Physics 206b
Homework Assignment XV
SOLUTIONS

1. **A pair of narrow slits is illuminated by a monochromatic (i.e., a single wavelength of light) source of coherent light with a wavelength of 546 nm. The slits are 3 microns apart. A screen is placed 2 meters from the pair of slits. How far from the central bright spot (n=0) will the first dark region appear on the screen? How far from the central bright spot will the next bright spot appear on the screen?**

We didn't derive this explicitly in class, but I stated it for you. It is derived in your text. The formula for the angle at which the dark region will appear is $\left(n+\frac{1}{2}\right)\lambda = d\sin(\theta)$ where n is any integer, λ is the wavelength of the light, and d is the distance between the slits. Let's call the distance from the central bright region to the point of interest x and the distance from the slits to the screen L. We have (for small angles) $\sin(\theta) = \frac{x}{L}$. (The limitation to small angles is needed to avoid having to use $\tan(\theta)$ here. The approximation is an excellent one for this system!) Thus we have $\left(n+\frac{1}{2}\right)\lambda = d\sin(\theta) = d\frac{x}{L}$. Taking $n=0$ (since we're interested in the first dark region) and using the numbers given in the problem, we have (after solving for x) $x = \frac{\lambda L}{2d} = \frac{5.46\times10^{-7}m\times 2m}{2\times3\times10^{-6}m} = 18.2cm$.

The formula for the first bright spot after the central one is $n\lambda = d\sin(\theta)$. Following the same development as before but taking $n=1$ (after all, $n=0$ is the central bright region), we have $x = \frac{\lambda L}{d} = \frac{5.46\times10^{-7}m\times 2m}{3\times10^{-6}m} = 36.4cm$.

2. **The work function of potassium is 2.3 eV. What is the maximum wavelength of light for which potassium will emit electrons via the photoelectric effect?**

We have $E_{electron} = hf - W$. To find the minimum energy photon (maximum wavelength), we will set this equal to zero. So we have $E_{electron} = hf_{\min} - W = 0$ which gives $hf_{\min} = \frac{hc}{\lambda_{\max}} = W$ (using $\lambda f = c$). All we need to finish the problem is the conversion factor (make sure you understand where this comes from!!!) $1eV = 1.6\times10^{-19} J$ and we can write

$$\lambda_{\max} = \frac{hc}{W} = \frac{6.626\times10^{-34} J\cdot s\times3\times10^{8}\,\frac{m}{s}}{2.3eV\times1.6\times10^{-19}\,\frac{J}{eV}} = 540\,nm.$$

3. **Again taking the work function of potassium to be 2.3 eV. If a sheet of potassium is illuminated with light with a wavelength of 400 nm (just at the threshold between visible and ultraviolet), with what speed will electrons be emitted from its surface?**

The energy of the electrons is given by $E_{electron} = hf - W = \frac{hc}{\lambda} - W$ so we have

$$E_{electron} = \frac{hc}{\lambda} - W = \frac{6.626\times10^{-34} J\cdot s\times3\times10^{8}\,\frac{m}{s}}{4\times10^{-7} m} - 2.3eV\times1.6\times10^{-19}\,\frac{J}{eV} = 1.29\times10^{-19} J$$

(It is *far* easier to just work in eV rather than converting to joules all the time, by the way, but since this is a new unit for many of you, it is far *safer* to stick with something familiar and just do the conversion to joules.)

Now this is the kinetic energy of the electrons. We want their speed, so we use our old friend relating *K.E.* to speed $v = \sqrt{\frac{2E}{m}}$. This gives $v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2.58\times10^{-19} J}{9.11\times10^{-31} kg}} = 5.32\times10^{5}\,\frac{m}{s}$.

4. **A man is walking slowly. His speed is $1\frac{m}{s}$. His mass is $80\,kg$. What is his de Broglie wavelength?**

This is $\lambda=\frac{h}{p}$. Finding the momentum first, we have $p=mv=80\,kg\times 1\frac{m}{s}=80\,kg\cdot\frac{m}{s}$. Thus $\lambda=\frac{h}{p}=\frac{6.626\times 10^{-34}\,J\cdot s}{80\,kg\cdot\frac{m}{s}}=8.28\times 10^{-36}\,m$.

For comparison, the diameter of the *nucleus* of an atom is about $10^{-15}\,m$. The diameter of an atom is about $10^{-10}\,m$.

5. **Consider again the experiment described in problem #1. The slit separation is the same and the distance from the slits to the screen is the same. One wishes to have the same pattern of "dark" and "light" but this time using electrons ("light" would mean many electrons while "dark" would mean few or none, in this case). If one wanted the same distance from the center to the first dark and light spots, what energy of electrons would one use? Express this energy in electron volts.**

I hope you recognized my typo and that I meant problem #2! If not, go back and redo it, I'll wait here...

Now, for the same pattern, we'll just need the same wavelength. Thus we expect to have $\lambda=546nm$ (I could have left most of the verbiage out of this problem and just told you the wavelength of the electrons right off, but I wanted you to reason through this yourselves). From this, we can find the momentum using de Broglie's relation $p=\frac{h}{\lambda}$. The kinetic energy is related to the momentum by $E=\frac{p^2}{2m}$ so we have $E=\frac{h^2}{2m\lambda^2}=\frac{\left(6.626\times 10^{-34}\,J\cdot s\right)^2}{2\times 9.11\times 10^{-31}\,kg\times\left(546nm\right)^2}=8.08\times 10^{-25}\,J$. Now we convert this to electron volts using the scale factor $1.6\times 10^{-19}\,\frac{J}{eV}$ and we have $E=\frac{8.08\times 10^{-25}\,J}{1.6\times 10^{-19}\,\frac{J}{eV}}=5.05\times 10^{-6}\,eV$. By comparison the energy of a photon with the same wavelength would be $E=\frac{hc}{\lambda}=2.28eV$ — quite a difference between de Broglie wavelengths of massive objects and the wavelengths of electromagnetic waves!

6. **A beam of electrons with an energy of 20 eV passes through a small hole with a diameter of 500 nm. If the electrons are initially all traveling in the $\hat{z}$ direction, what will their approximate momentum be in the $\hat{y}$ direction after they pass through the hole?**

The passage through the hole constitutes a measurement of their y position. Since there is an uncertainty of this measurement of $\Delta y = 5\times10^{-7}m$, the diameter of the hole, there will be a concomitant uncertainty in the momentum in the $\hat{y}$ direction. The *minimum* value of this is set by Heisenberg's Uncertainty relation $\Delta y \Delta p_y \geq \frac{\hbar}{2}$. (You may well see slightly different values given for this. The quantity is an estimate, not a precise value.) Please note that this is a *minimum*! One can certainly have uncertainties larger than those predicted by Heisenberg's relation, just nothing smaller than them.

Now, since the electrons have *no* momentum in the $\hat{y}$ direction before passing through the hole, any momentum they have in that direction after passing through the hole is due to the measurement made by the hole. Using this, we have $p_y = \frac{\hbar}{2\times5\times10^{-7}m} = \frac{6.626\times10^{-34}J\cdot s}{4\pi\times5\times10^{-7}m} = 1.05\times10^{-28}kg\frac{m}{s}$.

Now, if you want to push yourselves: The total momentum and energy are still conserved. Use the fact that the total momentum is unchanged to find the final *angle* at which the electrons emerge.

To do this, we realize that $\frac{p_y}{p_{total}} = \sin(\theta)$ and, using conservation of energy, we have

$$p_{total} = \sqrt{2mE} = \sqrt{2\times9.11\times10^{-31}kg\times20eV\times1.6\times10^{-19}\frac{J}{eV}} = 2.41\times10^{-24}kg\frac{m}{s}.$$

This gives $\frac{p_y}{p_{total}} = \sin(\theta) = \frac{1.05\times10^{-28}kg\frac{m}{s}}{2.41\times10^{-24}kg\frac{m}{s}} = 4.35\times10^{-5}$, an angle of about $\theta = 2.5\times10^{-3}$ degrees. This is small, but nowhere near zero!

7. **The Paschen series is the set of spectral "lines" emitted from hydrogen atoms undergoing transitions terminating on the n=3 level. What are the wavelengths of the light emitted from the three most energetic transitions in the Paschen series?**

The energies of the Paschen series are given by $E = -13.6eV \times \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$ with $n_f = 3$—the principal quantum number of the terminal orbital. Now, the energy of a photon is related to its frequency via Planck's relation, $E = hf$. But the frequency is related to the wavelength (in vacuum) by $\lambda f = c$. So the energy of a photon is related to its wavelength by $E = \frac{hc}{\lambda}$. This allows us to rewrite the above formula in terms of wavelength and just lump all the constants together into a new constant. This is knows as the "Rydberg constant," denoted by the letter R, and it has the value $R = 1.09737 \times 10^7 \frac{1}{m}$. Thus, we have $\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$.

I made an error in the statement of the problem. I meant to say *least* energetic transitions. There is no *set* of most energetic transitions, although, as we'll see in a moment, there is a limit for the absolute most energetic transition. My apologies for any confusion.

The most energetic photon is what's known as the "band edge." It is the limit for the wavelength as n_i approaches infinity. This gives $\frac{1}{\lambda} = R\frac{1}{n_f^2} = \frac{R}{9} = 1.2193 \times 10^6 m^{-1}$ which, in turn, yields $\lambda = 8.2 \times 10^{-7} m$ or 820 nm.

Solving for the *least* energetic transitions (as I'd intended), we would have $n_i = 4, 5, \text{or } 6$. These give, respectively, $\lambda_4 = 1.874 \times 10^{-6} m$, $\lambda_5 = 1.281 \times 10^{-6} m$, and $\lambda_6 = 1.0935 \times 10^{-6} m$.

8. **How fast would someone need to move relative to an observer for that person's watch to appear to move at half the speed of the watch of the observer?**

We need to define an "event" with a duration. Let's call the event the motion of a clock's hands through 1 hour. Now, there will be two frames in which the event is observed. In one of those frames, the beginning and end of the event will occur at the same spatial point. I.e., if the clock is hanging on a wall in a ship, its position in the ship's frame will not change in that hour. On the other hand, that same clock's position *will* change in the frame of an observer on the shore. The time as measured in the frame in which the position doesn't change is called the "proper time" and is denoted by t_0. The time as measured in the other frame is denoted by t and has no special name.

We want the time as measured in the other frame to be twice the proper time. Using the Lorentz formula for time dilation, this is $t = 2t_0 = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$. Thus $\sqrt{1-\frac{v^2}{c^2}} = \frac{1}{2}$. This gives $\frac{v}{c} = .87$ or $v = 2.6\times 10^8 \frac{m}{s}$.