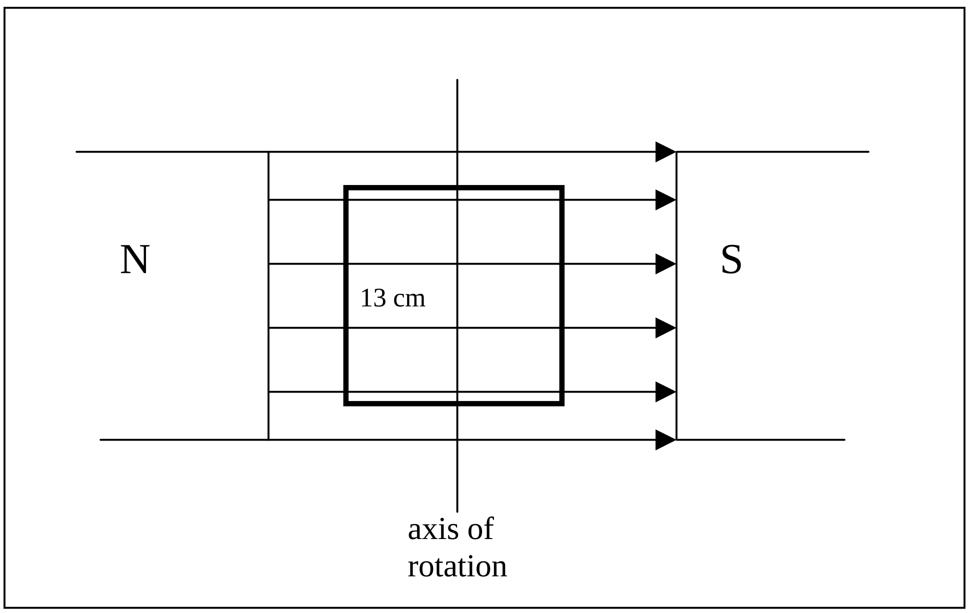
# Physics 206b
## Homework Assignment XI
## *SOLUTIONS*

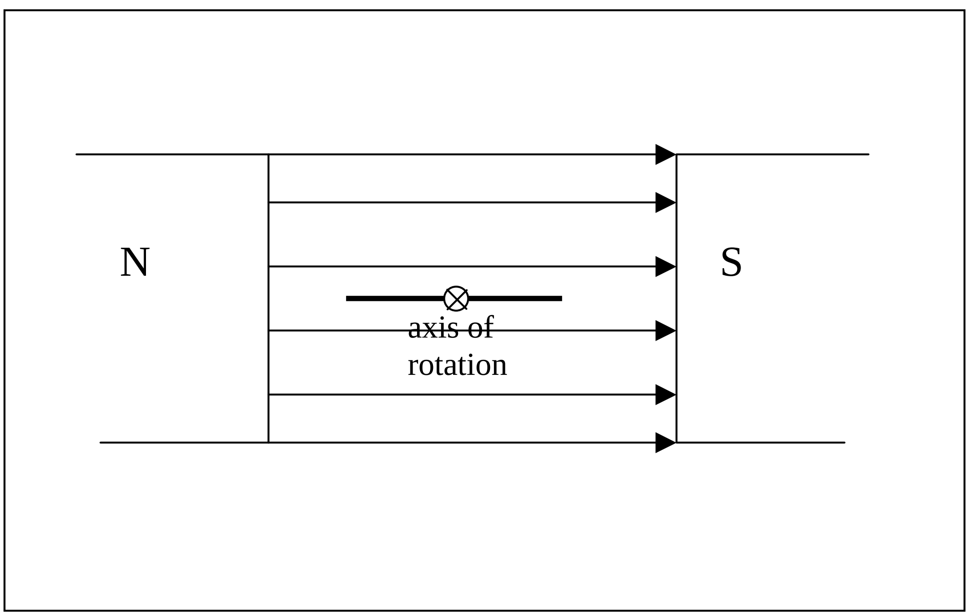
**1. A square loop of wire with a side length of 13 cm is in a constant magnetic field of $\vec{B} = 2.1T\,\hat{x}$, as shown. The wire has an overall resistance of $3\Omega$. What average torque is needed to rotate this loop with a constant rotational speed of $20\pi \frac{rad}{s}$ (i.e., $10\frac{rev}{s}$)? For this problem, it is sufficient to take $\Delta\Phi_B = \Phi_{B\max} - \Phi_{B\min}$. That is, the difference between the maximum flux and the minimum flux over the entire rotational range. Then, $\Delta t$ would be the time over which that change happens.**



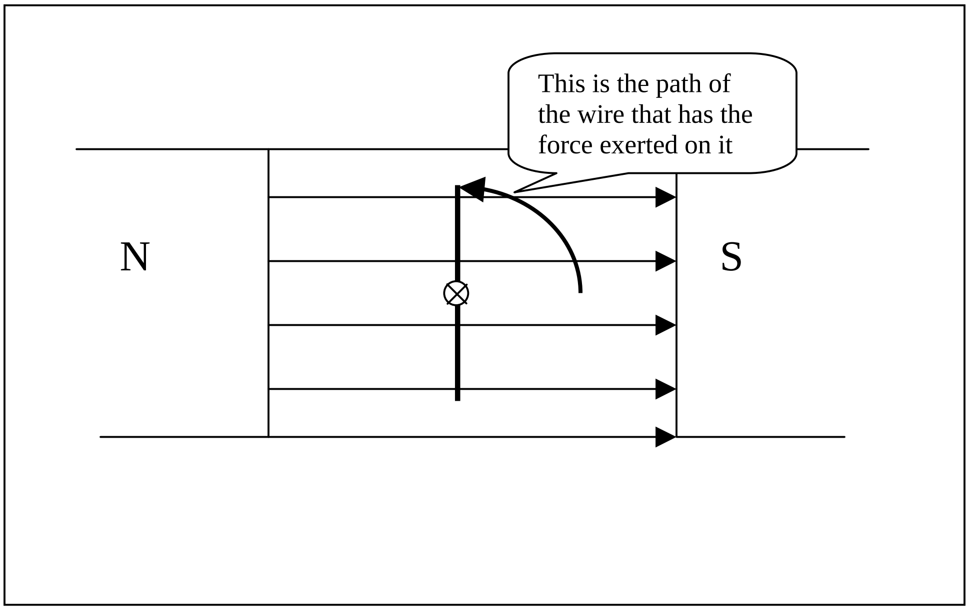
I was a bit surprised to see people doing this problem a different way than I'd intended. Many of you used a formula from the book for torque and adapted it to this situation. Although this method leads to a slightly different answer than the method that I used, it is, nevertheless, still valid. I'll go ahead and do it my way first. Then I'll do this the way that some of you did it. Finally, I'll show you the answer that one gets doing it the "real" way, without the gross approximation that both your way and my way used—we'll see that all three methods yield very similar results, as they must if they are at all valid.

First, let's think about the Physics in this problem: Rotating the loop results in a change in the magnetic flux through the loop—it's a maximum when the magnetic field strikes the open part of the loop "head on" and a minimum (zero) when the magnetic field strikes the loop edge-on. Since this change occurs over some interval of time, Faraday's law predicts that there will be an EMF induced in the wire. An EMF in a wire results in a current flowing in the wire. A current flowing in a wire with a nonzero resistance in it dissipates energy at some rate, this is the power. Since energy must be conserved, overall, the energy dissipated must be replaced. It is replaced by the torque, which does work.

So, let's do this one step at a time: First, let's find the change in magnetic flux through the loop. Let's also make another picture. Let's look at the loop along the axis of rotation, for clarity. This gives us:



This picture shows the situation when the loop is at the minimum-flux position. When it rotates 90° from this position, the flux is a maximum, as shown in the next picture.




Now, in making the transition from minimum flux (zero) to maximum flux, the loop must go through a ¼ rotation. Since the rotational speed is $\omega = 20\pi \frac{rad}{s}$, this takes $t = \frac{\theta}{\omega} = \frac{\frac{\pi}{2} rad}{20\pi \frac{rad}{s}} = .025s$.

But I prefer to use symbols. Recalling that the time for ¼ rotation is ¼ of the period and recalling the relationship between angular frequency, circular frequency, and period, $\frac{\omega}{2\pi} = f = \frac{1}{T}$, we can just say $t = \frac{1}{4f}$.

The maximum flux is $\Phi_B = BA = 2.1T \times (.13m)^2 = 3.55 \times 10^{-2} Wb$. (The unit of magnetic flux is the “weber,” abbreviated *Wb*.) Combining these, we have, according to Faraday’s law (but leaving out the minus sign in this case because we are not concerned with direction in this problem) $\mathcal{E} = \frac{\Delta\Phi_B}{\Delta t} = 4ABf$. Up to this point, my method and yours coincide. We get a value of $\mathcal{E} = \frac{3.55 \times 10^{-2} Wb}{.025 s} = 1.42V$.

Knowing the EMF, we can readily calculate the power dissipated. This is given by $P = \frac{V^2}{R} = \frac{\mathcal{E}^2}{R} = \frac{16A^2B^2f^2}{R}$. (For the number-obsessed, this is $P = \frac{(1.42V)^2}{3\Omega} = .672W$ Of course, since we used an average EMF, this is the average power. Power is defined as work done divided by the time over which that work was done. Since, in this case, the work is done over ¼ of a revolution (we could average over a longer interval, but we'd get the same answer), we can say $P = \frac{W}{t} = .672W$, but the answer is a lot more interesting if you save the numbers for the end). This can easily be solved for the total work in ¼ cycle, to give $W = Pt = \frac{P}{4f} = \frac{4A^2B^2f}{R}$. Again, if you really want numbers, this is $W = .672W \times .025s = .017J$.

Now, we use the definition of work: $W = \vec{F} \cdot \vec{d}$, or in words, "the work done is the product of the distance traveled while the force is acting and the component of the force in the direction of that distance." We once again use an average and assume the force acts continuously. The distance traveled is the arc shown in the picture above. We are only concerned with the force that is along the direction of that arc. The length of the arc is $d = \frac{2\pi r}{4} = \frac{2\pi \frac{13cm}{2}}{4} = .102m$. So the average force is $F = \frac{W}{d} = \frac{\frac{4A^2B^2f}{R}}{\left(\frac{2\pi r}{4}\right)} = \frac{8A^2B^2f}{\pi Rr}$. This is the force which must be exerted (on average) by an outside entity to make the loop spin as described.

Since the force found above is acting at the end of a "moment arm," it exerts a torque. Now, examine the definition of torque, $\vec{\tau} = \vec{d} \times \vec{F}$. Here is a real trap for those who plug into formulas blindly: The $\vec{d}$ in this formula is *not* the same as the $\vec{d}$ in the work formula! In this case, $\vec{d}$ is the length of the moment arm, not the distance traveled. In fact, it's precisely perpendicular to the distance traveled. Here, we have $d = r$ (we can ignore the direction because (a)we don't care about the direction of the torque in this case, and (b)the force will always be perpendicular to the moment arm in this situation). Combining this with the average force found above, we find that the average torque is $\tau = rF = \frac{8A^2B^2f}{\pi R} = \frac{4A^2B^2\omega}{\pi^2 R}$.

Now is a good time to stick some numbers in. The area of the loop is $A = \left(1.3\times10^{-1}m\right)^2 = 1.69\times10^{-2}m^2$. Inserting the rest of the numbers, we get $\tau = \frac{4A^2B^2\omega}{\pi^2 R} = \frac{4\times\left(1.69\times10^{-2}m^2\right)^2\times\left(2.1T\right)^2\times20\frac{1}{s}}{\pi\times3\Omega} = 1.07\times10^{-2}N\cdot m$.

Now, to do it "your" way. Many of you used the formula $\tau = IAB\cos(\theta)$. The biggest problem with the use of formulas is that we tend to forget how they were arrived at in the first place. This leads to embarrassing errors! In this case, the torque was found using the force on the wire caused by the interaction of the magnetic field and the current in the wire—this is the same force as we discussed in problems #1 and #4. This is $\vec{F} = I\vec{L}\times\vec{B}$. As the wire rotates, the flux changes *and* the torque changes because the direction of the force experienced by the wire changes relative to the direction the wire is moving. (Notice that the actual direction of the force *doesn't* change until the current flips direction—after ½ cycle.) This is the reason for the cosine function in the formula above. But we are finding an average value for the torque which eliminates the dependence of the torque on θ from moment to moment. The average value for $\cos(\theta)$ over ¼ cycle is $\frac{2}{\pi}$, but any other reasonably estimated value will be almost as valid in this context.

We need to begin by finding the current in the wire. The *average* current can be found from the average EMF using the method I showed above. This gives (as stated previously) $\mathcal{E} = \frac{\Delta\Phi_B}{\Delta t} = 4ABf$. Inserting this into Ohm's law, we get $I = \frac{\mathcal{E}}{R} = \frac{4ABf}{R}$ which can then be substituted, along with our approximation for the angular dependence, into the formula for torque, giving $\tau = IAB\cos(\theta) = \frac{4ABf}{R}\times AB\times\frac{2}{\pi} = \frac{8A^2B^2f}{\pi R}$, which is identical to the result that I got previously. Had you used a different averaging method, you would have gotten a slightly different result, of course.

Just for giggles, I did this problem using calculus. In this case, the result obtained is $\tau = \frac{\pi A^2 B^2 f}{R}$, which isn't terribly different from the result that we obtained here.

**2. A sinusoidal electric potential with a peak strength of 120V oscillating at 60 Hz (i.e., U.S. household current) is applied to solenoid with 1100 windings per meter. The total resistance of the wire is $7\Omega$. The solenoid has an air-core (i.e., no chunk of metal running down its middle). A single wire loop with a diameter of 0.6 cm is placed just outside of the solenoid facing it. (Assume the diameter of the solenoid is much greater than that of the wire loop.) The resistance of the wire loop is $R = 0.3\Omega$. How much power is dissipated by the wire loop? You may consider the average current in the loop to be ½ of the peak current. Neglect mutual inductance in this problem.**

This is actually a very easy problem except that it *must* be done in multiple steps. I constructed it that way intentionally: Many problems, even quite complex ones, are readily doable if one attacks them in small enough bites.

In order to do this, we first need to understand the Physics of the problem. This will guide us in figuring out the steps to its solution. What's happening here? Well, a current, sinusoidal or otherwise, flowing through a wire will create a magnetic field. This is stated by Ampere's law. Wrapping that wire into the shape of a coil creates a solenoid which has a very "smooth" magnetic field and one which can be quite strong. If the current is time-varying, then so will be the magnetic field. A fixed loop of wire outside of the solenoid will have that field passing through it and so will have a nonzero flux. If the field is time-varying then so will be the flux. A time-varying flux, according to Faraday's law, will create an EMF in a loop of wire. That EMF will create a current in the loop. The flow of charge in the loop (i.e., the current) will cause the energy in it to be dissipated via (typically) heating.

Read all that again a couple of times until you have a picture of what's going on. Now, let's break down our problem solving strategy step by step:

I. Use the applied voltage and solenoid's resistance to find the current in the solenoid (which will be a function of time).

II. Use the current found and the parameters of the solenoid to find the magnetic field of the solenoid. This will also be a function of time.

III. Find the flux through the external loop. Again, this will be a function of time.

IV. Find the rate of change of the flux through the loop and set this equal to the EMF.

V. This step is crucial: Now, we find the time-average of the EMF. It’s critical that we don’t do the time average until *after* we’ve found the EMF. If we were to do it at an earlier step, we’d obliterate the time-dependence of the flux and Faraday’s law would predict zero for the EMF, which isn’t true!

VI. Having found the time averaged EMF, we can use this along with the resistance in the loop to find the power dissipated.

Now that we have our strategy, let’s do it. I’ll retain the steps laid out above:

I. $$I(t) = \frac{V(t)}{R} = \frac{V_{peak}\sin(\omega t)}{R} = \frac{120V \times \sin(120\pi t)}{7\Omega} = 17.14\,A \times \sin(120\pi t)$$

II. $$B = \mu_0 n I = 4\pi \times 10^{-7}\frac{N}{A^2} \times 1100\frac{1}{m} \times 17.14\,A \times \sin(120\pi t) = 2.37\times 10^{-2}\,T \times \sin(120\pi t)$$

III. $$\Phi_B = BA = 2.37\times 10^{-2} \times \sin(120\pi t) \times \pi \times \left(3\times 10^{-3}\,m\right)^2 = 6.7\times 10^{-7}\,Wb \times \sin(120\pi t)$$

IV. A bit of explanation is needed on this step. When we calculate the rate of change of something, we can ignore anything that *doesn’t* change with time. The only thing in this expression that changes with time is the factor $\sin(\omega t)$. We know (because I declared it to you in class—a direct revelation from Sinai) that $\frac{\Delta \sin(\omega t)}{\Delta t} = \omega\cos(\omega t)$ that factor of $\omega$ that comes out of the procedure is very important! Don’t leave it out! Without it, not only will the numeric value of the answer be wrong, the units will be totally wrong (webers instead of volts). Using this, we have

$$\frac{\Delta \Phi_B}{\Delta t} = 6.7\times 10^{-7}\,Wb \times 120\pi \times \cos(120\pi t) = 2.53\times 10^{-4}\,V \times \cos(120\pi t) = \mathcal{E}(t)$$

V. Using the averaging method stated in the problem, we can just replace $\cos(120\pi t)$ with ½. This gives us $\mathcal{E}_{average} = \frac{2.53\times10^{-4}V}{2} = 1.26\times10^{-4}V$

VI. Finally, we have $P = \frac{V^2}{R} = \frac{\mathcal{E}^2_{average}}{R} = \frac{\left(1.26^{-4}V\right)}{.3\Omega} = 5.3\times10^{-8}W$ .

Clearly, this is a tiny number. To get a bigger EMF, we could use a bigger loop or, better, a coil with many turns.

**3. The primary coil in a transformer has 1000 windings. The secondary coil in the transformer has 37 windings. If a peak potential of 120V at 60 Hz is applied to the primary, when the secondary delivers a peak current of 13 A, what current will pass through the primary coil? What peak potential will appear between the leads of the secondary?**

Easy stuff, here! I don't recommend that you memorize a formula for this—I sure don't have it memorized. Just remember two things: First, power is conserved. This is because energy is conserved. Second, the ratio of the voltages will be the same as the ratio of the windings between the primary and the secondary coils. This is a direct result of Faraday's law: Since the total flux is proportional to the number of windings (after all, the *B* field is the same for both coils and the area is the same for both coils, so the flux must be proportional to the number of windings), the EMF must, likewise, be proportional to the number of windings. Thus we must have $\frac{V_{primary}}{V_{secondary}} = \frac{N_{primary}}{N_{secondary}}$. Since the power is given by $P = VI$ on both sides, we also know that $\frac{I_{secondary}}{I_{primary}} = \frac{N_{primary}}{N_{secondary}}$. Combining these two, we have $\frac{I_{secondary}}{I_{primary}} = \frac{V_{primary}}{V_{secondary}} = \frac{N_{primary}}{N_{secondary}}$.

Now, we know that $V_{primary} = 120V$ and $I_{secondary} = 13A$. Also $\frac{N_{primary}}{N_{secondary}} = \frac{1000}{37}$. Therefore, $I_{primary} = \frac{N_{secondary}}{N_{primary}} I_{secondary} = \frac{37}{1000}\times13A = .481A$. And $V_{secondary} = \frac{N_{secondary}}{N_{primary}} V_{primary} = \frac{37}{1000}\times120V = 4.44V$. This is what would be called a "step down" transformer. It takes a high voltage at a small current and turns it into a low voltage at a high current.

4. **What is the total energy stored in a 17 nF capacitor when fully charged with a potential difference of 11V between its faces? Assume the capacitor is air-spaced. If this is a parallel-plate capacitor with a separation of 0.01 mm, what is the stored energy density? (Yes, this deals with material from earlier this semester. Consider this just a reminder.)**

This is just application of a formula (yuck—you know my feelings on *that*!). $U = \frac{1}{2}CV^2$. Substituting numbers, we have $U = \frac{1}{2}CV^2 = \frac{1}{2}\times 1.7\times 10^{-8}F \times (11V)^2 = 1.03\times 10^{-6}J$.

To find the energy density, we need to do a *little* bit of work: We'll need the volume of the capacitor. Since, for a parallel plate capacitor, $C = \varepsilon_0 \frac{A}{d}$. We can use this to solve for the area. This is $A = \frac{dC}{\varepsilon_0} = \frac{1\times 10^{-5}m \times 1.7\times 10^{-8}F}{8.85\times 10^{-12}\frac{C^2}{N\cdot m^2}} = 1.92\times 10^{-2}m^2$. (This is *huge*, by the way!) Therefore, the volume of the region between the plates is $Vol. = Ad = 1.39\times 10^{-6}m^3$. Using this, the energy density is $u = \frac{U}{Vol.} = \frac{1.03\times 10^{-6}J}{1.39\times 10^{-6}m^3} = 0.743\frac{J}{m^3}$.

5. **Consider again the solenoid described in problem #2. Take the diameter of the solenoid to be 2 cm and its length to be 11 cm. What is the maximum energy stored in its magnetic field under the conditions described in that problem? What is the inductance of the solenoid?**

We must begin by calculating the inductance. The formula for this is given in your text and was also derived in class. It is $L = \mu_0 n^2 \pi r^2 l$. Inserting numbers, we have

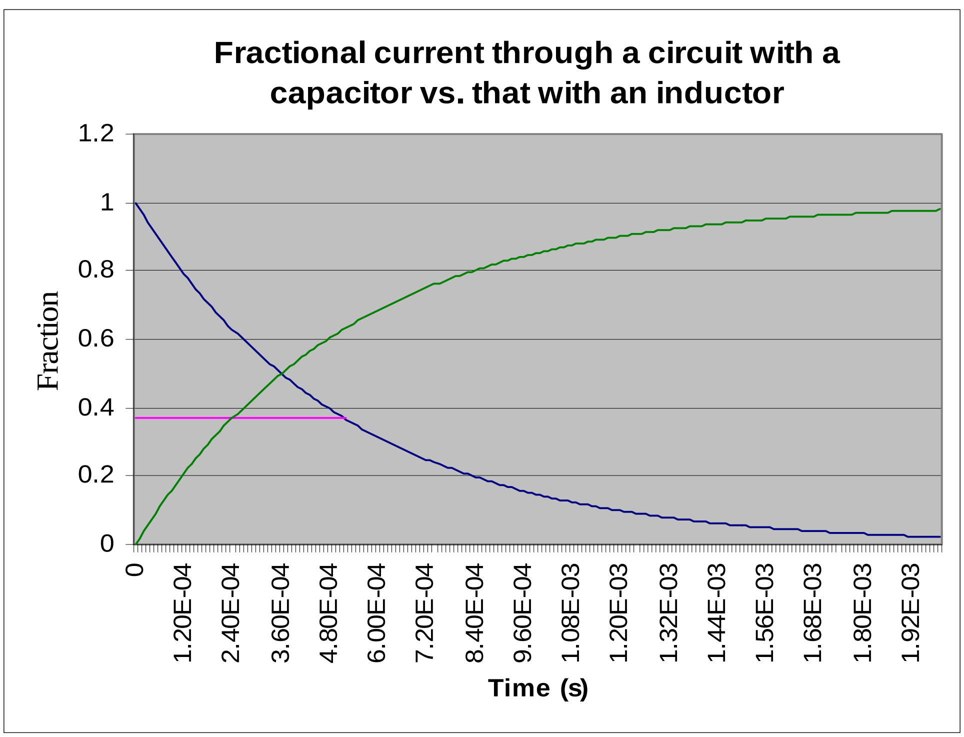
$$\begin{aligned} L &= \mu_0 n^2 \pi r^2 l = 4\pi^2 \times 10^{-7}\frac{T\cdot m}{A}\times 1.21\times 10^6 \frac{1}{m^2}\times 1\times 10^{-4}m^2 \times .11m \\ &= 5.25\times 10^{-5}H \end{aligned}.$$

Now, we just apply another formula for the energy: $U = \frac{1}{2}LI^2$. Since the magnetic field varies sinusoidally, so will the stored energy (more on that later). However, the peak of the stored energy will be found when the current is at a peak. From Problem #2, this occurs when $I = 17.14A$. Thus we have $U = \frac{1}{2}LI^2 = \frac{1}{2}\times 5.25\times 10^{-5}H \times (17.14A)^2 = 7.72\times 10^{-3}J$.

6. **In words (a sketch or two or three or four might not hurt either!), contrast the current flowing through a circuit with an inductor in it and that of a circuit with a capacitor in it. Similarly, contrast the induced EMF in an inductor with the back EMF of a charging capacitor in a circuit with a current flowing in it.**

Inductors fight change. Just keep coming back to that. If there is initially no current in an inductor, the current will grow slowly until it finally approaches some asymptotic value. Once it reaches that current, it will try to stay there. So an attempt to turn the current off will also be met with a tendency to stay on and the current will keep flowing, although it will decay slowly. In the long-time limit, an inductor is just a wire—this fact is important! On the other hand a capacitor starts out acting like “just a wire.” Initially, current flows through the circuit as though the capacitor weren’t there and there were just a conductor in its place. As it charges up, the capacitor starts to impede the flow of additional current, however. Ultimately, the capacitor acts like a break in the wire and current slows to an arbitrarily small value. The currents through a circuit with inductor and one with a capacitor are shown in the plot below. Each assumed the same time constant ($\tau$). The inductor’s current is shown in green while that for the circuit with a capacitor is shown in blue. The 1/e position is indicated with the purple horizontal line.

The induced EMF in an inductor starts out high. It is



caused by a rapid change in current. Since, when we initially turn on a circuit, the change is from nothing to something (whatever it happens to be), the fractional change is huge so the induced EMF is very large. Remember that the induced EMF is created by the *change* in current, not the current itself. (This is analogous to recognizing that a force creates a *change* in a velocity, not a velocity.) As the current grows and approaches its asymptotic value, the rate at which it changes diminishes. Thus, the induced EMF diminishes as the current grows. Ultimately, the induced EMF becomes arbitrarily small as the current becomes arbitrarily close to its asymptotic value. On the other hand, the back EMF of a charging capacitor is caused by the accumulation of charge on the capacitor. Initially, there is no charge on the capacitor so it doesn't fight the addition of new charge and new charge accumulates rapidly. As time passes, the total charge grows and so does the back EMF. Ultimately, the potential across the capacitor becomes arbitrarily close to the potential of the EMF source responsible for charging it. Thus there is no net potential difference and so the current slows to an arbitrarily small value.

7. **A D.C. potential, *V*, is applied to a solenoid with *n* windings per meter and a total resistance *R*. The solenoid has a total length *L* and a diameter *D*. Assume the magnetic field inside the solenoid to be the same everywhere. There will be a force exerted on the solenoid by its own magnetic field that looks like a pressure. I.e., it is a force distributed over the area of the solenoid. Note: The magnetic field inside of a solenoid can be considered to be constant both in size and direction everywhere and is in the $\pm\hat{z}$ direction. (*Hint: See problems #5 and #6 of Assignment #10.*)**

    I. **Calculate the magnetic pressure of this system.**

    II. **Is the pressure outward or inward?**

    III. **If the solenoid were a container holding a monatomic ideal gas, it would take some amount of mechanical work to create that same net pressure. Calculate the mechanical work of a "tin can" with the same dimensions as the solenoid needed to create the same pressure in the gas. Compare this to the energy stored in the magnetic field of the solenoid. What current, *I*, would be needed to have these numbers agree?**

James Clerk Maxwell, who wrote down a self-consistent set of laws governing the intermingled behaviors of magnetic and electric fields, had an interest in Hydrodynamics, the study of the flow of fluids, long before he became interested in electricity and magnetism. He was fascinated by the similarities in the behaviors of mechanical fluids (e.g., flowing water) and electromagnetic systems. This fascination continues to this day in the vary active field of "magnetohydrodynamics." This question is your first introduction to this rich field!

Recall that the pressure experienced by something due to some force is the total force divided by the area over which that force acts. In this case, we have a force created when the current traveling in the solenoid interacts with the magnetic field inside the solenoid. That force acts on the entire area of the solenoid. So, we have a pressure.

The magnetic field in the solenoid is the same everywhere inside of it. This is given by $B = \mu_0 n I$. This is exclusively in some direction along the *z* axis, although we don't know right off whether it is +*z* or –*z*. In order to figure this out, consider that the solenoid is just a series of "rings" of current. Pick a direction for the current, we'll see in a moment that, for this problem, it doesn't matter which direction one chooses—clockwise or counter-clockwise. Now use the right hand rule to figure out the direction of the field just as you did in a previous assignment. Let's pick counterclockwise for now. If the loop is considered to lie in the plane of this page, this would put the magnetic field pointing in the +*z* direction.

We again use the right hand rule to find the force on the current-carrying wire using this. The force on a length of such wire is given by $\vec{F} = I\vec{L} \times \vec{B}$. Now, of course the wire is in a loop, so the *L* vector doesn't point in a single direction (in a Cartesian system). However, think of it this way: We know that whenever we have a cross product the resultant points in a direction perpendicular to both vectors in the product. The *L* vector will always be tangent to the loop, everywhere on it. The *B* vector is along the axis of the loop. The only direction that is perpendicular to both of these is along the radius of the loop. This doesn't tell us whether the force is inward or outward, but it does tell us that it is radial. To find whether it's in or out, we use the right hand rule. My favorite variation on this rule for $\vec{F} = I\vec{L} \times \vec{B}$ is this: Point your thumb in the direction of the current. Point your fingers in the direction of the magnetic field. The palm of your hand will point in the direction of the force. Or, you can do it algebraically—but you'll have to pick a single point on the loop and break out the *L* vector at that point and then extrapolate to all other points. (In case you're wondering [and you should be!]: This can all be done quite rigorously by introducing a coordinate system consisting of $r, \theta, z$. Here, *r* is in the radial direction, $\theta$ is along the loop, and *z* is along the axis. This is known as a “cylindrical” coordinate system. There are, in fact, 11 different coordinate systems, including the Cartesian system that you know about. Picking the right coordinate system can make an insoluble problem almost trivial. Of course, this means first learning about the properties of this assortment of coordinate systems.)

Using the version of the rule I describe above, you should be able to convince yourself that the force experienced by the wire is always outward, independent of the direction of the current. (If the current reverses, $\vec{L}$ reverses but so does $\vec{B}$.) We now must find the total size of the force. To do this, all we need to do is “unwind” the coil.

The solenoid consists of $N$ loops, total. Each loop is, to a very good approximation, a circle. Thus, the circumference of each loop is $2\pi r$ and the total length of wire is $L = 2N\pi r$. (It’s worth repeating that the thing that allows us to “unwind” the solenoid this way is the fact that the force experienced by it is *outward* everywhere. Now, “outward” changes direction at each point, so we can’t use vectors here without switching coordinate systems as mentioned above. But we *can* use this to calculate the full force acting on the wire.) So the total force is $F = 2NI\pi rB$. And, substituting our expression for the magnetic field, $B = \mu_0 nI$, into this, we have $F = 2NI\pi rB = 2NI\pi r\mu_0 nI$. The only thing left is to recall that $n = \frac{N}{l}$, where $l$ is the total length of the solenoid. (Sorry about the proliferation of quantities represented by the same letter! Sometimes it just works out that way.) So we have $F = 2n^2 I^2 \pi r l \mu_0$.

To find the pressure, we just divide the total force by the area over which that force acts. The area in this case is the total area of the solenoid. This is $A = 2\pi r l$. Thus we are left with $P = \frac{F}{A} = \frac{2n^2 I^2 \pi r l \mu_0}{2\pi r l} = n^2 I^2 \mu_0$. Now we can do some thermodynamics.

Recall from the ideal gas law $PV = Nk_B T$ (don’t be confused by the reappearance of a quantity with the letter *N*—this is the total number of molecules in a sample of gas, in this case). Recall, also, that the total energy per molecule of a monatomic ideal gas is given by $E = \frac{3}{2}k_B T$. Thus, the total energy of a sample of monatomic ideal gas is $E_{tot} = \frac{3}{2}Nk_B T = \frac{3}{2}PV$.

Well, we know that our “ideal gas” has a pressure of $P = n^2 I^2 \mu_0$. This is stored in a “tin can” with the volume of the solenoid. This volume is $V = \pi r^2 l$, so we have $E_{tot} = \frac{3}{2}PV = \frac{3}{2}n^2 I^2 \mu_0 \pi r^2 l$.

Now remember what it is that we’ve just calculated: This is the energy that would be present in a sample of monatomic ideal gas occupying a volume the same as that of the solenoid exerting an outward pressure the same as that experienced by the solenoid due to the magnetic field interacting with the current in the wire that forms the solenoid. Of course, the magnetic field has an energy itself, without analogy to an ideal gas. This energy is given by $U = \frac{1}{2}LI^2$ where I’ve, once again, used a letter to mean something different just to confuse you (really, I didn’t—sometimes things just work out this way; if we were truly being careful, we’d relabel things so that there’d be some way to distinguish). Here, $L$ is the inductance of the solenoid. Well, we know the inductance of a solenoid. It is $L = \mu_0 n^2 \pi r^2 l$. Thus the total energy in the solenoid is $U = \frac{1}{2}LI^2 = \frac{1}{2}\mu_0 n^2 \pi r^2 lI$. Comparing this to the analogous ideal-gas system, we have $\frac{E_{tot}}{U} = \frac{\frac{3}{2}n^2 I^2 \mu_0 \pi r^2 l}{\frac{1}{2}\mu_0 n^2 \pi r^2 l I^2} = 3$.

I think this is a really cool result (of course, I’ve got the job that I have because I think all sorts of weird things are cool—this is a way of keeping folks like me off the streets without filling up the jails): If we send a current down the wire of a solenoid, there will be an outward force created on the solenoid. This force acts like there’s something “trapped” in the solenoid that’s trying to escape and that some amount of work would have had to be done to get it there. Creating this much pressure mechanically would take three times as much work as creating the same situation in a purely electromagnetic way. One point that this drives home is that giving the endpoint of a system (in this case, the fact that the solenoid experiences an outward pressure) does *not* give all the information needed to know how much energy went into getting it there.