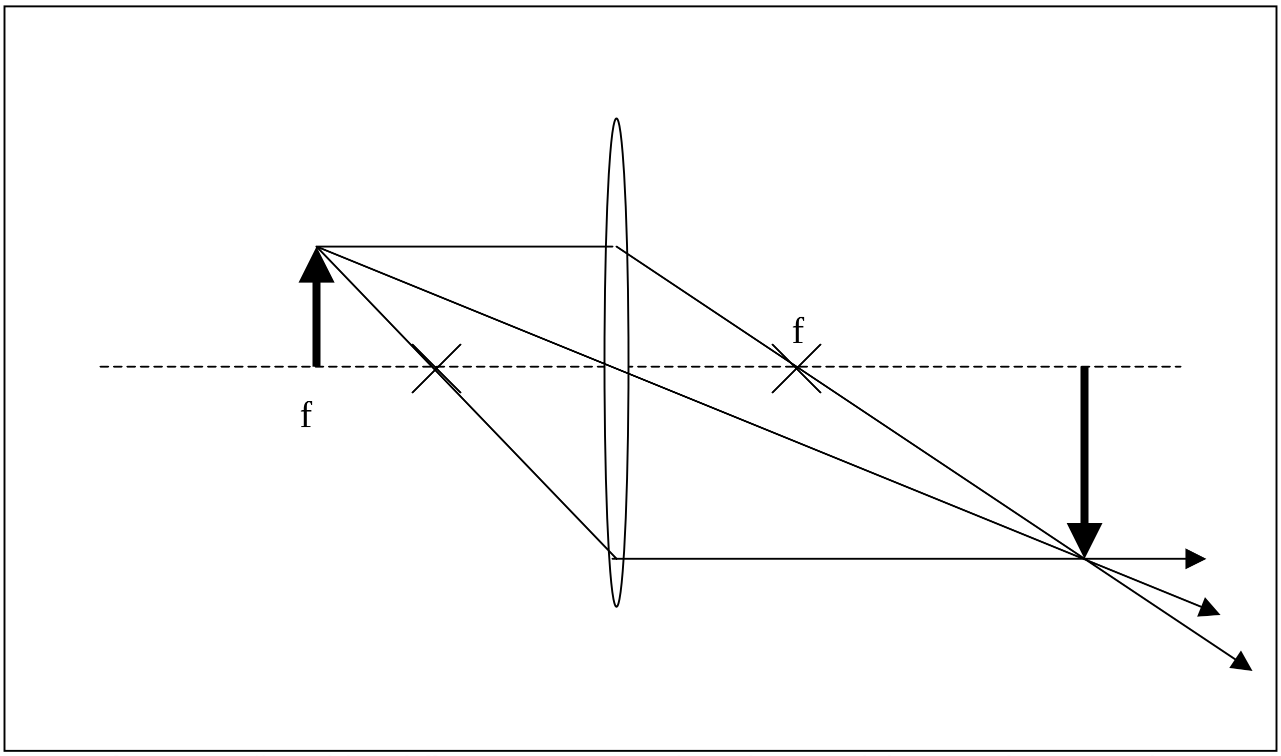
Physics 206b
Homework Assignment XIV
SOLUTIONS

1. **An object is placed 14 cm away from a lens with a focal length of 9 cm. Use ray tracing to determine, qualitatively, the location of the image. (I.e., you needn't be ultra-precise in this, but do keep the distances as close to proportional as you can.)**



Here's a "cookbook" way of drawing this: First, draw a centerline for reference. Then, draw an icon representing the lens. Remember that we are using a "thin lens" approximation, which means that we don't need the lens to be accurately represented. We'll consider the thickness to be zero and act like all the bends that it makes in the light happen along a plane. So we just sketch in something to remind us where the lens is. Since lenses are symmetric in the thin lens approximation, light from the left traveling parallel to the optic axis will cross the axis a distance f from the lens and light from the right traveling parallel to the optic axis will also cross the axis a distance f from the lens. We indicate these distances by drawing an X on either side of the lens a distance f from it (appropriately scaled, of course). The locations of the Xs are called the "focal points." Note that a serious source of error is to confuse a focal point with an image point! The focal point is the image point for an object at infinity. All other object distances have their own image distances, but the focal points for a lens are independent of the object locations.

So far, everything can be done pretty loosely. The next step has to be done with some care. That is, putting in the object. An arrow makes a dandy object, since it has a definite top and bottom and a Physicist can draw one pretty well. This needs to get drawn the same scale of distance from the lens as the focal length. So, for example, in this case the object is about 50% greater distance from the lens than the focal length. This should be drawn to the right ratio.

The next step is to put in some rays. We have three special rays for this—please bear in mind that there are an infinite number of rays going from the object, through the lens, to the image! But if it's a true image, then all the rays will go there (that's the whole "one-to-one" thing). So, if three rays go there, or even two, then all the others will as well. Sometimes, only two are possible, which is fine. The third does give a nice bit of redundancy. You'll *always* be able to do at least two, however. The three rays we have are:

i. The ray that comes off the object parallel to the optic axis (the centerline): This ray bends at the lens and then crosses the optic axis a focal length from the lens.

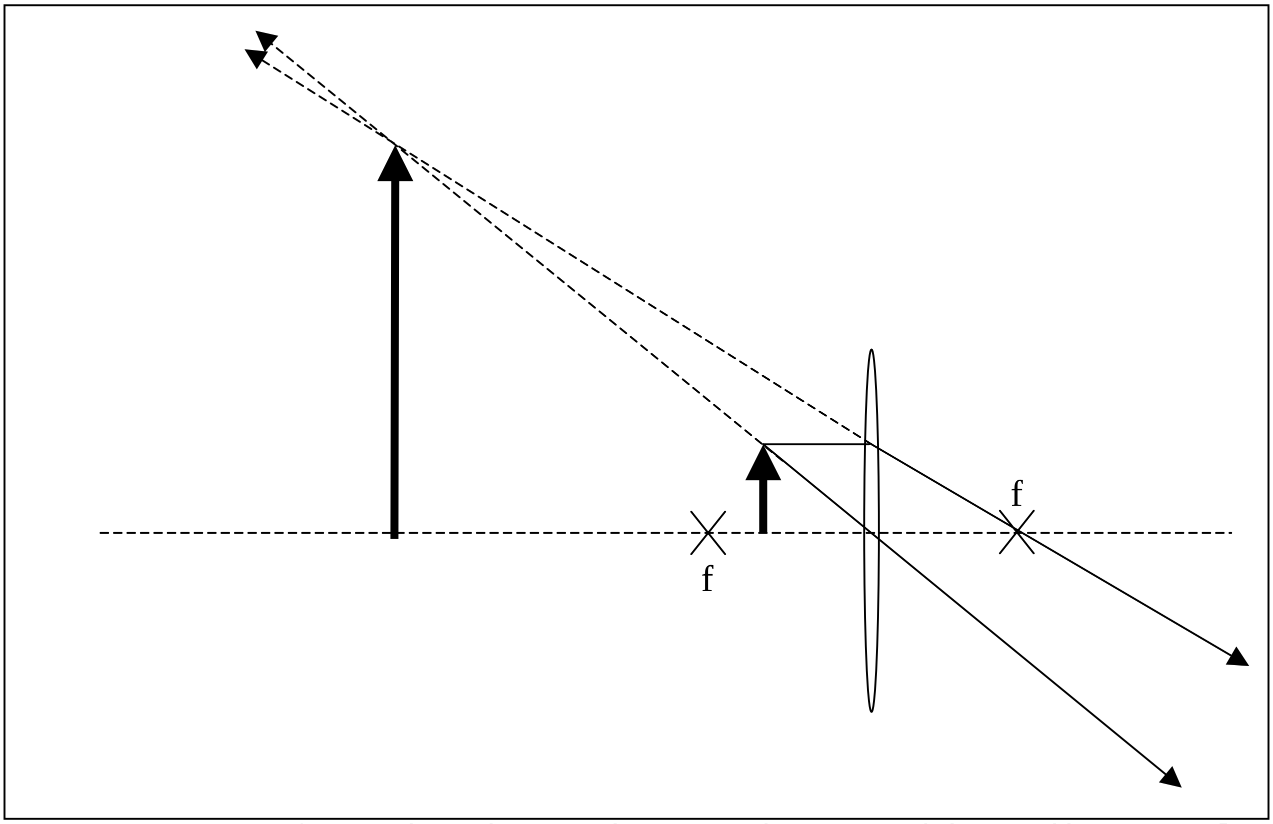
ii. The ray which passes through the center of the lens: This ray continues undeviated—just draw a line from the object through the center of the lens and keep on going.

iii. The ray which crosses the optic axis a focal length from the lens on the same side as the object: The ray is the analog to the one in #i. It hits the lens and then gets bent so that it's parallel to the centerline. Note that if the object is closer to the lens than the focal length, this ray cannot be drawn!

Draw these three rays and their continuations on the other side of the lens. See where they cross? That's the image point! We did this for rays from the very tip of the arrow. Try picking a couple of points along its shaft and do the same thing for those, following all the rules above. Convince yourself that each point on the object will be "mapped" to a point on the image.

Note that the rays all actually go to the image point. This point would pass the "paper test," so the image is real.

2. Using ray tracing, indicate how a single, thin lens can act as a "magnifying glass." What condition must be met by the object distance for this to work?



We can see from the above picture that an object distance less than the focal length of a lens with a positive focal length will produce a virtual image that is larger than the original object. Let's go through this systematically. Let's also introduce some numbers to make it a bit more concrete. Let's take an object distance of 40 cm and a focal length of 50 cm.

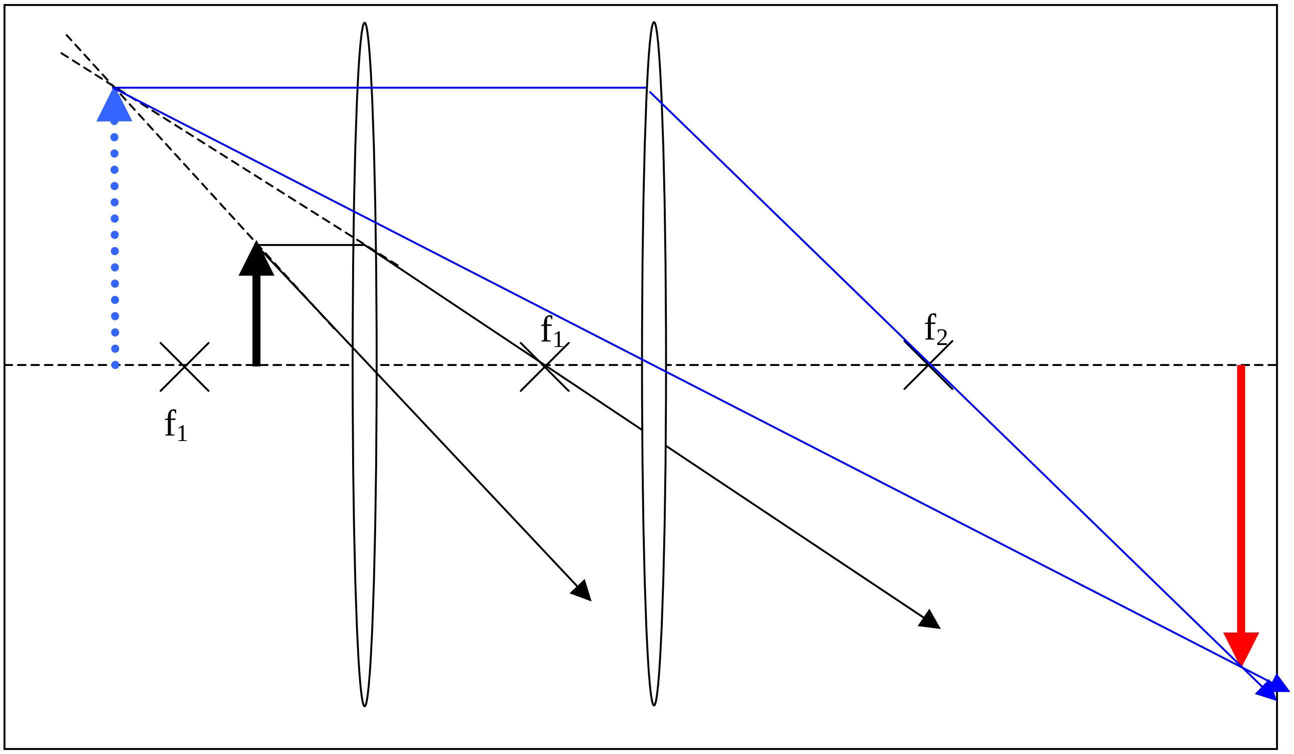
Now, since the object is closer than the focal length, we can't use special ray #iii. (We actually can, it just requires another little technique. But let's leave that alone for now.) But only two rays are truly essential since wherever they cross will be the image for all rays if our image is perfect. So let's just not worry about it. (This graphical technique can be used even for non-perfect images [i.e., not a one-to-one mapping], so we'd have to use the more complicated method if we were dealing with that situation.)

Using the two rays that we *can* draw, we quickly see that the rays diverge from each other after passing through the lens. There is no way they will form a real image. Not to worry, though—we know about virtual images. Notice that the rays do meet up, but at a point on the same side of the lens as the object. Anybody looking at the rays coming out of the lens would think that they came from an object much bigger and farther away from the lens than the original object. This is precisely what we mean by a "magnifying glass." The lens creates a virtual image that's many times larger than the object. You may notice that the image is also farther away than the object was, but this isn't nearly enough to offset the increase in size.

Let's look at the math on this, now. For clarity, I'll use d_i and d_o instead of p and q. Just as in the previous problem, we have $d_i = \frac{d_o \times f}{d_o - f}$. Putting in the numbers, we have $d_i = \frac{40 \times 50\ cm}{40\ cm - 50\ cm} = -200\ cm$. Notice the minus sign. This is how you know that the image is virtual, even without doing the drawing.

The ratio of the size of the virtual image to the object is the same as the ratio of the distance between the virtual image and the lens and the distance between the object and the lens. The ratio of the sizes is called the "magnification" and this can be expressed mathematically as $m = \frac{h_i}{h_o} = \frac{d_i}{d_o}$ where h_i and h_o are the heights of the image and the object, respectively. Using this formula and the image and object distances, we have $m = \frac{d_i}{d_o} = \frac{200}{40} = 5$. We write this as "5x" and would say "this has a 5x magnification."

3. **An object is placed 7 cm to the left of a lens with a focal length of 12 cm. A lens with a focal length of 19 cm is placed 17 cm to the *right* of the first lens. What is the location of the final image? Sketch this system with appropriate raytracing from the initial object through any intermediate steps to the final image.**



The key to this problem is to remember that the image created by the first lens is the object for the second lens. In this case, the image created by the first lens is virtual. No matter: We just put it where it belongs (the dotted blue arrow). This is the source of a set of rays (the blue rays) that pass through the second lens. Notice that these rays are unaffected by the first lens. Why? Well because they don't actually exist at the location of the first lens! The wonderful thing about images is that the very definition of "image" demands that an infinite variety of rays will exist for any object/image pair. All of these rays have the same property: They start at the object (virtual or otherwise) and pass through the image (likewise, virtual or otherwise). Basically, this means that if I can draw a line, it can represent a ray. Having found the location of the first image, I just use a different set of rays to find the second image. Piece of cake!

Just for yucks, let's use the thin lens formula to check the graphical result. The first image distance is $d_{i1} = \frac{d_{o1} \times f_1}{d_{o1} - f_1} = \frac{7cm \times 12cm}{7cm - 12cm} = -16.8cm$. This means that the object distance for the second lens will be $d_{o2} = 17cm + 16.8cm = 33.8cm$. This gives

$d_{i2} = \frac{d_{o2} \times f_2}{d_{o2} - f_2} = \frac{33.8cm \times 19cm}{33.8cm - 19cm} = 43.4cm$. Note that this distance is measured relative to the second lens. This is consistent with our drawing.

4. Explain why the following statement is *both* correct and incorrect: "Red light added to green light makes yellow light."

The word "color" is more than a little ambiguous: While it is true that different wavelengths of light have different colors when viewed by a human being, it is possible to stimulate the response of a human eye to (almost) any visible wavelength by a suitable combination of different brightnesses of three so-called "primary colors" selected to stimulate the three kinds of cone cells in a human retina just the right amount. So we have to distinguish between yellow light and a combination of colors which looks the same as yellow to a human being.

Mixing red light with green light does result in a human being seeing yellow. That is, everything in the human's visual system will react as though yellow light were hitting their eye. So red light mixed with green light makes yellow, from a purely visual perspective. (To prove this to yourself, go to your computer monitor and, using something like MS Word create an image that is yellow. Then, use a magnifying glass to get a good look at the "pixels" on the screen that make up the image. There will be red ones and green ones, but no yellow ones!)

However, the phrase "yellow light" implies that the light itself is yellow (roughly 570 nm) and that we're referring to the light and not just a human being's reaction to it. So the statement is false: You do not make yellow light by mixing green light with red light. You stimulate a response in a human being equivalent to that of yellow light if you do this, but you don't make the light itself.

5. Assume a human eye is 25 mm in diameter. A myopic (nearsighted) person can clearly see an infinitely distant object when wearing corrective lenses (eyeglasses). Lenses are often characterized by their "power" rather than their focal length. The power of a lens is simply defined as $P = \frac{1}{f}$ and the unit for this is the "diopter" where $1 diopter = 1\frac{1}{meter}$. A lens of this person's eyeglasses has a power of –5 diopters (this is a strong but not extreme prescription). What is the focal length of the biological lens in the person's eye?

This is a really convoluted way of asking a very straightforward question. I did it this way because I wanted to drive home an important point: The lens of a human eye *should* form a real image on the retina. This means that the focal length of the lens must be equal to the eye's diameter when the person is looking at a distant object.

In the case of the person described in this problem, a lens with a focal length of $f_{eyeglass} = \frac{1}{-5\,diopter} = -0.2\,m$ when placed in front of the person's eye will result in clear vision of an infinitely distant object. Well, an infinitely distant object will create a virtual image $0.2\,m$ in front of the person's eye. This will act as the object for the lens in the person's eye, so we now know the object distance *and* the image distance (the diameter of the eye) and so can use the thin lens formula to find the focal length.

Using the above information, we have $\frac{1}{f_{person}} = -\frac{1}{f_{eyeglass}} + \frac{1}{diameter} = 5\frac{1}{m} + \frac{1}{0.025\,m} = 5\frac{1}{m} + 40\frac{1}{m} = 45\frac{1}{m}$ thus $f_{person} = \frac{1}{45\frac{1}{m}} = 22.2\,mm$. Note several things in the above: I used the negative of the focal length because the *object* distance for the second lens (the eye's lens) is positive. Also, we did ignore an important fact: We ignored the distance between the eyeglass's lens and the eye's lens. What we really solved for is the situation where the prescription is for a contact lens. People wear their eyeglasses roughly 1 cm from the front of their eyes and this *is* important. (Those of you who wear glasses: Slide them forward and back relative to your eyes. Note how the correction they offer changes.). We really should have used an object distance of $0.21m$ rather than $0.2m$ to take this into account. Notice that this distance difference changes the power of the prescription needed by about ¼ diopter—this *is* significant!

6. Unpolarized light is passed through three polarizers. The second and third polarizers are oriented at 30 and 60 degrees, respectively, with respect to the orientation of the first. What fraction of the original light is passed at the end?

The first polarizer delivers light with a single polarization direction. In doing so, it reduces the intensity by 50%. Next, each of the successive polarizers reduces the remaining intensity as prescribed by the Law of Malus: $I_{out} = I_{in}\cos^2\theta$. The chief source of error in applying this is in determining the angle. θ is the angle made by the axis of the polarizer measured relative to the polarization direction of the light. What's tough to keep a grasp on when working with polarizers is the fact that the light that *emerges* from a polarizer has its polarization pointed in the direction of the axis of the polarizer it has just passed through. Thus, the angles made by the successive polarizers relative to the first polarizer are not relevant. What's important is that each one is oriented at an angle of 30° relative to the one just before it.

Using this, we find that each successive polarizer reduces the intensity it receives by a factor of $\cos^2(30°) = 0.75$. Thus the final intensity will be $I = I_0 \times \frac{1}{2} \times \frac{3}{4} \times \frac{3}{4} = \frac{9}{32} I_0 = 0.28 I_0$.

Note that polarization questions come in two broad "flavors": Sometimes we start with unpolarized light and polarize it. Other times, the problem starts with the light already polarized. Be careful to note which of these categories a particular problem falls into.