Physics 206b
Homework Assignment XIII
SOLUTIONS

1. Place the following in order of either increasing or decreasing wavelength (be sure to state which way you're going). In each case, give an approximate wavelength *and* frequency for the entity:
   a. Yellow light
   b. Blue light
   c. Radio waves
   d. Ultraviolet
   e. X-Rays
   f. Green light
   g. Infrared
   h. Gamma rays
   i. Microwaves

In increasing order of wavelength, with some representative wavelengths and (circular) frequencies, we have:

h. Gamma rays (γ rays) $\lambda = .1\,nm$ and shorter, $f = 3\times10^{18}\ Hz$ and higher.

e. X-Rays $\lambda = 50\,nm$ and shorter, $f = 6\times10^{15}\ Hz$ and higher.
Note that the difference between X-Rays and γ rays lies more in their origin than their wavelength. γ rays originate in nuclear events while X-Rays originate in the electrons in atoms.

d. Ultraviolet $\lambda = 400\,nm - 10\,nm$, $f = 7.5\times10^{14}\ Hz$ – $f = 3\times10^{16}\ Hz$.

b. Blue light $\lambda \approx 450\,nm$, $f = 7\times10^{14}\ Hz$

f. Green light $\lambda \approx 550\,nm$, $f = 5.5\times10^{14}\ Hz$

a. Yellow light $\lambda \approx 590\,nm$, $f = 5\times10^{14}\ Hz$

g. Infrared $\lambda = 750\,nm - 100{,}000\,nm$
($100{,}000\,nm = 100\,microns = 100\,\mu m$), $f = 4\times10^{14}\ Hz - f = 3\times10^{12}\ Hz$

i. Microwaves $\lambda = 100\,\mu m - 10\,cm$, $f = 3\times10^{12}\ Hz - f = 3\times10^{9}\ Hz$

c. Radio waves $\lambda = 10\,cm$ and longer, $f = 3\times10^{9}\ Hz$ and lower

I found an excellent reference for this on the web at http://en.wikipedia.org/wiki/Electromagnetic_spectrum

Here it is (we'll talk about the last column, "energy" later this semester):

CLASS	FREQUENCY	WAVELENGTH	ENERGY
γ	300 EHz	1 pm	1.24 MeV
HX	30 EHz	10 pm	124 keV
SX	3 EHz	100 pm	12.4 keV
	300 PHz	1 nm	1.24 keV
EUV	30 PHz	10 nm	124 eV
NUV	3 PHz	100 nm	12.4 eV
NIR	300 THz	1 μm	1.24 eV
MIR	30 THz	10 μm	124 meV
FIR	3 THz	100 μm	12.4 meV
EHF	300 GHz	1 mm	1.24 meV
SHF	30 GHz	1 cm	124 μeV
UHF	3 GHz	1 dm	12.4 μeV
VHF	300 MHz	1 m	1.24 μeV
HF	30 MHz	1 dam	124 neV
MF	3 MHz	1 hm	12.4 neV
LF	300 kHz	1 km	1.24 neV
VLF	30 kHz	10 km	124 peV
VF	3 kHz	100 km	12.4 peV
ELF	300 Hz	1 Mm	1.24 peV
	30 Hz	10 Mm	124 feV

Legend:

γ = Gamma rays

HX = Hard X-rays

SX = Soft X-Rays

EUV = Extreme ultraviolet

NUV = Near ultraviolet

Visible light

NIR = Near infrared

MIR = Moderate infrared

FIR = Far infrared

Radio waves:

EHF = Extremely high frequency (Microwaves)

SHF = Super high frequency (Microwaves)

UHF = Ultrahigh frequency

VHF = Very high frequency

HF = High frequency

MF = Medium frequency

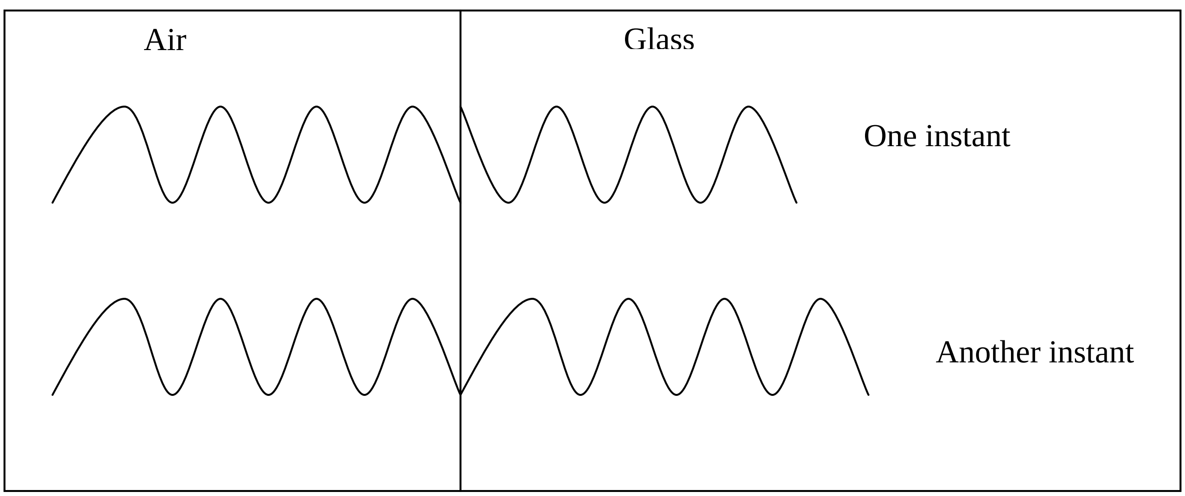
LF = Low frequency

VLF = Very low frequency

VF = Voice frequency

ELF = Extremely low frequency

2. **When light enters a piece of glass, it slows down. Since we know that $\lambda \times f = \mathrm{v}$, where v is the speed of light inside the glass, either the wavelength or the frequency (or both!) must change, since v is always smaller than c. Explain why it is that the wavelength changes and not the frequency.**



Remember the cause of the electromagnetic wave: Jiggling charge. The thing which is waving is the force experienced by a charge due to the jiggling of some other charge somewhere else in the universe. When this time-varying force hits a charge, it will jiggle in response. That charge then becomes the source of the wave. Now, imagine two charges: One right on the outside of a piece of glass and one right on the inside. Light strikes the outside charge and makes it jiggle. This jiggling creates the wave that will make the inside charge jiggle. Since one is causing the other, there must be a fixed relationship between them—that’s just the way the universe works: Things don’t react differently at different times when subjected to the same influences! (Note: Sometimes things seem to do just that, but, in a scientific view of things, this is because there was some influence that was neglected but which is really important. Tracking down cases like this can be very valuable for the advancement of knowledge!) If the charge on the inside moved at a different frequency, then there’d be no fixed relationship between what the outside charge was doing and what the inside charge was doing. So the frequencies must be the same. If v changes and f doesn’t change, then λ must change. The picture above may help. This illustrates the situation if f were the thing that changed.

3. **A ray of light enters a sheet of glass, like a windowpane. The two faces of the piece of glass are flat and parallel to each other. The angle of incidence of the ray is 30°. The piece of glass is 5 millimeters thick. Neglect the reflected portion of this ray in this problem.**

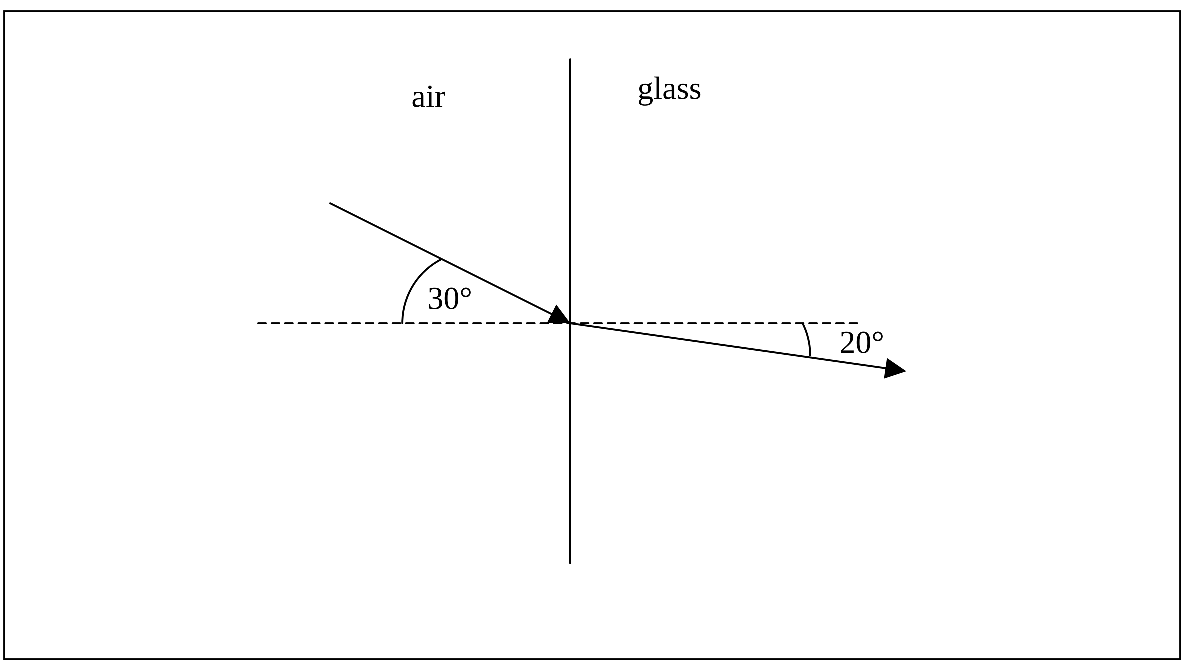
   a. **Make a sketch showing the incident ray and the refracted ray. Also, show the ray striking the far side of the glass on its way out and the ray leaving the sheet of glass.**

   b. **What is the angle the refracted ray makes with the sheet of glass? Be sure to indicate this on the sketch you made in (a).**

   c. **What angle will the ray make with the far side of the sheet? Again, indicate this on your sketch.**

   d. **What angle will the ray make with the glass upon leaving it? Once again, indicate this clearly on your sketch.**

The key to refraction problems is to deal with them one surface at a time. The ray strikes one surface, gets refracted, travels through the medium, and strikes the next surface. Let's deal with each of these in turn. At the first surface, we have the situation pictured below:



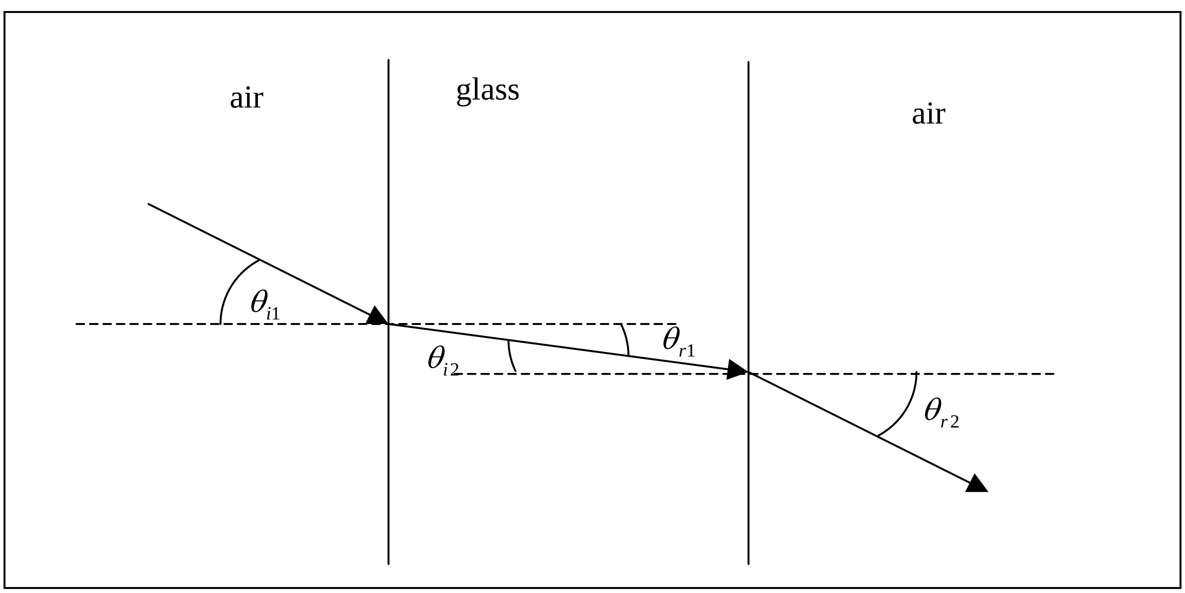
Notice what I've done: I started out by sketching the surface normal at the location where the ray strikes the glass. Always do this! This imaginary line (shown here dotted) is a tool which will really come in handy. Next, I sketched in the incident ray and the refracted ray. Glass has a higher refractive index than glass, so the angle of refraction will be lower than the incident angle. (Remember: When light goes from something with a low refractive index to something with a high refractive index, the angle is smaller in the substance with the higher refractive index.)

We can directly calculate the refracted angle. For this, we need the Law of Refraction: $n_i \sin(\theta_i) = n_r \sin(\theta_r)$. Now, it's *very* important that you recognize that "incident" and "refracted" are just ways of keeping track. Notice that both sides of the equations are the same, just with different numbers.

Taking the refractive index of glass to be 1.5, this gives us $n_i \sin(\theta_i) = 1.0 \times \sin(30°) = n_r \sin(\theta_r) = 1.5 \times \sin(\theta_r)$. This can easily be solved to give $\theta_r = \sin^{-1}\left(\frac{\sin(30°)}{1.5}\right) = \sin^{-1}\left(\frac{0.5}{1.5}\right) = \sin^{-1}(0.333) = 19.5°$.

For convenience and generality, I'm just going to call this "θ_{r1}"—remember, if we do a problem with numbers, we have one answer; if we do it with symbols, we have an infinite number of answers.

Now, let's add the second surface to the picture. First, realize that "rectilinear propagation" means that the light will keep going in a straight line after it gets bent at the first boundary. We don't need to worry about anything between the two faces of the glass. So we have:



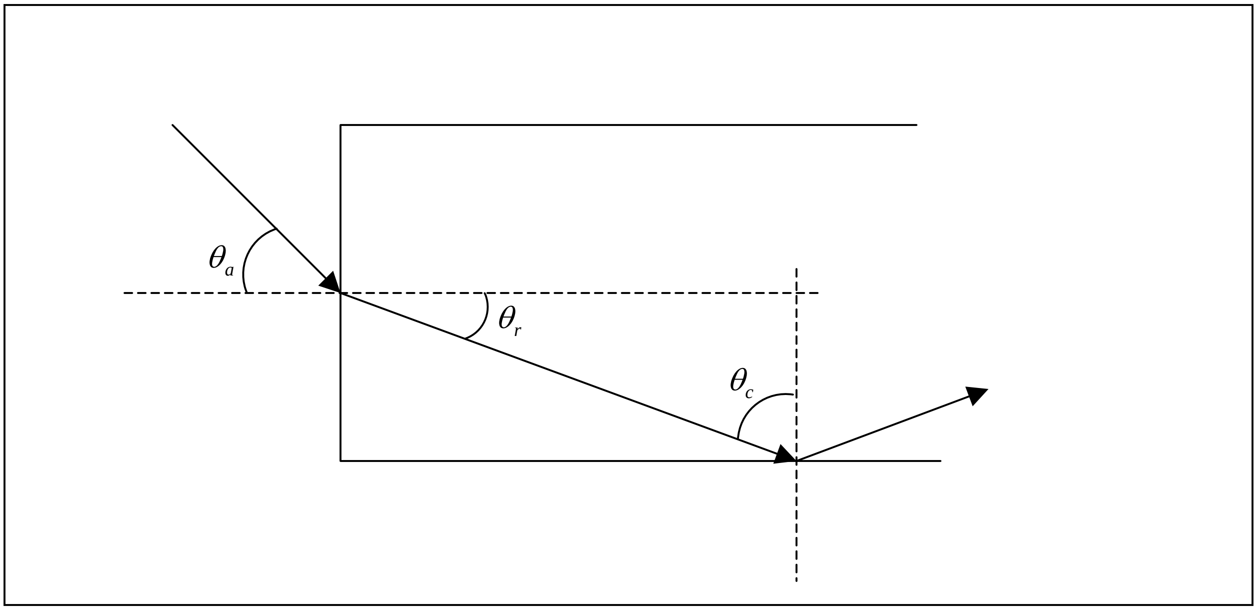
Now comes the one hard part of the whole problem: Notice that there are two angles inside the glass. We've already identified one as "θ_{r1}". The other is the angle that the ray makes on its way out. This angle, θ_{i2}, is identical to θ_{r1}. This is due to one of the fundamental axioms of geometry. ("Alternate, interior angles.") Be careful, though: This is *only* true because the two surface normals that I've drawn are parallel to each other. In turn, they are only parallel to each other because the two faces of the piece of glass are parallel to each other! If the faces of the glass weren't parallel, this wouldn't be true. In that case, we'd have to do some geometry to find the answer.

Once we recognize that the angle of incidence with the far face is θ_{r1}, we can see that the refracted ray *leaves* the glass at the same angle at which it entered in the first place: $\theta_{r2} = \theta_{i1} = 30°$. Thus, the ray leaves the glass along a path parallel to the path it had when it originally entered. It *is* displaced by a little bit, however. (For an interesting additional problem, determine by how much the incoming and outgoing rays are displaced from each other.)

4. A ray of light enters a glass fiber (n=1.5). The fiber is cylindrical and the end is exactly perpendicular to the axis. The fiber is "clad" with a material with an index of refraction of n_{clad}=1.47. (The cladding completely surrounds the fiber except for the ends.) What is the "acceptance angle" of this fiber? The "acceptance angle" is the maximum angle of incidence for which rays will remain trapped in the fiber.

Fibers are essentially never used without a "cladding." That is, a fiber, under real conditions, is always encased in a sheath of glass that is usually quite thick. (Indeed, real fibers are first clad and then coated with plastic and then the whole set is frequently armored to protect the delicate, hair-thin fiber from the elements.) The cladding has a lower refractive index than the fiber. This ensures that fibers *will* be able to achieve total internal reflection but that this reflection will not be defeated (the technical term for this is "frustrated") by contact with other fibers, specks of dust, or other environmental features.

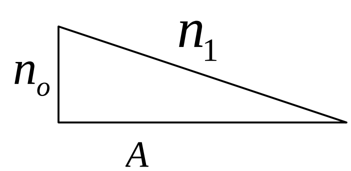
So let's see the problem. We have the geometry shown below:



Notice that the incident angle is, at most, the acceptance angle. So I've called it θ_a. The incident ray is refracted according to good ol' Snell's law to give θ_r inside the fiber. This ray travels until it hits the side of the fiber. It makes an angle θ_c with the normal for the side of the fiber. This will be the critical angle when the original ray was incident at the acceptance angle. So far, so good.

Now, I intentionally drew the surface normal to the front face extra long to show you that $\theta_c + \theta_r = \frac{\pi}{2} = 90°$. (It's close to the end of the year so I'll be nice and use degrees as a gift.) Thus $\theta_r = 90° - \theta_c$ and $\sin(\theta_r) = \sin(90° - \theta_c) = \cos(\theta_c)$.

We have a relationship already for the critical angle. Remember, the critical angle is the angle of incidence for which the angle of refraction is 90°. Thus, if the fiber is clad with a material with a refractive index of n_o and the fiber has a refractive index n_1, we must have $n_1 \sin(\theta_c) = n_o$ which gives $\theta_c = \sin^{-1}\left(\frac{n_o}{n_1}\right)$. This leaves us with a bit of a quandary: We want to find the cosine of something for which we know the sine. The result is kinda cute. We call up our old friend: SOHCAHTOA. We know that $\sin(\theta_c)=\frac{n_o}{n_1}$. We also know that $\sin(\theta_c)=\frac{O}{H}$ for some triangle. We *want* to find $\cos(\theta_c)$. No problem! Since we know the “adjacent” side of the triangle (“what triangle?” you’re probably asking—well, it doesn’t actually exist, but we can pretend that it does; it’s a triangle with n_o as one side and n_1 as its hypotenuse with the angle θ_c across from n_o—I’ll draw a picture in a moment) and its hypotenuse, we can find the other side. Making a picture (see, I told you I’d do that), we have



and we can easily find, using the theorem of Pythagoras, that the side labeled “*A*” is given by $A=\sqrt{n_1^2-n_o^2}$. Thus, $\cos(\theta_c)=\frac{\sqrt{n_1^2-n_o^2}}{n_1}$. (This is a fine trick to add to your “toolbox.”)

So we have $\sin(\theta_r)=\sin(90°-\theta_c)=\cos(\theta_c)=\frac{\sqrt{n_1^2-n_o^2}}{n_1}$ and we want to find θ_a. Let’s just assume that the incident medium (air, in this case) has a refractive index of n_i. We can always set this equal to 1 if we decide that the medium is air. Thus we have $n_i\sin(\theta_a)=n_1\sin(\theta_r)$. We now know $\sin(\theta_r)$ in terms of various refractive indices, so we can just plug it in. This gives $n_i\sin(\theta_a)=n_1\sin(\theta_r)=n_1\times\frac{\sqrt{n_1^2-n_o^2}}{n_1}=\sqrt{n_1^2-n_o^2}$.

Finally, we can solve this for the acceptance angle to get $\theta_a=\sin^{-1}\left(\frac{\sqrt{n_1^2-n_o^2}}{n_i}\right)$. Now we can plug in some numbers. For the problem as stated, $n_o=1.47$. This gives $\theta_a=\sin^{-1}\left(\frac{\sqrt{1.5^2-1.47^2}}{1}\right)=\sin^{-1}\left(\sqrt{0.0891}\right)=17.4°$.

4. An object is placed 14 cm away from a lens with a focal length of 9 cm. Determine the location of the image using the thin lens formula. Also, determine the magnification of the system.

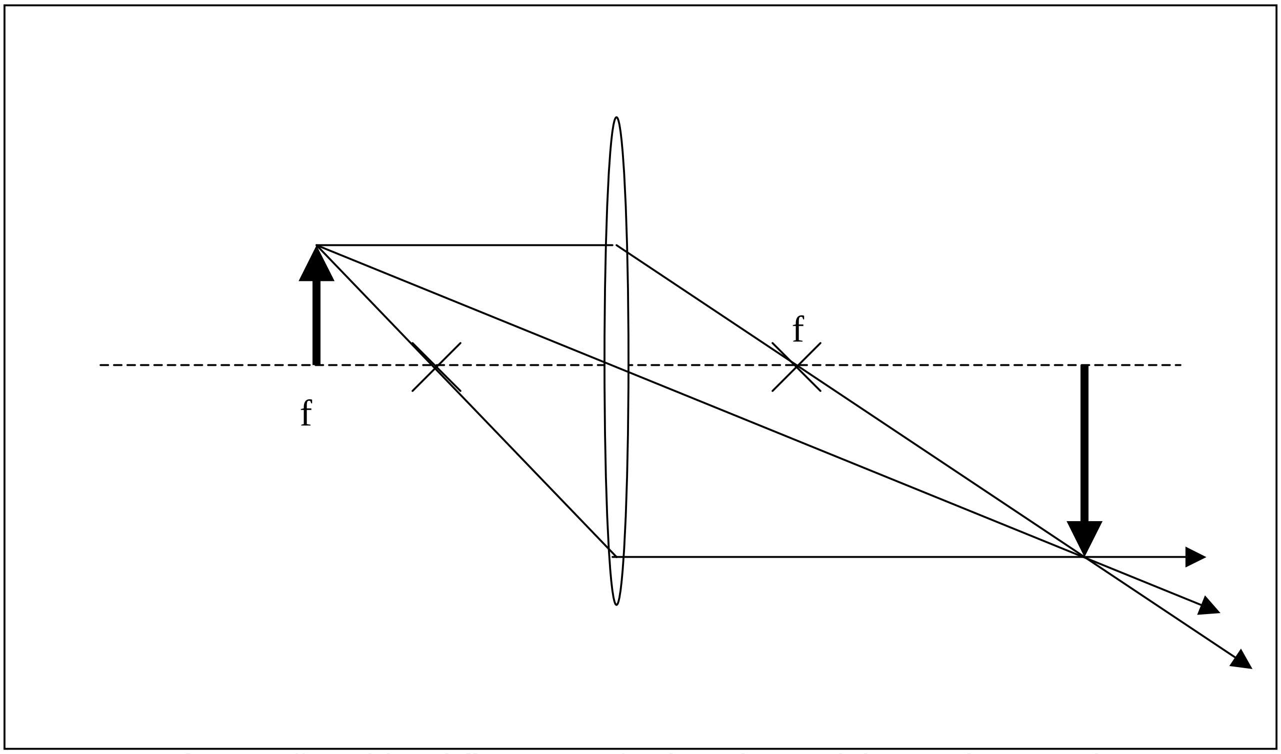
Another plug-and-chug problem (too easy!). The thin lens formula is $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$ where f is the focal length of the lens and p and q are the object and image distances. It doesn't matter which of these two you pick for the image and the object since their roles in the formula are identical. This is why I prefer this version of the formula to a variation which is very common $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$. This second version does help one remember what the variables are, but it obscures the symmetry. Feel free to use whichever version you feel most comfortable with.

A quick bit of algebra gives us $p = \frac{fq}{q-f}$ into which we stick the numbers provided and get $p = \frac{fq}{q-f} = \frac{9cm \times 14cm}{14cm - 9cm} = \frac{126cm^2}{5cm} = 25.2cm$.

Note where the symmetry might help us with this: The distance we found is *either* the image distance for an object at the location specified *or* the object distance needed to yield an image at 14 cm. You observed this phenomenon for yourselves in the lab exercise on lenses.

The magnification is simply the ratio of the image distance to the object distance: $m = \frac{p}{q} = \frac{25.2cm}{14cm} = 1.8$.

5. **For the object and lens described above, at the separation stated above, Use ray tracing to determine, qualitatively, the location of the image. (I.e., you needn't be ultra-precise in this, but do keep the distances as close to proportional as you can.)**



Here's a "cookbook" way of drawing this: First, draw a centerline for reference. Then, draw an icon representing the lens. Remember that we are using a "thin lens" approximation, which means that we don't need the lens to be accurately represented. We'll consider the thickness to be zero and act like all the bends that it makes in the light happen along a plane. So we just sketch in something to remind us where the lens is. Since lenses are symmetric in the thin lens approximation, light from the left traveling parallel to the optic axis will cross the axis a distance f from the lens and light from the right traveling parallel to the optic axis will also cross the axis a distance f from the lens. We indicate these distances by drawing an X on either side of the lens a distance f from it (appropriately scaled, of course). The locations of the Xs are called the "focal points." Note that a serious source of error is to confuse a focal point with an image point! The focal point is the image point for an object at infinity. All other object distances have their own image distances, but the focal points for a lens are independent of the object locations.

So far, everything can be done pretty loosely. The next step has to be done with some care. That is, putting in the object. An arrow makes a dandy object, since it has a definite top and bottom and a Physicist can draw one pretty well. This needs to get drawn the same scale of distance from the lens as the focal length. So, for example, in this case the object is about 50% greater distance from the lens than the focal length. This should be drawn to the right ratio.

The next step is to put in some rays. We have three special rays for this—please bear in mind that there are an infinite number of rays going from the object, through the lens, to the image! But if it's a true image, then all the rays will go there (that's the whole "one-to-one" thing). So, if three rays go there, or even two, then all the others will as well. Sometimes, only two are possible, which is fine. The third does give a nice bit of redundancy. You'll *always* be able to do at least two, however. The three rays we have are:

i. The ray that comes off the object parallel to the optic axis (the centerline): This ray bends at the lens and then crosses the optic axis a focal length from the lens.

ii. The ray which passes through the center of the lens: This ray continues undeviated—just draw a line from the object through the center of the lens and keep on going.

iii. The ray which crosses the optic axis a focal length from the lens on the same side as the object: The ray is the analog to the one in #i. It hits the lens and then gets bent so that it's parallel to the centerline. Note that if the object is closer to the lens than the focal length, this ray cannot be drawn!

Draw these three rays and their continuations on the other side of the lens. See where they cross? That's the image point! We did this for rays from the very tip of the arrow. Try picking a couple of points along its shaft and do the same thing for those, following all the rules above. Convince yourself that each point on the object will be "mapped" to a point on the image.

Note that the rays all actually go to the image point. This point would pass the "paper test," so the image is real.