

Physics 206a
HOMEWORK #8
SOLUTIONS

1. A molecule of carbon monoxide (CO), which consists of one atom of carbon and one atom of oxygen, is forced to break apart by the addition of a certain amount of energy. There is a net excess of energy of 3×10^{-19} joules which all goes to kinetic energy of the “fragments” (i.e., the atoms). What are the speeds of each of the two fragments? (Carbon has a mass of 12 amu and oxygen has a mass of 16 amu.)

Well, as many of you noticed, I inadvertently left the solution to this problem in the solution set to assignment #7 when I posted it. Oops! To save space, I won't repeat the solutions here. I simply refer you to the solution to Problem #9 of Assignment #7.

2. A man with a mass of 199 kg stands on a skateboard which has a mass of 1 kg. There is no friction between the skateboard and the ground. He throws a baseball with a mass of 210 grams perfectly horizontally with a speed of $35 \frac{\text{meters}}{\text{second}}$ in a direction 12° North of East. What is the man's final velocity? Express this as a vector in component form. Take North to be the $+\hat{y}$ direction and East to be the $+\hat{x}$ direction.

Here, there's only one thing we don't know: The final velocity of the man. We *could* treat this as a one-dimensional problem and recognize that the final momentum of the man will be 180° away from the final momentum of the ball. Then we could solve for the speed of the man and simply plop that in at an angle of 192° . But this is a good opportunity for some practice with vectors, so let's treat it, systematically, as a 2-d problem.

The initial momentum of the system is zero. Momentum is conserved in the system, so the final momentum will also be zero. The skateboard effectively removes the Earth from the problem, since it is frictionless, so we need consider only the momenta of the man (including the skateboard) and the ball. There's a crucial insight buried in that judgment: Whenever we do a problem involving momentum, we must consider what objects are part of “the system.” This can be very difficult! Momentum for “the system” is conserved, but only if we get the components of “the system” correct. Here's how you decide: Conservation of momentum results directly from Newton's third law. This says that whenever object *A* experiences a force from object *B*, object *B* must experience a force of identical size but opposite direction from object *A*. “The system” must include every *pair* of objects exerting forces on each other. We'll see an implication of this in the next problem.

Assuming the man and the skateboard stay together, we can write $(m_{\text{man}} + m_{\text{skateboard}})\vec{v}_{\text{man}} = -m_{\text{ball}}\vec{v}_{\text{ball}}$. Using our SOHCAHTOA definitions, we can write this out in component form as

$$(m_{\text{man}} + m_{\text{skateboard}})v_{\text{man } x} = -m_{\text{ball}}v_{\text{ball}} \cos(\theta)$$

and

$$(m_{man} + m_{skateboard})v_{man y} = -m_{ball}v_{ball} \sin(\theta)$$

where $\theta = 12^\circ$.

These give

$$v_{man x} = -\frac{m_{ball}v_{ball} \cos(\theta)}{(m_{man} + m_{skateboard})} = -3.6 \times 10^{-2} \frac{\text{meters}}{\text{second}}$$

and

$$v_{man y} = -\frac{m_{ball}v_{ball} \sin(\theta)}{(m_{man} + m_{skateboard})} = -7.6 \times 10^{-3} \frac{\text{meters}}{\text{second}}.$$

Combining these, we get

$$\vec{v}_{man} = -3.6 \times 10^{-2} \frac{\text{meters}}{\text{second}} \hat{x} - 7.6 \times 10^{-3} \frac{\text{meters}}{\text{second}} \hat{y}.$$

As a final note I will point out that many of you persist in leaving the vector nature out of discussions of vector quantities, such as velocity and momentum. As I've mentioned many times before, this is dead wrong! I will be *merciless* in grading answers on exams or homeworks that call for vector quantities if the vector nature is left out! We've been doing this long enough, at this point, that such oversights are no longer forgivable.

- 3. In the previous problem, the initial velocity of the baseball was purely horizontal. However, we know that it will fall toward the earth as it flies. If the ball was thrown from an initial height of 1.5 meters, in the instant before it strikes the ground, what is the momentum change that was experienced by the planet earth due to the ball?**

Understanding this problem is very important. Let's think about it: Before the man throws the ball, the total momentum of the system is zero. The instant after he throws it, the total momentum of the *ball* is in the \hat{x} direction. However, after he throws it there is a net force that acts on the ball. This is the force of gravity pulling it down. We know that it will accelerate downward until it hits the ground. The acceleration due to gravity will act exclusively in the \hat{z} direction. We know how to handle that acceleration from many problems we dealt with earlier this semester. Just before the ball hits the ground, it will have a velocity with x , y , and z components. This means that its momentum will have an x component, a y component, and a z component. Just as the interaction between the ball and the man that resulted in the ball having a velocity in the \hat{x} and \hat{y} directions also resulted (because of Newton's third law) in the man having a velocity in the \hat{x} and \hat{y} directions, the interaction between the ball and the earth that results in the ball having a velocity in the \hat{z} direction also must result in the earth having a velocity in the \hat{z} direction.

Reread that last sentence over again until it makes sense! What matters is that the earth exerts a force on the ball. By Newton's third law, if the earth exerts a force on the ball then the ball must exert an equally-sized, opposite directed force on the earth. Since the ball accelerates downward, the earth must accelerate upward. If it didn't, momentum would not be conserved! Whatever momentum the ball has in the downward direction, the earth must have in the upward direction. We can easily figure this one out: Since we're just concerned with the \hat{z} direction, we just need to find the speed of the ball when dropped from a height of 1.5 meters. (Once again: We split the velocity vector into its components and then treat the components independently. So far as the z component is concerned, the x and y components might as well not exist at all! A ball thrown horizontally from a height of 1.5 meters is identical to a ball dropped from a height of 1.5 meters in terms of its vertical behavior.) From prior work, we know that the final speed of something which accelerates from an initial speed of zero over a distance d is $v = \sqrt{2ad}$.

Thus $v = \sqrt{2ad} = \sqrt{2 \times 9.8 \frac{\text{meters}}{\text{second}^2} \times 1.5 \text{ meters}} = 5.4 \frac{\text{meters}}{\text{second}}$. But it's not the speed of the ball we're concerned with, it's its momentum. The magnitude of the momentum of the ball is $p = mv = 0.210 \text{ kg} \times 5.4 \frac{\text{meters}}{\text{second}} = 1.13 \frac{\text{kilogram} \times \text{meters}}{\text{second}}$. This is in the negative \hat{z} direction. Thus, the momentum change of the earth due to the ball is $1.13 \frac{\text{kilogram} \times \text{meters}}{\text{second}}$. This is in the positive \hat{z} direction.