used something with a higher specific heat. The thing that made this so inefficient is the fact that the ethanol has such a high specific heat and the glass has such a low specific heat: There's a lot of energy to be gotten rid of in the ethanol, but the glass just can't accept that much energy without heating up significantly and the process stops as soon as they are at the same temperature.

8. Consider again the candy bar in problem #6. If the energy is used to heat up a sample of ice at an initial temperature of –10°C, what is the maximum mass of ice that could be completely melted with the burning candy bar?

It takes energy to warm up ice to a particular temperature. It also takes energy to change the phase of the ice from solid to liquid. In order to melt the ice, we must first add enough energy to it to raise its temperature by 10 degrees, to get it to the melting point, and then we must add even more energy to do the actual melting.

To get to the melting point, we use energy found using the same formula as in previous problems. This gives $Q_{melt} = mc\Delta T = \mathrm{m} \times 1 \frac{\text{calorie}}{\text{gram} \cdot \text{kelvin}} \times 10\,\text{kelvin}$. This is just the energy to get the ice to the melting point. To get it to melt, we must add an additional $Q_{fusion} = mL_f$ where *m* is the mass of the ice and L_f is the heat of fusion. For ice, $L_f = 333.7\frac{J}{gm} = 79.72\frac{cal}{gm}$. So $Q = Q_{melt} + Q_{fusion} = mc\Delta T + mL_f$. Solving for the mass, we have $m = \frac{Q}{c\Delta T + L_f} = \frac{2.8 \times 10^5\, cal}{1\frac{cal}{gm \cdot K} \times 10\,K + 79.72\frac{cal}{gm}} = 3.12 \times 10^3\,\text{gram}$.

9. A bar of aluminum is used as a heating element. It is a cylinder with a diameter of 5 mm and a length of 7 cm. An electric current is run through it so that it heats up to a desired temperature. When it reaches its final temperature, it dissipates 1000 W. At what wavelength will the “glow” from this object peak?

We pretty much did this completely in class using the filament of a lightbulb instead of the bar of aluminum. According to the Stefan-Boltzmann law, the hotter something is, the more power each region on the object’s surface will emit in the form of “electromagnetic radiation.” Now, you don’t know what electromagnetic radiation is, so there’s no way that this can be truly meaningful for you at this point! But I assure you that we will cover this subject more fully later this semester. For now, let’s be content with a simple level of understanding.

The key relation needed to work this problem is not in your book (which is a defect! it belongs there—I’ll be sending a statement to this effect to the publisher), but we discussed it in class. It is called “Wien’s displacement law." A nice little discussion of it can be found at http://en.wikipedia.org/wiki/Wien's_displacement_law. Basically, Wien's displacement law says that the intensity of the electromagnetic radiation emitted by an object at a temperature T will be greatest at a wavelength given by

$$\lambda = \frac{2.898\times10^{-3}\,m\cdot K}{T}.$$

So, if we can find the temperature of the object, we can find the desired wavelength.

Now, the temperature of an object and the energy radiated from it is given by the Stefan-Boltzmann equation:

$$Q = e\sigma T^4 A t.$$

Here, Q is the total heat lost to the object via radiation in a time t. The material parameter, e, is the "emissivity." This can take values between 0 and 1, but for most solids it is on the large side and for metals it is quite reasonable to approximate it as being 1. The temperature (on an absolute scale) is denoted by T, A is the surface area of the object, and $\sigma = 5.67\times10^{-8}\frac{J}{s\cdot m^2\cdot K^4}$ is the Stefan-Boltzmann constant—a universal constant. If the temperature is constant, then the power is simply the energy (heat) dissipated divided by the time over which this happens, so

$$P = e\sigma T^4 A.$$

The Stefan-Boltzmann equation can easily be solved for the temperature with a tiny bit of algebra. We have

$$T = \sqrt[4]{\frac{P}{\sigma A}}$$

where we approximate $e \approx 1$. The surface area of the cylinder can be calculated using the numbers given in the statement of the problem. Since the cylinder is very long compared to its diameter, we can ignore the energy radiated from the "caps" of the cylinder and just say $A = 2\pi RL$. Substituting the numbers given, we get $A = 2\pi RL = 1.1\times10^{-3}m^2$. Now all we need to do is put this number and the others given into our equation for the temperature. This gives

$$T = \sqrt[4]{\frac{P}{\sigma A}} = \sqrt[4]{\frac{1000W}{5.67\times10^{-8}\frac{J}{s\cdot m^2\cdot K^4}\times1.1\times10^{-3}m^2}} = \sqrt[4]{1.6\times10^{13}K^4} = 2001K\ .$$

We now need only insert this temperature into Wien's displacement law to get

$$\lambda = \frac{2.898\times10^{-3}\,m\cdot K}{T} = \frac{2.898\times10^{-3}\,m\cdot K}{2001K} = 1.45\times10^{-6}\,m\ .$$

As we'll learn later in the semester, what we call "light" is just electromagnetic radiation with wavelengths in a particular, narrow range. The color of light depends on its particular wavelength. Electromagnetic radiation with a wavelength of $1.45\times10^{-6}m$ cannot be seen directly by human eyes, but it's pretty close. This is light that we call "infrared." It can easily be detected with simple instruments. Also, Wien's displacement law tells us that the maximum intensity of the radiation will be at this wavelength. But a very wide range of wavelengths will also be produced at quite high intensities as well. So this bar of aluminum would certainly be seen to glow if you were to look at it.

Problem #10 has been moved to the next homework assignment.